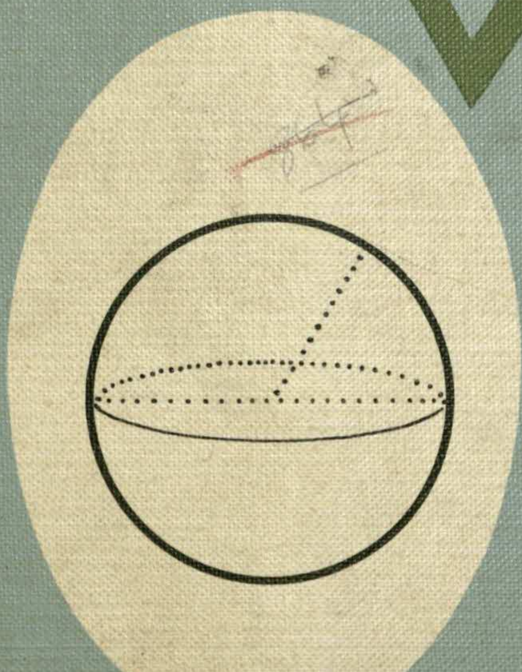
MASTERING MATHEMATICS

$$h^2 = a^2 + b^2$$



TEACHER'S EDITION — ANNOTATED AND KEYED

How to Use the TEACHER'S EDITION—Annotated and Keyed

FOR

AMERICAN ARITHMETIC

SECOND EDITION

This Teacher's Edition is designed to be the greatest possible help to the busy teacher. Within one cover, it presents these distinctive features.

1. Pupil's Book. The full text of the pupil's book is printed in black and color exactly as it appears in the book used by the pupil.

2. Annotation. Concise statements of objectives and practical and helpful suggestions to the teacher are overprinted in a distinctive and legible color directly on the pages of the pupil's text.

3. Answers and Keys. Answers to all concrete problems are overprinted in color on the pages of the pupil's text. They are placed in the normal positions or as near to the problem as possible. There is no need for the teacher to work these problems or to turn to an answer section.

The keys to the more involved problems with their answers are given in the Teacher's Guide and Key at the end of the book.

4. Teacher's Guide and Key. For the convenience of the teacher, the Teacher's Guide and Key is printed on tinted paper immediately following the index in the pupil's text. The tinted paper is different enough from the white paper of the pupil's text to assist the teacher in turning easily to guide or key pages. A tint has been used that has been found scientifically easy on the eyes. The pages of the Teacher's Guide and Key are numbered separately with the letter G pointing out that guide and key pages are meant.

The Teacher's Guide and Key provides a full statement of the aims and detailed practical suggestions for each chapter and each page or group of pages in the pupil's text. Following the discussion of the treatment of each page is the key, working the more involved problems in full. References to pages in the workbook AMERICAN WORKSHOP are provided for those teachers using these workbooks. A list of audio-visual aids is provided at the end of the guide.

For advance study in preparation for the class session and for quick reference in the classroom, the Teacher's Edition—Annotated and Keyed offers the teacher all the above in this one volume, very little larger than the pupil's text itself.

MASTERING MATHEMATICS

American Arithmetic
Second Edition

Clifford B. Upton
Formerly Professor of Mathematics
Teachers College, Columbia University

Kenneth G. Fuller
Mathematics Department
California State Polytechnic College

Milo E. Whitson
Mathematics Department
California State Polytechnic College

AMERICAN BOOK COMPANY

MASTERING MATHEMATICS

American Arithmetic, Second Edition

Illustrations by Charles Dougherty and Sylvia Haggander
Covers by John Horton

EP 10 9 8 7 6 5 4 3 2 1

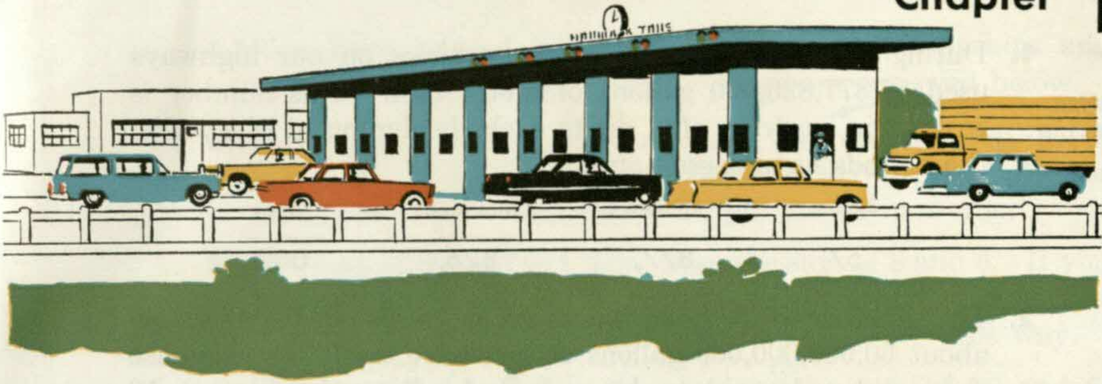
TEACHER'S EDITION - Annotated and Keyed
SL 10 9 8 7 6 5 4 3 2 1

Contents

Before beginning this book, refer to "Suggestions to Teachers," pages 375-377, and the Introduction to the Teacher's Guide and Key. These sections contain a discussion of number relationships and principles, long division, improvement tests, an outline of Grade 8 work, and so on.

See Teacher's Guide and Key for specific aims of Chapter 1.

Chapter 1



Our Automobile Industry

Review use of large numbers and previously taught operations.

1. In 1900 only 4192 automobiles were manufactured in the United States. In 1955 our automobile factories set a new production record, making 7,920,200 passenger cars and 1,249,100 trucks and busses. In all, how many motor vehicles were made in the United States during that year? 9,169,300*

2. In 1960 the number of passenger cars made in the United States was 6,674,800 and the number of trucks and busses made was 1,194,500. How many motor vehicles in all were made during that year? 7,869,300 How many more passenger cars were made in 1955 than in 1960? 1,245,400 How many more trucks and busses were made in 1955 than in 1960? 54,600

3. The average factory value of the 6,674,800 passenger cars made in 1960 was $1822. What was the total factory value of all the passenger cars made that year? $12,161,485,600

4. The average factory value of the 1,194,500 trucks and busses made in 1960 was $1968. What was the total factory value of all the trucks and busses made that year? $2,350,776,000

5. In 1960 about $\frac{2}{5}$ of the passenger cars made were 4-door sedans, about $\frac{1}{6}$ were station wagons, and about $\frac{1}{4}$ were hard tops and convertibles. Of the 6,674,800 passenger cars made that year, about how many were 4-door sedans? 2,669,920 station wagons? 1,112,467 hard tops and convertibles? 1,668,700

*For convenience here, only the numerical part of the answer is given. Pupils should give complete answers and process steps, as shown in the Teacher's Guide and Key for each page. This procedure is used throughout the text.

Review two ways of writing numbers. Some pupils may need review of rounding off numbers to the nearest thousand, ten thousand, million, and so on, before doing ex. 2.

Round Numbers

1. During a recent year the motor vehicles on our highways used 57,877,826,000 gallons of motor fuel. This number is written in **periods** of 3 digits each, beginning at the right; the periods have these names:

Stress meaning.

Billions	Millions	Thousands	Ones
57,	877,	826,	000

2. The number 57,877,826,000 is close to 60,000,000,000; so about 60,000,000,000 gallons of fuel were used. In this case the number is said to be **rounded off** to the nearest 10 billion. The number 57,877,826,000 can also be rounded off to the nearest billion, which gives 58,000,000,000, or to the nearest 100 million, which gives 57,900,000,000. Another way to write the number 57,900,000,000 is 57.9 billion, which means $57.9 \times 1{,}000{,}000{,}000$. Emphasize the difference between rounding off to the nearest 10 billion and the nearest billion.

3. The number 1,700,000 can be written as 1.7 million, which means $1.7 \times 1{,}000{,}000$. The decimal forms of writing numbers, such as 1.7 million and $59.4 billion, are often used in newspapers to express large numbers that represent millions or billions. To write the number 5,827,000 in this new form, first round off the number to the nearest 100 thousand, which gives 5,800,000; then write the number as 5.8 million. The number 14,750,000, which is just halfway between 14,700,000 and 14,800,000, is rounded off to 14,800,000 or 14.8 million. Emphasize how the number is rounded off when it is halfway between two numbers.

Stress.

4. Write each number, using only digits: 74.3 million; $1.2 billion; 424.5 million; $126.8 billion; $1.1 million. 74,300,000 $1,200,000,000 424,500,000 $126,800,000,000 $1,100,000

5. Round off each number to the nearest 100 thousand and then write it in the new decimal form as explained in ex. 2: 2,715,600; 4,683,240; 16,538,000; 9,093,316; 8,105,122; 92,573,000; 7,150,000; 3,401,120; 9,854,000. See Guide.

6. Round off each of these numbers to the nearest 100 million and then write it in the new decimal form: 4,790,000,000; 138,742,000,000; $37,390,000,000; $12,850,000,000. See Guide.

Discuss uses of round numbers. Have pupils bring to class newspaper clippings showing numbers written in short form (57.9 billion). Discuss the difference between place value and period value.

Review commutative and associative principles, and other principles used in addition. Provide a diagnostic test on fundamental addition facts.

Principles for Addition

1. You have learned interesting and useful relationships and principles in addition. Some of these are reviewed below.

2. **If a and b stand for any two numbers, then $a + b = b + a$.** This is called the **commutative principle for addition.** Tell what this principle means in words.

3. The sum of 6 and 7 is 1 more than the sum of 6 and 6. If you know that $6 + 6 = 12$, this relationship tells you that $6 + 7 = 12 + 1 = 13$. Tell how to find $7 + 8$ this way.

$7 + 8 = 7 + (7 + 1) = (7 + 7) + 1 = 14 + 1 = 15$

4. The sum of 9 and 7 is the same as the sum of 10 and 6. $9 + 7 = 10 + 6$. To find the sum of 9 and 7 you can subtract 1 from 7 and add 1 to 9. Then you can add 10 and 6, which gives 16. Tell how to find $9 + 4$ this way.

$9 + 4 = 9 + (1 + 3) = (9 + 1) + 3 = 10 + 3 = 13$

5. **If n stands for any number, then $n + 0 = n$.** What does $0 + n$ equal? n Tell in words how to find the sum of 0 and any other number.

6. You can find the sum $4 + 3 + 7$ these two ways:

$$(4 + 3) + 7 = 7 + 7 = 14$$
$$4 + (3 + 7) = 4 + 10 = 14$$

The way you group these three numbers when you add them does not affect your answer. **If a, b, and c stand for any three numbers, then $(a + b) + c = a + (b + c)$.** This is called the **associative principle for addition.** Tell what this principle means in words. Write each of these sums two ways with parentheses to illustrate the associative principle: $6 + 9 + 2$; $5 + 8 + 1$; $4 + 7 + 9$; $7 + 6 + 5$. See Guide.

7. When you add up the column on the right, and check your answer by adding down the column, you are using both the commutative and associative principles.

$$\begin{array}{r} 2 \\ 6 \\ \underline{3} \\ 11 \end{array}$$

For: $\underbrace{(3 + 6) + 2}_{\text{associative}} = \underbrace{2 + (3 + 6)}_{\text{commutative}} = \underbrace{2 + (6 + 3)}_{\text{commutative}} = (2 + 6) + 3$. Which principle is used in each step?

associative

8. On page 355 give the sums in the time stated by your teacher. Relationships and principles will help you.

See page 377 for directions for giving the diagnostic test. Discuss with the pupils errors discovered and different ways of recalling these facts using relationships and principles.

Review column addition.

Problems and Practice

$5.10

1. John keeps a cash account. He is adding to find out how much money he received this week. Add the numbers at the right to find John's answer. If you find the sum by adding up the columns, then check your answer by adding down the columns.

$1.25
.85
.90
1.30
.45
.35

2. The first week of school Mary had to buy school supplies. These are the amounts she spent: $.75, $.08, $.19, $.48, $.98, $1.25, $.05, $.22, $.03, $1.09. Find the total amount Mary spent on school supplies. $5.12

3. For one week keep an account of all the money you spend during the week and find the total at the end of the week. Perhaps you will decide to do this every week.

Copy, add, and check:

4. 13 + 79 + 50 + 97 + 78 + 17 + 63 + 75 + 11 + 47 530
5. 93 + 48 + 52 + 99 + 28 + 61 + 14 + 78 + 45 + 89 607
6. 43 + 31 + 68 + 24 + 49 + 90 + 47 + 23 + 66 + 29 470
7. 90 + 66 + 38 + 91 + 62 + 46 + 85 + 72 + 13 + 99 662
8. 21 + 75 + 94 + 62 + 37 + 54 + 88 + 16 + 38 + 77 562
9. 31 + 57 + 46 + 15 + 78 + 71 + 97 + 25 + 30 + 16 466
10. $33.96 + $47.14 + $76.24 + $69.27 + $3.74 + $94.80 $325.15
11. $16.79 + $15.85 + $49.75 + $3.89 + $45.37 + $9.89 $141.54
12. $805.72 + $39.69 + $3.17 + $753.98 + $695.49 + $88.27 $2386.32
13. $531.64 + $758.80 + $688.23 + $67.11 + $834.90 + $12.22 $2892.90

More Practice. See 1 on page 358. See Guide for use.

Instruct pupils to add up when finding a sum and to add down when checking. Some pupils may need a re-explanation of checking. Have pupils explain some examples to check understandings. Emphasize the importance of checking.

Discuss the relationship of subtraction and addition. Provide a diagnostic test on fundamental subtraction facts.

Subtraction

1. The relationship between a subtraction fact and an addition fact can be expressed this way. If you let x represent the number you get when you subtract 5 from 13, then $13 - 5 = x$ and $5 + x = 13$. You can see that x is 8 because $5 + 8 = 13$. So $13 - 5 = 8$. Tell what number x must be in each of these:

$12 - 8 = x$ 4 $\quad 14 - 9 = x$ 5 $\quad 15 - 7 = x$ 8 $\quad 11 - 5 = x$ 6

$8 + x = 12$ 4 $\quad 9 + x = 14$ 5 $\quad 7 + x = 15$ 8 $\quad x + 5 = 11$ 6

2. You have learned these principles:

If n stands for any number, then $n - 0 = n$.
If n stands for any number, then $n - n = 0$.

Tell what each principle means in words. What is $7 - 0$? 7 $4 - 0$? 4 $5 - 5$? 0 $9 - 9$? 0

3. You should be able to give the answers to the 100 subtraction facts rapidly and correctly. Turn to page 356 and give the answers within the time stated by your teacher. If you have difficulty, relationships and principles will help you. Emphasize.

Subtract, and check your work by adding up:

4.	572	9000	8665	81394	112321	\$931.00
	245	159	2936	15746	42059	463.93
	327	8841	5729	65,648	70,262	\$467.07
5.	563	6007	1638	15006	400000	\$296.12
	268	2999	885	10397	259276	107.87
	295	3008	753	4609	140,724	\$188.25

More Practice. See 2 on page 359.

6. Susan is treasurer of the Camera Club. Today when she had balanced the account she had \$36.53. Of this amount \$15.75 came from club dues and the rest from selling prints of pictures taken by members. How much money came from the sale of prints? \$20.78

See the Guide for an explanation of the additive method of subtraction. Discuss the importance and method of checking.

Review the commutative and associative principles for multiplication, the distributive principle, and other principles used in multiplication. Provide a diagnostic test on multiplication facts.

Principles for Multiplication

1. You have learned interesting relationships and principles in multiplication. Some of these are reviewed below.

2. Most of the multiplication facts go together in pairs. For example, $4 \times 7 = 28$ and $7 \times 4 = 28$. So $4 \times 7 = 7 \times 4$. Does $3 \times 5 = 5 \times 3$? Yes Does $8 \times 9 = 9 \times 8$? Yes **If a and b stand for any two numbers, then $a \times b = b \times a$.** This is called the **commutative principle for multiplication.** Tell what this principle means in words. Explain how this principle helps you find 9×6 if you know that $6 \times 9 = 54$.

3. It is easy to multiply 0 by any number. $5 \times 0 = 0$, because $0 + 0 + 0 + 0 + 0 = 0$. What is 4×0? 0 8×0? 0 0×3? 0 **If n stands for any number, then $n \times 0 = 0$.** Tell what $n \times 0 = 0$ means in words. What does $0 \times n$ equal? 0

4. It is also easy to multiply 1 by any number. $5 \times 1 = 5$, because $1 + 1 + 1 + 1 + 1 = 5$. What is 3×1? 3 7×1? 7 4×1? 4 1×6? 6 1×2? 2 **If n stands for any number, then $n \times 1 = n$.** Tell what $n \times 1 = n$ means in words. What does $1 \times n$ equal? n

5. On page 355 give the products within the time stated by your teacher. The above principles will help you.

6. You can find the product $3 \times 5 \times 6$ in these two ways:

$$(3 \times 5) \times 6 = 15 \times 6 = 90$$
$$3 \times (5 \times 6) = 3 \times 30 = 90$$

The way that you group these three numbers when you multiply them does not affect your answer. Find each of these products by grouping the numbers two ways: See Guide.

$2 \times 4 \times 7$ $\quad$ $5 \times 6 \times 8$ $\quad$ $2 \times 9 \times 8$ $\quad$ $7 \times 5 \times 9$

7. If a, b, and c stand for any three numbers, then $(a \times b) \times c = a \times (b \times c)$. This is called the **associative principle for multiplication.** Tell what this principle means in words. Write each of these products two ways with parentheses to illustrate the associative principle: See Guide.

$4 \times 9 \times 7$ $\quad$ $5 \times 1 \times 8$ $\quad$ $3 \times 4 \times 11$ $\quad$ $12 \times 3 \times 4$

See page 377 for directions for giving the diagnostic test. Discuss facts which cause difficulty, and principles and relationships that can be used to recall or check these facts.

Emphasize the meaning of the term factor.

8. When two or more numbers are multiplied together, each of these numbers is called a **factor** of the product. Thus, 3 and 5 are factors of 15; and 2, 5, and 7 are factors of 70. 35 is also a factor of 70. Why? Find as many factors as you can of these numbers: 21; 55; 24; 45.

See the Guide for the answers to the exercises on this page.

9. The associative principle can be used to show how 4×7 is related to 2×7. $4 \times 7 = (2 \times 2) \times 7 = 2 \times (2 \times 7)$. So you can find 4×7 by **doubling** 2×7, and $4 \times 7 = 2 \times 14 = 28$. Since $6 \times 8 = (2 \times 3) \times 8 = 2 \times (3 \times 8)$, you can find 6×8 by doubling 3×8. Explain how to find the following products by this plan: 4×9, 6×7, 4×8, 8×9.

10. You have learned how to find $4 \times (5 + 8)$ by doing the work inside the parentheses first, as shown below:

$$4 \times (5 + 8) = 4 \times 13 = 52$$

You can also find $4 \times (5 + 8)$ this way:

$$4 \times (5 + 8) = (4 \times 5) + (4 \times 8) = 20 + 32 = 52$$

Each number in the sum that is inside the parentheses is multiplied by 4, and then these products are added. Find each of these two ways:

$2 \times (4 + 8)$ $7 \times (2 + 3)$ $8 \times (7 + 9)$ $5 \times (6 + 8)$

11. **If a, b, and c stand for any three numbers, then $a \times (b + c) = (a \times b) + (a \times c)$.** This is called the **distributive principle.** Notice that both addition and multiplication are involved in this principle. Write each of the following with parentheses to illustrate the distributive principle, and find the value of each two ways:

$3 \times (6 + 2)$ $7 \times (3 + 5)$ $4 \times (8 + 3)$ $11 \times (7 + 2)$

12. When you multiply 14 by 2 as on the right you are using the distributive principle, for you really multiply 4 by 2 and 10 by 2 and then add these products. The distributive principle allows you to do this because:

$$\begin{array}{r} 14 \\ 2 \\ \hline 28 \end{array}$$

$$2 \times 14 = 2 \times (10 + 4) = (2 \times 10) + (2 \times 4) = 20 + 8 = 28$$

13. Explain how you are using the distributive principle when you multiply 16 by 4.

Point out that a product of two factors can be found several ways by expressing one factor as a sum of two numbers and then using the distributive principle. See the **Guide** for an illustration.

Review multiplication of whole numbers. Emphasize the reason for the correct placement of partial products. Make sure the students understand the examples in ex. 2.

Multiplication

1. Study the multiplication example on the right. You can check a multiplication example by going over the work carefully; or you can interchange the multiplicand and the multiplier and multiply again as shown. If the products are the same, the work is considered correct.

	Check
483	265
265	483
2415	795
2898	2120
966	1060
127995	127995

2. Study these multiplication examples carefully:

42	37	29	74	91	58
30	50	80	300	700	400
1260	1850	2320	22200	63700	23200

Multiply. Check by interchanging the factors:

3.	245 × 382 (93,590)	128 × 406 (51,968)	361 × 547 (197,467)	279 × 590 (164,610)
4.	402 × 983 (395,166)	150 × 866 (129,900)	516 × 516 (266,256)	623 × 685 (426,755)
5.	314 × 825 (259,050)	672 × 853 (573,216)	800 × 896 (716,800)	350 × 375 (131,250)
6.	502 × 502 (252,004)	268 × 300 (80,400)	179 × 240 (42,960)	441 × 565 (249,165)

7. Mrs. King bought 2 yd. of linen at \$.89 a yard and 5 yd. of cotton cloth at \$.75 a yard. How much in all did Mrs. King have to pay? \$5.53

8. A bus ticket from Washington to Fairfield costs \$2.77. If 44 persons made the trip on the bus, how much did they pay all together? \$121.88

9. A truck carried a load of 125 boxes, each weighing 75 lb. Find the weight of the entire load of boxes. Was the load more or less than 5 tons and how much more or less? 9375 lb. 625 lb. less

More Practice. See (3) on page 359.

Review the relationship of division to multiplication, and principles used in division. Provide a diagnostic test on division facts. Review division with 1-figure divisors.

1. The relationship between a division fact and a multiplication fact can be expressed this way. If you let the letter y represent the number you get when you divide 24 by 3, then $24 \div 3 = y$ and $y \times 3 = 24$. You can see that y is 8 because $8 \times 3 = 24$. So $24 \div 3 = 8$. Tell what y must be in each of these:

$18 \div 3 = y$ 6 $\quad 30 \div 5 = y$ 6 $\quad 56 \div 8 = y$ 7 $\quad 42 \div 7 = y$ 6

$y \times 3 = 18$ 6 $\quad y \times 5 = 30$ 6 $\quad y \times 8 = 56$ 7 $\quad 7 \times y = 42$ 6

2. You have learned these principles:

If n stands for any number other than 0, then $0 \div n = 0$.
If n stands for any number, then $n \div 1 = n$.
If n stands for any number other than 0, then $n \div n = 1$.

Tell what each principle means in words. What is $0 \div 4$? 0 $0 \div 8$? 0 $7 \div 1$? 7 $5 \div 1$? 5 $6 \div 6$? 1 $3 \div 3$? 1

3. Turn to page 357 and give the answers to the 90 division facts. You should be able to give the correct answers within the time stated by your teacher. If you have difficulty with these facts, review the multiplication facts. The above principles will also help you.

4. You can use division to find an unknown factor of a number. If you know that one of two factors which have a product of 63 is 7, then the division fact $7\overline{)63}$ tells you that the other factor is 9. Find the unknown factor:

$2 \times$ ..7. $= 14 \quad$ ..5. $\times 9 = 45 \quad 4 \times$ .21. $= 84 \quad$.$40\frac{1}{3}$. $\times 6 = 242$

Divide. To check, multiply the quotient by the divisor and add the remainder.

5.	$5\overline{)600}$ 120	$9\overline{)8325}$ 925	$7\overline{)6005}$ 857 R6	$8\overline{)7858}$ 982 R2	$4\overline{)36728}$ 9182
6.	$6\overline{)145}$ 24 R1	$8\overline{)5924}$ 740 R4	$9\overline{)6381}$ 709	$5\overline{)3695}$ 739	$7\overline{)21749}$ 3107
7.	$9\overline{)387}$ 43	$5\overline{)3439}$ 687 R4	$7\overline{)4221}$ 603	$8\overline{)9216}$ 1152	$4\overline{)32072}$ 8018
8.	$7\overline{)925}$ 132 R1	$9\overline{)5244}$ 582 R6	$6\overline{)3314}$ 552 R2	$5\overline{)1879}$ 375 R4	$8\overline{)94832}$ 11,854

More Practice. See 5 on page 360.

See page 377 for directions for giving the test. If any facts cause difficulty, discuss how relationship in ex. 1 and principles in ex. 2 can be used for help. Point out that 0 cannot be used as a divisor.

Review what it means to cast out the 9's from a number and how to find the residue easily.

Casting Out Nines

1. Find the remainders when these numbers are divided by 2, and then find the remainders when they are divided by 5: See Guide.

42   33   61   54   95   67,982   45,853

Did you have to do each division to find the remainder?

2. Divide these numbers by 9 and study the remainders: See Guide.

10   100   20   200   30   300   301   302   320   303

Did you see a relationship between the remainder and the digits in the number that was divided?

3. When 321 is divided by 9, the quotient is 35 and the remainder is 6. So $321 = (35 \times 9) + 6$. You see that 321 contains 35 nines. If all the 9's are "cast out" of 321, the remainder or **residue** is 6. Cast out the 9's from these numbers and study the residues: See Guide.

23   32   26   62   45   54   121   211   232   322

4. When the 9's are cast out of 32, the residue is 5, which equals the sum of the digits 3 and 2. You can find the residue without dividing by adding the digits in the number. Find the residues after the 9's are cast out of these numbers by adding the digits of each number. Then check the residue by division.

21 3   43 7   71 8   122 5   341 8   401 5   510 6   3121 7

5. The sum of the digits in 853 is 16, which is greater than 9. If you add the digits in 16, you get 7, which is the residue after *all* the 9's are cast out of 853. You should continue to cast out 9's until the final residue is less than 9. Find the residues if the 9's are cast out of these numbers:

75 3   86 5   99 0   257 5   428 5   2454 6   7884 0

6. The sum of the digits in 993 is 21. The sum of the digits in 21 is 3. So the residue is 3 if the 9's are cast out of 993. You can get the residue immediately if you skip the two 9's when finding the sum of the digits in 993. When casting out 9's, you can skip any digit which is 9 or any group of digits which has a sum of 9. Find the residues if the 9's are cast out of these numbers:

291 3   911 2   236 2   572 5   9273 3   12,612 3   16,363 1

Give the students considerable practice in casting out 9's. Have them make up examples for the class to use. Encourage discovery and the use of short cuts.

Reteach casting out 9's as another method of checking multiplication. See the Guide for a discussion of this check.

Another Way to Check Multiplication

1. You can easily and quickly check multiplication by casting out 9's. The steps in this check are shown below:

1. Cast the 9's out of 2544, which gives a residue of 6. This residue is called the **check number** of the multiplicand.

2. Cast the 9's out of 476. This gives 8 for the check number of the multiplier.

3. Multiply the check number 6 by the check number 8, which gives 48. Then cast the 9's out of 48, which gives 3. This is the **final** check number.

	9's Check
2544	6
476	8
15264	48 (3)
17808	
10176	
1210944 (3)	

4. Cast the 9's out of 1,210,944, which gives 3. This is the check number of the product. Since this is the same number as the final check number in step 3, the product is considered correct. When the final check numbers are not the same, the work is wrong. You must go over it to find the error.

2. Study the work on the right carefully. You find that the product is incorrect although the final check numbers are the same. The partial products are correct. But the partial product 4748 is misplaced; you can see that 4748 is written one place too far to the right. This gave the wrong product, but casting out 9's did not detect this error. Whenever a partial product is misplaced, checking by casting out 9's will not detect the error, so it is important to see that each partial product is in the correct place before casting out 9's.

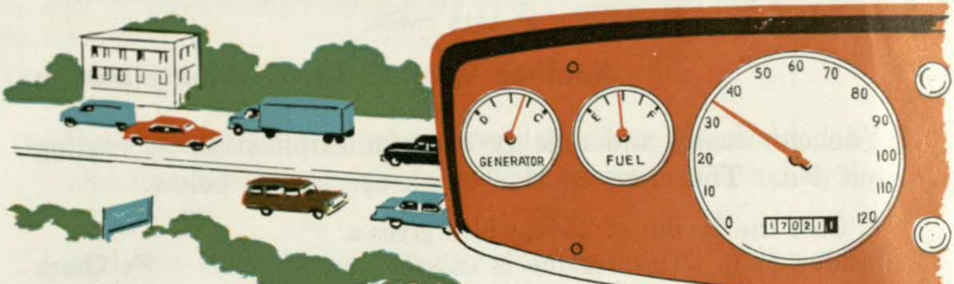
2374	7
2013	6
7122	42 (6)
2374	
4748	
505662 (6)	

Multiply. Check by casting out 9's:

	30,498	41,385	330,906	170,848
3.	78 × 391	89 × 465	421 × 786	304 × 562
	57,036	25,714	342,333	570,024
4.	97 × 588	86 × 299	409 × 837	702 × 812

More Practice. See 4 on page 359.

Discuss ex. 1-2 carefully with the class. Emphasize the fact that students should continue to cast out 9's until the final residue or check number is less than 9.



Division

1. Problem Mr. Wood finds that his average speed on long trips in his car is 35 mi. an hour. Next week he will start on a trip of 1490 mi. About how many hours of driving will Mr. Wood allow? 43

Explanation To estimate the first quotient figure, divide 14 by 3. This gives 4. When you multiply 35 by 4, you get 140. Subtract 140 from 149, which leaves 9. Why is 4 correct?

$$\begin{array}{r} 42\frac{4}{7} \\ 35\overline{)1490} \\ \underline{140} \\ 90 \\ \underline{70} \\ 20 \end{array}$$

To find the second quotient figure, divide 9 by 3. The quotient is 3. When you multiply 35 by 3, you get 105, which is more than 90. Try 2 as the quotient figure. Why is 2 correct?

There is a remainder of 20. Write 20 over the divisor and change the fraction to lowest terms. The quotient is $42\frac{4}{7}$. Then $42\frac{4}{7}$ hr. are needed for the trip. This is about 43 hr.

Check by multiplying 42 by 35 and adding 20.

Emphasize.

When the second figure of the divisor is 1, 2, 3, 4, or 5, divide by the first figure of the divisor to estimate each quotient figure.

Divide. Check the work by multiplying:

2. $83\overline{)19842}$ $239\frac{5}{83}$ $44\overline{)23499}$ $534\frac{3}{44}$ $318\overline{)48619}$ $152\frac{283}{318}$ $631\overline{)159643}$ 253

3. $54\overline{)19823}$ $367\frac{5}{54}$ $22\overline{)13955}$ $634\frac{7}{22}$ $756\overline{)63504}$ 84 $725\overline{)171100}$ 236

4. $31\overline{)19997}$ $645\frac{2}{31}$ $35\overline{)25584}$ $730\frac{34}{35}$ $432\overline{)40338}$ $93\frac{3}{8}$ $819\overline{)181035}$ $221\frac{4}{91}$

More Practice. See 6 on page 360.

Reteach long division with 2-figure and 3-figure divisors when the second figure of the divisor is 1, 2, 3, 4, or 5. Emphasize the fact that the first quotient figure must always be written over the last figure of the first partial dividend.

Reteach long division with 2-figure and 3-figure divisors when the second figure of the divisor is 6, 7, 8, or 9.

More Work in Division

1. Problem In a large city 28,477 babies were born last year. That was an average of about how many born per day? 78 babies

Explanation Since the second figure of 365 is 6, divide by 4 instead of 3 to estimate each quotient figure. You divide by 4 because 365 is nearer to 400 than it is to 300. This gives 7 as the quotient figure. Then multiply 365 by 7 and subtract the product from 2847. This leaves 292, which is less than 365.

$$\begin{array}{r} 78 \\ 365\overline{)28477} \\ \underline{2555} \\ 2927 \\ \underline{2920} \\ 7 \end{array}$$

To find the second quotient figure, divide 29 by 4. The quotient is 7. But when this step is finished the remainder is 372, which is larger than 365. This shows that 7 is too small for the quotient figure. Try 8. Why is 8 right?

The quotient is 78 with a remainder of 7. So an average of 78 babies were born per day.

Emphasize.

When the second figure of the divisor is 6, 7, 8, or 9, divide by 1 more than the first figure of the divisor to estimate each quotient figure.

Divide. Check the work by multiplying:

2.	87)26470 304 R22	49)196309 4006 R15	467)78674 168 R218	675)158760 235 R135
3.	46)34500 750	88)618651 7030 R11	288)33128 115 R8	273)222495 815
4.	19)16214 853 R7	67)134098 2001 R31	561)14586 26	291)140262 482
5.	29)82099 2831	38)103126 2713 R32	483)14007 29	396)121176 306
6.	37)68221 1843 R30	86)425110 4943 R12	279)18996 68 R24	682)341682 501
7.	78)65676 842	28)190156 6791 R8	471)40540 86 R34	975)726425 745 R50

More Practice. See 7 on page 360. Use for individual assistance.

Emphasize and show that when the remainder is greater than the divisor, the quotient figure is too small and should be increased by 1. In ex. 1 ask the students why 7 in the quotient is in tens place (2847 "tens" were divided).

Reteach casting out 9's as another method of checking long division. Notice that this method of checking is strongly recommended for division in Grades 7 and 8.

Another Way to Check Division

1. Division examples can also be checked by casting out 9's. In the example at the right, first find the check numbers for 237, 492, 32, and 116,636, as was done in checking multiplication. Then multiply the check number 6 (quotient) by the check number 3 (divisor), which gives 18. Add to 18 the check number 5 (remainder); this gives 23, which has the check number 5. Since 5 is the same as the check number for the dividend, consider the work correct.

```
       (6)
(3)    492
237)116636 (5)     9's Check
     948
     2183              6
     2133              3
      506             18
      474              5
       32 (5)         23 (5)
```

2. Study carefully these examples and their checks:

```
      (8)
(4)   368     9's Check
85)31296 (3)
   255           8
    579          4
    510         32
     696         7
     680        39 (3)
      16 (7)
```

```
      (5)
(2)   968     9's Check
29)27086 (5)
   251           5
    198          2
    174         10
     246         5
     232        15 (6)
      14 (5)
```

The second example above does not check because the final check number (6) does not agree with the check number of the dividend, which is 5; so this example contains a mistake. Correct the mistake and then see if the example checks. 934 See Guide.

Divide. Check by casting out 9's:

3. 10,418 ÷ 19 548 R6   392,288 ÷ 736 533   238,405 ÷ 305 781 R200

4. 15,656 ÷ 24 652 R8   238,439 ÷ 484 492 R311   153,000 ÷ 525 291 R225

5. 42,021 ÷ 69 609   194,906 ÷ 298 654 R14   246,795 ÷ 666 370 R375

More Practice. See 8 on page 361.

Discuss ex. 1-2 carefully and have volunteers explain the check in ex. 3. Remind the students that we continue to cast out 9's until we have a check number less than 9.

Review addition and subtraction of mixed numbers containing like and unlike fractions (pages 15–16).

Adding and Subtracting Fractions

1. If you multiply or divide both terms of a fraction by the same number, you get another fraction having the same value.

So $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ and $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$.

2. An improper fraction can be changed to a whole number or a mixed number by dividing the numerator by the denominator. So $\frac{11}{3} = 3\frac{2}{3}$ and $\frac{24}{4} = 6$.

3. Study these addition examples:

$$\begin{array}{l} 4\frac{5}{6} \\ \underline{3\frac{1}{6}} \\ 7\frac{6}{6} = 8 \end{array} \qquad \begin{array}{l} 1\frac{9}{16} = 1\frac{9}{16} \\ \underline{7\frac{5}{8}} = \underline{7\frac{10}{16}} \\ \phantom{7\frac{5}{8} =\ } 8\frac{19}{16} = 9\frac{3}{16} \end{array} \qquad \begin{array}{l} 6\frac{1}{2} = 6\frac{3}{6} \\ \underline{2\frac{5}{6}} = \underline{2\frac{5}{6}} \\ \phantom{2\frac{5}{6} =\ } 8\frac{8}{6} = 9\frac{2}{6} = 9\frac{1}{3} \end{array}$$

4. Study these subtraction examples:

$$\begin{array}{l} 6\frac{1}{8} = 5\frac{9}{8} \\ \underline{3\frac{5}{8}} = \underline{3\frac{5}{8}} \\ \phantom{3\frac{5}{8} =\ } 2\frac{4}{8} = 2\frac{1}{2} \end{array} \qquad \begin{array}{l} 9\frac{3}{4} = 9\frac{3}{4} \\ \underline{4\frac{1}{2}} = \underline{4\frac{2}{4}} \\ \phantom{4\frac{1}{2} =\ } 5\frac{1}{4} \end{array} \qquad \begin{array}{l} 2\frac{1}{4} = 2\frac{3}{12} = 1\frac{15}{12} \\ \underline{1\frac{5}{12}} = \underline{1\frac{5}{12}} = \underline{1\frac{5}{12}} \\ \phantom{1\frac{5}{12} = 1\frac{5}{12} =\ } \frac{10}{12} = \frac{5}{6} \end{array}$$

Add in ex. 5–7.(1) Subtract in ex. 5 and 6:(2)

5.	6	$9\frac{1}{2}$	$7\frac{5}{6}$	$6\frac{1}{8}$	$8\frac{3}{16}$	$7\frac{1}{6}$	$2\frac{1}{2}$
	$4\frac{3}{8}$	$7\frac{3}{4}$	$2\frac{1}{2}$	$4\frac{5}{8}$	$3\frac{5}{8}$	$4\frac{5}{12}$	$1\frac{9}{16}$
(1), (2)	$10\frac{3}{8}$, $1\frac{5}{8}$	$17\frac{1}{4}$, $1\frac{3}{4}$	$10\frac{1}{3}$, $5\frac{1}{3}$	$10\frac{3}{4}$, $1\frac{1}{2}$	$11\frac{13}{16}$, $4\frac{9}{16}$	$11\frac{7}{12}$, $2\frac{3}{4}$	$4\frac{1}{16}$, $\frac{15}{16}$
6.	$4\frac{2}{3}$	$8\frac{1}{3}$	$7\frac{1}{4}$	$9\frac{3}{4}$	$5\frac{1}{4}$	$7\frac{3}{5}$	$4\frac{1}{2}$
	$1\frac{5}{6}$	$6\frac{1}{2}$	$5\frac{5}{8}$	$5\frac{2}{3}$	$2\frac{11}{16}$	$1\frac{9}{10}$	$2\frac{7}{10}$
(1), (2)	$6\frac{1}{2}$, $2\frac{5}{6}$	$14\frac{5}{6}$, $1\frac{5}{6}$	$12\frac{7}{8}$, $1\frac{5}{8}$	$15\frac{5}{12}$, $4\frac{1}{12}$	$7\frac{15}{16}$, $2\frac{9}{16}$	$9\frac{1}{2}$, $5\frac{7}{10}$	$7\frac{1}{5}$, $1\frac{4}{5}$
7.	$3\frac{2}{3}$	$1\frac{3}{4}$	$6\frac{5}{8}$	$9\frac{5}{6}$	$5\frac{1}{12}$	$6\frac{7}{16}$	$8\frac{1}{10}$
	$9\frac{2}{3}$	$7\frac{3}{4}$	$3\frac{1}{8}$	$2\frac{1}{2}$	$3\frac{2}{3}$	$2\frac{3}{8}$	$4\frac{3}{10}$
	$4\frac{2}{3}$	$8\frac{1}{2}$	$2\frac{3}{4}$	$7\frac{2}{3}$	$8\frac{5}{6}$	$5\frac{1}{4}$	$2\frac{7}{10}$
(1)	18	18	$12\frac{1}{2}$	20	$17\frac{7}{12}$	$14\frac{1}{16}$	$15\frac{1}{10}$

More Practice. See 9 and 10 on page 361.

Use diagrams to illustrate the principle in ex. 1. Model examples in ex. 3–4 should be discussed thoroughly with the class (use diagrams and/or materials as needed). Be sure the students understand how $6\frac{1}{8}$ is changed to $5\frac{9}{8}$, and so on.

First review the meaning of least common denominator. Then discuss ex. 1 with the class. Do ex. 2 with the class, letting different students explain how they found the least common denominator.

Least Common Denominator

1. Problem Add $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{3}{8}$. $\frac{7}{8}$

Explanation There is an easy way to find the least common denominator for the fractions shown above. First see if the largest denominator, which is 8, contains as factors both 6 and 3; 8 **does not** contain either 6 or 3. Next multiply 8 by 2, which gives 16, and see if 16 contains as factors both 6 and 3; 16 **does not** contain them. Then multiply 8 by 3, which gives 24. You see that 24 **does** contain as factors both 6 and 3, so 24 is the least common denominator. Tell how to complete the work.

$$\frac{1}{6} = \frac{4}{24}$$
$$\frac{1}{3} = \frac{8}{24}$$
$$\frac{3}{8} = \frac{9}{24}$$
$$\frac{21}{24} = \frac{7}{8}$$

Emphasize.

To find the least common denominator, see if the largest denominator contains as factors the other denominators. If not, multiply it in turn by 2, 3, 4, and so on, until you get a number that contains as a factor each denominator.

Add in ex. 2–5.(1) Subtract in ex. 3–5:(2)

2.	$4\frac{1}{2}$	$5\frac{3}{4}$	$9\frac{1}{5}$	$7\frac{3}{8}$	$6\frac{7}{10}$	$1\frac{3}{16}$	$2\frac{11}{12}$
	$1\frac{7}{8}$	$2\frac{5}{6}$	$4\frac{5}{6}$	$1\frac{1}{3}$	$4\frac{1}{3}$	$8\frac{5}{6}$	$3\frac{3}{8}$
	$3\frac{1}{6}$	$1\frac{1}{2}$	$6\frac{2}{3}$	$4\frac{7}{12}$	$1\frac{1}{2}$	$7\frac{3}{8}$	$5\frac{1}{16}$
(1)	$9\frac{13}{24}$	$10\frac{1}{12}$	$20\frac{7}{10}$	$13\frac{7}{24}$	$12\frac{8}{15}$	$17\frac{19}{48}$	$11\frac{17}{48}$
3.	$7\frac{1}{4}$	$9\frac{3}{8}$	$6\frac{3}{4}$	$7\frac{3}{8}$	$8\frac{5}{8}$	$6\frac{3}{4}$	$9\frac{11}{12}$
	$1\frac{1}{6}$	$4\frac{5}{6}$	$2\frac{5}{6}$	$4\frac{5}{12}$	$4\frac{1}{10}$	$5\frac{1}{10}$	$5\frac{3}{8}$
(1), (2)	$8\frac{5}{12}$, $6\frac{1}{12}$	$14\frac{5}{24}$, $4\frac{13}{24}$	$9\frac{7}{12}$, $3\frac{11}{12}$	$11\frac{19}{24}$, $2\frac{23}{24}$	$12\frac{29}{40}$, $4\frac{21}{40}$	$11\frac{17}{20}$, $1\frac{13}{20}$	$15\frac{7}{24}$, $4\frac{13}{24}$
4.	$5\frac{1}{6}$	$3\frac{5}{6}$	$7\frac{1}{8}$	$9\frac{9}{10}$	$8\frac{1}{6}$	$6\frac{7}{10}$	$4\frac{7}{12}$
	$2\frac{5}{8}$	$1\frac{3}{4}$	$5\frac{1}{6}$	$3\frac{5}{6}$	$4\frac{3}{10}$	$1\frac{1}{4}$	$2\frac{1}{8}$
	$7\frac{19}{24}$, $2\frac{13}{24}$	$5\frac{7}{12}$, $2\frac{1}{12}$	$12\frac{7}{24}$, $1\frac{23}{24}$	$13\frac{11}{15}$, $6\frac{1}{15}$	$12\frac{7}{15}$, $3\frac{13}{15}$	$7\frac{19}{20}$, $5\frac{9}{20}$	$6\frac{17}{24}$, $2\frac{11}{24}$
5.	$8\frac{1}{6}$	$9\frac{3}{4}$	$1\frac{5}{8}$	$7\frac{7}{8}$	$9\frac{1}{10}$	$4\frac{3}{16}$	$7\frac{1}{8}$
	$5\frac{3}{8}$	$5\frac{1}{6}$	$\frac{5}{6}$	$4\frac{7}{12}$	$2\frac{3}{4}$	$1\frac{1}{12}$	$6\frac{1}{12}$
	$13\frac{13}{24}$, $2\frac{19}{24}$	$14\frac{11}{12}$, $4\frac{7}{12}$	$2\frac{11}{24}$, $\frac{19}{24}$	$12\frac{11}{24}$, $3\frac{7}{24}$	$11\frac{17}{20}$, $6\frac{7}{20}$	$5\frac{13}{48}$, $3\frac{5}{48}$	$13\frac{5}{24}$, $1\frac{1}{24}$

More Practice. See 11 on page 362. Use for reteaching.

Not all students need do all the examples. Group students who need help for reteaching.

Use the diagrams given in the Guide when discussing ex. 1. Emphasize the meaning of multiplication when the multiplier is a fraction.

Multiplying Fractions

1. Study these examples carefully:

$$\frac{2}{3} \times 11 = \frac{22}{3} = 7\tfrac{1}{3} \qquad \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} = \frac{1}{2} \qquad 5\tfrac{1}{3} \times \frac{3}{8} = \frac{\overset{2}{\cancel{16}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{8}}} = 2$$

$$\frac{4}{5} \times 3\tfrac{1}{8} = \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{5}{\cancel{25}}}{\underset{2}{\cancel{8}}} = \frac{5}{2} = 2\tfrac{1}{2} \qquad 7\tfrac{1}{2} \times 1\tfrac{1}{10} = \frac{\overset{3}{\cancel{15}}}{2} \times \frac{11}{\underset{2}{\cancel{10}}} = \frac{33}{4} = 8\tfrac{1}{4}$$

Point out that a mixed number is often changed to an improper fraction before multiplying.

Multiply. Cancel when you can. Check by going over the work:

2. $\frac{1}{2} \times \frac{1}{2}$ $\frac{1}{4}$	$8 \times 3\frac{1}{2}$ 28	$7\frac{1}{2} \times 11$ $82\frac{1}{2}$	$1\frac{1}{8} \times 5\frac{1}{3}$ 6
3. $\frac{3}{4} \times 8$ 6	$\frac{7}{8} \times 1\frac{1}{3}$ $1\frac{1}{6}$	$3\frac{1}{8} \times 7\frac{1}{5}$ $22\frac{1}{2}$	$2\frac{3}{5} \times 3\frac{1}{2}$ $9\frac{1}{10}$
4. $9 \times \frac{3}{8}$ $3\frac{3}{8}$	$\frac{3}{4} \times 36$ 27	$2\frac{1}{4} \times 3\frac{1}{3}$ $7\frac{1}{2}$	$1\frac{1}{4} \times 3\frac{1}{2}$ $4\frac{3}{8}$
5. $\frac{1}{6} \times \frac{3}{5}$ $\frac{1}{10}$	$\frac{5}{6} \times 1\frac{1}{2}$ $1\frac{1}{4}$	$4\frac{1}{2} \times 4\frac{1}{2}$ $20\frac{1}{4}$	$1\frac{1}{5} \times 1\frac{1}{4}$ $1\frac{1}{2}$
6. $\frac{5}{6} \times \frac{2}{5}$ $\frac{1}{3}$	$\frac{1}{6} \times 2\frac{1}{4}$ $\frac{3}{8}$	$3\frac{3}{4} \times 1\frac{3}{5}$ 6	$2\frac{2}{3} \times 1\frac{3}{4}$ $4\frac{2}{3}$
7. $9 \times \frac{5}{6}$ $7\frac{1}{2}$	$\frac{4}{5} \times 6\frac{1}{4}$ 5	$2\frac{2}{3} \times 4\frac{1}{2}$ 12	$2\frac{2}{5} \times 6\frac{1}{4}$ 15

More Practice. See 12 on page 363.

8. Jerry and his father picked $7\frac{1}{2}$ bu. of corn. Jerry had $\frac{1}{3}$ of this corn to sell for himself. How many bushels of corn did Jerry have for himself? $2\frac{1}{2}$

Dividing by a Fraction

1. Study these examples carefully:

$$4 \div \frac{2}{3} = \frac{\overset{2}{\cancel{4}}}{1} \times \frac{3}{\underset{1}{\cancel{2}}} = 6 \qquad \frac{7}{8} \div \frac{3}{4} = \frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{3} = \frac{7}{6} = 1\frac{1}{6}$$

$$3\frac{3}{4} \div \frac{5}{8} = \frac{\overset{3}{\cancel{15}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{5}}} = 6$$

To divide by a fraction, invert the divisor and multiply.

Divide. Check by going over the work:

2.	$\frac{7}{16} \div \frac{1}{12}$ $5\frac{1}{4}$	$18 \div \frac{1}{3}$ 54	$7\frac{1}{3} \div \frac{11}{12}$ 8	$4\frac{1}{5} \div \frac{2}{3}$ $6\frac{3}{10}$
3.	$\frac{7}{12} \div \frac{7}{16}$ $1\frac{1}{3}$	$10 \div \frac{5}{12}$ 24	$3\frac{3}{5} \div \frac{9}{10}$ 4	$1\frac{3}{4} \div \frac{1}{3}$ $5\frac{1}{4}$
4.	$\frac{3}{10} \div \frac{9}{16}$ $\frac{8}{15}$	$25 \div \frac{5}{16}$ 80	$4\frac{1}{6} \div \frac{5}{16}$ $13\frac{1}{3}$	$5\frac{1}{4} \div \frac{7}{8}$ 6
5.	$\frac{15}{16} \div \frac{3}{16}$ 5	$11 \div \frac{5}{8}$ $17\frac{3}{5}$	$4\frac{3}{8} \div \frac{7}{12}$ $7\frac{1}{2}$	$3\frac{1}{2} \div \frac{3}{5}$ $5\frac{5}{6}$
6.	$\frac{3}{16} \div \frac{5}{16}$ $\frac{3}{5}$	$14 \div \frac{7}{12}$ 24	$8\frac{1}{4} \div \frac{11}{12}$ 9	$1\frac{3}{8} \div \frac{1}{2}$ $2\frac{3}{4}$
7.	$\frac{15}{16} \div \frac{3}{8}$ $2\frac{1}{2}$	$16 \div \frac{3}{4}$ $21\frac{1}{3}$	$3\frac{3}{8} \div \frac{9}{10}$ $3\frac{3}{4}$	$2\frac{1}{4} \div \frac{3}{8}$ 6
8.	$\frac{9}{10} \div \frac{3}{16}$ $4\frac{4}{5}$	$15 \div \frac{2}{3}$ $22\frac{1}{2}$	$3\frac{3}{4} \div \frac{3}{10}$ $12\frac{1}{2}$	$1\frac{7}{8} \div \frac{5}{6}$ $2\frac{1}{4}$
9.	$\frac{9}{16} \div \frac{5}{12}$ $1\frac{7}{20}$	$18 \div \frac{1}{4}$ 72	$2\frac{1}{2} \div \frac{15}{16}$ $2\frac{2}{3}$	$3\frac{3}{4} \div \frac{5}{8}$ 6

10. Sally made $4\frac{1}{2}$ lb. of chocolate candy. She sold it in bags containing $\frac{3}{8}$ lb. each. How many bags of candy did Sally have all together? 12

11. On a bicycle trip Jack and Bob rode $6\frac{3}{4}$ mi. in $\frac{3}{4}$ hr. Find their average speed in miles per hour. 9

More Practice. See 13 on page 364.

Develop an understanding of what division by a fraction means and why the divisor is inverted. See the Guide for suggestions. Discuss why answers are sensible.

Dividing by Whole and Mixed Numbers

1. Study these examples carefully:

$$\text{A. } \frac{5}{8} \div 2 = \frac{5}{8} \div \frac{2}{1} = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$$

$$\text{B. } 3\tfrac{1}{3} \div 8 = \frac{10}{3} \div \frac{8}{1} = \frac{\overset{5}{\cancel{10}}}{3} \times \frac{1}{\underset{4}{\cancel{8}}} = \frac{5}{12}$$

$$\text{C. } \frac{1}{2} \div 1\tfrac{1}{4} = \frac{1}{2} \div \frac{5}{4} = \frac{1}{\underset{1}{\cancel{2}}} \times \frac{\overset{2}{\cancel{4}}}{5} = \frac{2}{5}$$

$$\text{D. } 4\tfrac{1}{8} \div 1\tfrac{5}{6} = \frac{33}{8} \div \frac{11}{6} = \frac{\overset{3}{\cancel{33}}}{\underset{4}{\cancel{8}}} \times \frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{11}}} = \frac{9}{4} = 2\tfrac{1}{4}$$

Stress.

When you divide by 2, write 2 over 1 to make a fraction. Then invert the divisor and multiply. The rule on page 18 can also be used when the divisor is a whole number, as in A and B above. In C and D, change the divisor from a mixed number to an improper fraction and follow the rule. Stress.

Divide. Check your work by going over it:

2. $48 \div 1\frac{3}{5}$ 30 $\quad \frac{7}{8} \div 1\frac{5}{16}$ $\frac{2}{3}$ $\quad 1\frac{7}{8} \div 1\frac{1}{2}$ $1\frac{1}{4}$ $\quad 2\frac{1}{4} \div 1\frac{3}{4}$ $1\frac{2}{7}$

3. $18 \div 2\frac{1}{4}$ 8 $\quad \frac{5}{8} \div 2\frac{13}{16}$ $\frac{2}{9}$ $\quad 4\frac{1}{2} \div 3\frac{3}{4}$ $1\frac{1}{5}$ $\quad 1\frac{3}{5} \div 1\frac{1}{3}$ $1\frac{1}{5}$

4. $40 \div 3\frac{3}{4}$ $10\frac{2}{3}$ $\quad \frac{1}{4} \div 2\frac{1}{2}$ $\frac{1}{10}$ $\quad 3\frac{3}{4} \div 1\frac{1}{2}$ $2\frac{1}{2}$ $\quad 1\frac{7}{8} \div 1\frac{1}{4}$ $1\frac{1}{2}$

5. $27 \div 3\frac{3}{5}$ $7\frac{1}{2}$ $\quad \frac{2}{3} \div 1\frac{2}{5}$ $\frac{10}{21}$ $\quad 2\frac{1}{3} \div 5\frac{5}{6}$ $\frac{2}{5}$ $\quad 5\frac{3}{5} \div 4\frac{1}{5}$ $1\frac{1}{3}$

6. $55 \div 3\frac{2}{3}$ 15 $\quad \frac{5}{6} \div 1\frac{2}{3}$ $\frac{1}{2}$ $\quad 3\frac{3}{8} \div 1\frac{4}{5}$ $1\frac{7}{8}$ $\quad 3\frac{1}{3} \div 4\frac{1}{6}$ $\frac{4}{5}$

7. $20 \div 7\frac{1}{2}$ $2\frac{2}{3}$ $\quad \frac{7}{8} \div 5\frac{1}{4}$ $\frac{1}{6}$ $\quad 5\frac{1}{3} \div 6\frac{2}{3}$ $\frac{4}{5}$ $\quad 3\frac{1}{2} \div 8\frac{3}{4}$ $\frac{2}{5}$

8. $42 \div 3\frac{1}{2}$ 12 $\quad \frac{5}{6} \div 3\frac{1}{8}$ $\frac{4}{15}$ $\quad 4\frac{1}{2} \div 1\frac{7}{8}$ $2\frac{2}{5}$ $\quad 4\frac{2}{3} \div 1\frac{1}{6}$ 4

More Practice. See 14 on page 364.

The model examples in ex. 1 should be put on the board and discussed thoroughly with the class. Emphasize how a whole-number divisor is inverted and that mixed numbers should be changed to improper fractions before dividing.

Review place value and the use of exponents.

Place Value

1. Our system of writing numbers is said to have **place value** because the value of each digit depends upon the place it occupies. Study the numbers below:

A. In 7000.004 the 4 represents 4 thousandths
B. In 7000.04 the 4 represents 4 hundredths
C. In 7000.4 the 4 represents 4 tenths
D. In 7004 the 4 represents 4 ones
E. In 7040 the 4 represents 4 tens
F. In 7400 the 4 represents 4 hundreds

In B the value of 4 is 10 times as great as in A; in C the value of 4 is 10 times as great as in B. You see that the value of 4 becomes 10 times as great if you move it one place to the left.

If any digit of a number is moved one place to the left, its value is multiplied by ten.

2. The values of the first five places in whole numbers are:

ten thousand	thousand	hundred	ten	one
10,000	1000	100	10	1
$10 \times 10 \times 10 \times 10$	$10 \times 10 \times 10$	10×10	10	1

These values are often written by using an **exponent**, which is a small number placed as shown below:

$$10 \times 10 \times 10 \times 10 = 10^4 \qquad 10 \times 10 = 10^2$$
$$10 \times 10 \times 10 = 10^3 \qquad 10 = 10^1$$

The exponent tells how many factors of 10 are in the product. 10^4 means a product of four 10's and is read "ten to the fourth power." 10^2 is often read "ten squared." Ten to the first power is usually written without the exponent 1.

3. Give the meaning of these numbers: 10^5 8^2 10^7 5^3 4^4 See Guide.

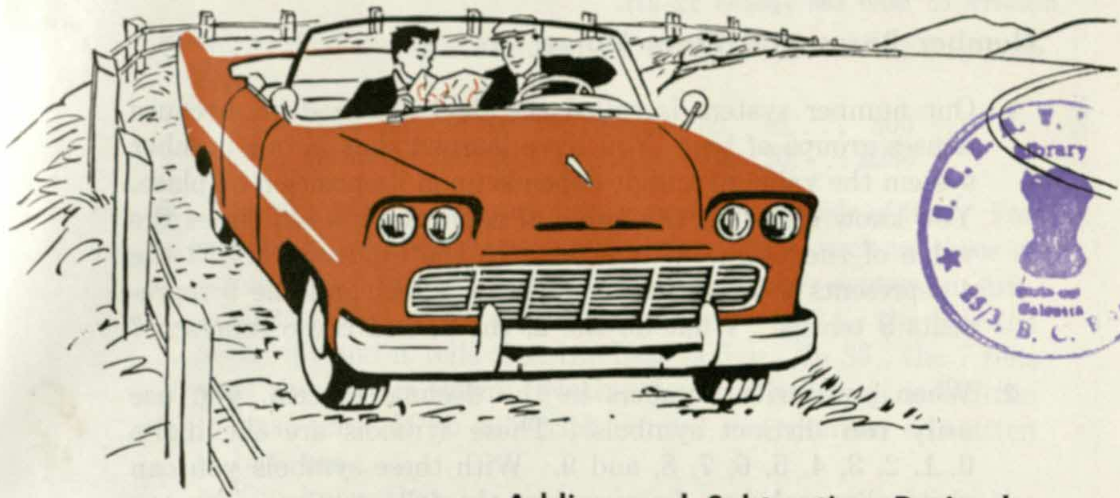
4. The value of 5827 can be expressed with exponents this way:

$$5827 = (5 \times 10 \times 10 \times 10) + (8 \times 10 \times 10) + (2 \times 10) + (7 \times 1)$$
$$= (5 \times 10^3) + (8 \times 10^2) + (2 \times 10^1) + (7 \times 1)$$

5. Write the values of these numbers using exponents:

729 4956 8253 46,824 See Guide.

Emphasize the comparison of place values and develop a need for exponents as the place value gets larger. Point out the relationship of the exponent of 10 to the number of 0's in the corresponding product.



Adding and Subtracting Decimals

1. The distance from Mayfield to Fairview is 37.5 mi. and from Mayfield to Jamestown is 21.7 mi. How much farther is the distance to Fairview than to Jamestown? 15.8 mi.

▶ When you subtract decimals, be sure to keep the decimal points under one another. The decimal point in the answer is placed under the decimal points above.

$$\begin{array}{r} 37.5 \\ \underline{21.7} \end{array}$$

2. Problem Betty bought a skirt for \$7, a pair of shoes for \$7.49, a sweater for \$5, and a blouse for \$2.74. How much did she spend for all of these things?

Explanation You can write these amounts as at A and add them, which gives \$22.23. You may prefer to write a decimal point and two zeros after \$7 and \$5 as at B before you add these amounts. When you have an addition example in decimals in which the numbers do not all have the same number of decimal places, you must be very careful to keep the decimal points under one another.

A	B
\$7.	\$7.00
7.49	7.49
5.	5.00
2.74	2.74
\$22.23	\$22.23

When you add or subtract decimals that represent measurements, they should all be to tenths, or all to hundredths, and so on. Emphasize.

Practice Exercises. See 15 on page 365.

See the Guide for a discussion of the addition and subtraction of decimals that represent measurements and of decimals that represent money.

Review writing numbers in systems having bases other than ten and changing numbers to base ten (pages 22–23).

Number Bases and Place Value

1. Our number system is called the **decimal system** because it uses groups of ten. You have learned that in our number system the value of a digit depends upon its position or place. You know also that the value of each place is ten times the value of the place on its right. In the number 8214.93 the 4 represents 4 ones, the 1 represents 1 ten, and the 9 represents 9 tenths. What do the 2, the 3, and the 8 represent?

2. When you write numbers in the decimal system, you use **only ten** distinct symbols. These symbols are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. With these symbols you can write all numbers. In counting, ten follows nine. For ten you have no digit. So you make a group of ten ones and write 10, which means 1 group of ten ones and no ones. 36 means 3 groups of ten ones and 6 ones. 467 means 4 hundreds, 6 tens, 7 ones. Notice that one hundred is a group of ten tens. Give the meaning of: See Guide.

376 2486 472.68 40,000 570.029

3. You can build a number system using only the symbols 0, 1, 2, 3, and 4. For one you can write 1, for two write 2, for three write 3, and for four write 4. For five you have no digit to use. So you make a group of five ones and write 10. In this new system 10 means 1 group of five ones and no ones, 14 means 1 group of five ones and 4 ones, and 43 means 4 groups of five ones and 3 ones. The value of each place is five times the value of the place on its right. So the 3 in 314 means 3 twenty-fives, since five times five is twenty-five.
▶ The decimal system, in which you use groups of ten, is called the **base ten** system. This new system, in which you use groups of five, is called the **base five** system.

4. Write one to fifty-two in base ten and in base five.

Base ten: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, . . . 52 See Guide.

Base five: 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, . . . 202

Be sure to bring out the fact that 10 means ten only when the base is ten. Emphasize also that in a given system the base is not represented by a single digit.

5. Write five to three hundred by fives in base ten and in base five. See Guide.

Base ten: 5, 10, 15, 20, 25, 30, 35, . . . 300
Base five: 10, 20, 30, 40, 100, 110, 120, . . . 2200

6. You have used a new number system with a base of five. You can build number systems with other bases, such as three or seven. In order to tell the base of a number system, a **subscript** is used when writing a number. In 14_5, the 5 is the subscript and it tells that the base is five. In 35_7, the 7 tells that the base is seven. Notice that the subscript is written in base ten. If no subscript is used, the number is written in base ten.

▶ You know that 14_5 means 1 five and 4 ones. Study the following carefully:

35_7 = 3 sevens and 5 ones
432_5 = 4 twenty-fives, 3 fives, 2 ones
213_4 = 2 sixteens, 1 four, 3 ones
2120_3 = 2 twenty-sevens, 1 nine, 2 threes, 0 ones

Give the value of each digit: See Guide.

7. 42_5 44_5 324_5 203_5 1400_5 2314_5

8. 12_7 73_8 113_4 295 1111_2 4253_6

9. 27_9 33_4 404_7 527_8 9746 1100_2

10. To change a number written in another base to base ten, you express the place values of this base in base ten:

$$32_5 = 3 \text{ fives and } 2 \text{ ones}$$
$$= (3 \times 5) + (2 \times 1) = 15 + 2 = 17$$
$$2435_6 = (2 \times 6 \times 6 \times 6) + (4 \times 6 \times 6) + (3 \times 6) + (5 \times 1)$$
$$= (2 \times 6^3) + (4 \times 6^2) + (3 \times 6) + (5 \times 1)$$
$$= 432 + 144 + 18 + 5 = 599$$

Change to base ten:

11. 42_5 22 44_5 24 324_5 89 203_5 53 1400_5 225 2314_5 334

12. 73_8 59 27_9 25 113_4 23 404_7 200 4253_6 969 1100_2 12

Change to base ten and determine which are even numbers:

13. 11_2 3 11_3 4 11_4 5 11_5 6 11_6 7 111_7 57 11_8 9 111_3 13

For use with this work, have objects available. Use counting and grouping of objects to illustrate the meaning of numerals in different bases. Also represent groups of objects by using numerals in bases other than ten.

Show how to add, subtract, and multiply numbers in bases other than ten without changing numbers to base ten (pages 24-25).

Computation in Other Number Bases

1. You know that $3 + 1 = 4$ (base five). To find $4 + 3$ (base five) you may think $4 + (1 + 2) = (4 + 1) + 2 = 12_5$, using the associative principle to make a group of five ones. Or you may think $(2 + 2) + 3 = 2 + (2 + 3) = 12_5$. So $4 + 3 = 12_5$. Add: $2 + 4$ (base five); $2 + 5$ (base eight); $4 + 4$ (base seven); $3 + 2 + 2$ (base four); $3 + 4 + 2$ (base six). See Guide.

2. Problem Add in base five: 13, 21, 14, and 33.

Base Five

$$\begin{array}{r} 13 \\ 21 \\ 14 \\ \underline{33} \\ 21 \\ \underline{12} \\ 141_5 \end{array}$$

Explanation You remember that in addition you add digits that have the same place value. First add the digits in ones place: $3 + 4 + 1 + 3 = 21_5$. This sum represents 21_5 ones, so it is placed as shown on the right. Then add the digits in fives place: $3 + 1 + 2 + 1 = 12_5$. This sum represents 12_5 fives. Why is it placed as shown? Adding 21_5 ones (2 fives and 1 one) and 12_5 fives, you get for the required sum 14_5 fives and 1 one, or 141_5.

Add: $42 + 22 + 31 + 13$ (base five); $14 + 35 + 42 + 50$ (base six). See Guide.

Add, giving the sum in the same base as the numbers:

3.

24_5	36_8	132_4	1762_8	101_2	2514_7	2503_6
32_5	25_8	213_4	3104_8	101_2	3062_7	4215_6
111_5	63_8	1011_4	5066_8	1010_2	5606_7	11122_6

4. You know that each subtraction fact is related to an addition fact. In base five, $4 - 1 = 3$, since $1 + 3 = 4$. To find $12_5 - 4$, think: "What added to 4 gives 12_5?" Since $4 + 3 = 12_5$, $12_5 - 4 = 3$.

Subtract: $11_5 - 4$; 2 $13_7 - 4$; 6 $12_8 - 5$; 5 $11_3 - 2$; 2 $15_8 - 6$. 7

5. Problem Subtract: $32_5 - 14_5$.

$$\begin{array}{r} 32_5 \\ \underline{14_5} \\ 13_5 \end{array}$$

Explanation First think: "What added to 4 gives 12_5?" Since $4 + 3 = 12_5$, 3 is the required addend. But $14_5 + 3 = 22_5$. Now think: "What added to 22_5 gives 32_5?" This addend is 10_5, or 1 five. So altogether 13_5 must be added to 14_5 to make 32_5; thus, $32_5 - 14_5 = 13_5$. To check, add 14_5 and 13_5.

Subtract, giving the answer in the same base as the numbers:

6.

34_7	42_8	31_7	124_5	111_2	312_4	712_8
12_7	15_8	14_7	40_5	101_2	103_4	435_8
22_7	25_8	14_7	34_5	10_2	203_4	255_8

Make sure the students understand thoroughly how to add and subtract numbers in other bases before assigning individual work. Encourage the students to make up additional practice examples.

7. You know that $2 \times 3 = 6$ (base eight). To find 6×5 (base eight), first write $6 \times 5 = 5 + 5 + 5 + 5 + 5 + 5$ $= 5 + (3 + 2) + 5 + (1 + 4) + (4 + 1) + 5$. Using the associative principle to make groups of eight ones, you have $(5 + 3) + (2 + 5 + 1) + (4 + 4) + 1 + 5 = 36_8$. So $6 \times 5 = 36_8$. The commutative principle tells that 5×6 also equals 36_8. Show that $5 \times 6 = 36_8$ by grouping into eights as was done for 6×5.
See Guide.

8. Some of the multiplication facts in base eight are shown on the right. One factor of a product is given in the left column and the other factor in the first row. To find 3×5 in the chart, locate 3 in the left column and then go across this row to the number in this row that is under the 5 in the first row. This number is 17, which is 3×5. Copy this chart and complete it by finding the missing products. Use the commutative principle when you can.

BASE EIGHT

×	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61

9. **Problem** Multiply: $41_8 \times 25_8$

Explanation The first partial product is $1 \times 25_8$, which is 25_8. It represents 25_8 ones. To find $4 \times 25_8$, you first get 4×5 from your chart, which is 24_8. Then you find $(4 \times 2) + 2$, which is $10_8 + 2$, or 12_8. Thus, the second partial product is 124_8. Why? What does it represent? Adding 25_8 ones and 124_8 eights, you get 1265_8 for the required product.

$$\begin{array}{r} 25_8 \\ \underline{41_8} \\ 25 \\ \underline{124} \\ 1265_8 \end{array}$$

Multiply:

10. $21_8 \times 32_8$ 672_8 $\quad$ $24_8 \times 41_8$ 1224_8 $\quad$ $32_8 \times 36_8$ 1414_8 $\quad$ $75_8 \times 124_8$ 12004_8

Make a chart for the multiplication facts in base five. Then multiply:

11. $13_5 \times 22_5$ 341_5 $\quad$ $24_5 \times 31_5$ 1344_5 $\quad$ $33_5 \times 14_5$ 1122_5 $\quad$ $32_5 \times 143_5$ 11231_5

Help students begin to complete their charts for ex. 8. Have them check products using commutative, associative, and distributive principles. (See the Guide for an example.) Encourage students to make up additional multiplication examples.

Multiplication of Decimals

1. Problem Jack can ride his bicycle at an average speed of 9.5 mi. per hour. At that rate, how far can he ride in 2.5 hr.? 23.75 mi.

Explanation Multiply 9.5 by 2.5 as you multiply whole numbers. The product is 2375. To find where to put the decimal point, make a rough estimate of the answer by multiplying 9 by 2. This gives 18. Since the answer should be a little more than 18, the decimal point should be placed after 23, which gives 23.75. Jack rode 23.75 mi.

```
  9.5
  2.5
  475
 190
 23.75
```

2. Helen bought 4.5 yd. of flannel at \$2.98 per yard. How much did the flannel cost? \$13.41

> **To multiply decimals, multiply as with whole numbers. Then, beginning at the right, point off as many decimal places in the product as there are decimal places in the multiplier and the multiplicand together.**

Estimate the answers. Then multiply the numbers:

3.	9 × 12.3 110.7	23 × 5.6 128.8	4.7 × .136 .6392	.012 × 93.8 1.1256
4.	6 × .067 .402	84 × .83 69.72	.81 × 5.67 4.5927	.043 × 5.42 .23306
5.	5 × 9.15 45.75	73 × 1.8 131.4	7.1 × 1.72 12.212	.125 × .125 .015625
6.	8 × .597 4.776	61 × .28 17.08	.35 × .048 .0168	.007 × 9.84 .06888

More Practice. See 16 on page 365.

After discussing ex. 1, do a few more examples, using estimation to locate decimal-point position. To increase understanding of the rule, use common fractions to verify answers ($.3 \times .4 = \frac{3}{10} \times \frac{4}{10} = \frac{12}{100} = .12$).

In re-explaining decimal-point placement in the quotient when the divisor is a whole number, have the students estimate answers to examples in ex. 1.

Dividing Decimals by Whole Numbers

1. Study these examples carefully:

A. $\begin{array}{r}1.19\\7\overline{)8.33}\end{array}$	B. $\begin{array}{r}.196\\4\overline{).784}\end{array}$	C. $\begin{array}{r}42.125\\8\overline{)337.000}\end{array}$

When you divide a decimal by a whole number, as in A and B above, be sure to place the decimal point in the quo- Stress. tient directly above the decimal point in the dividend. To avoid a remainder in division, as in C above, annex zeros* after the decimal point of the dividend and continue to divide until there is no remainder. *Refer to money ($10 and $10.00) to explain that the value of the quotient is not changed.

Divide until there is no remainder. Annex zeros to the dividend if necessary:

2.	$\begin{array}{r}13.7\\6\overline{)82.2}\end{array}$	$\begin{array}{r}.093\\4\overline{).372}\end{array}$	$\begin{array}{r}.52\\9\overline{)4.68}\end{array}$	$\begin{array}{r}15.8\\5\overline{)79.0}\end{array}$	$\begin{array}{r}139.5\\2\overline{)279.0}\end{array}$	$\begin{array}{r}39.625\\8\overline{)317.000}\end{array}$
3.	$\begin{array}{r}10.4\\9\overline{)93.6}\end{array}$	$\begin{array}{r}1.29\\6\overline{)7.74}\end{array}$	$\begin{array}{r}19.3\\3\overline{)57.9}\end{array}$	$\begin{array}{r}12.5\\4\overline{)50.0}\end{array}$	$\begin{array}{r}24.5\\6\overline{)147.0}\end{array}$	$\begin{array}{r}122.4\\5\overline{)612.0}\end{array}$
4.	$\begin{array}{r}.064\\8\overline{).512}\end{array}$	$\begin{array}{r}.042\\7\overline{).294}\end{array}$	$\begin{array}{r}.54\\6\overline{)3.24}\end{array}$	$\begin{array}{r}8.125\\8\overline{)65.000}\end{array}$	$\begin{array}{r}74.25\\4\overline{)297.00}\end{array}$	$\begin{array}{r}52.5\\6\overline{)315.0}\end{array}$
5.	$\begin{array}{r}.878\\5\overline{)4.390}\end{array}$	$\begin{array}{r}37.25\\2\overline{)74.50}\end{array}$	$\begin{array}{r}12.76\\5\overline{)63.80}\end{array}$	$\begin{array}{r}6.5\\6\overline{)39.0}\end{array}$	$\begin{array}{r}47.875\\8\overline{)383.000}\end{array}$	$\begin{array}{r}98.75\\4\overline{)395.00}\end{array}$

In ex. 2-5 have students verify the decimal-point position by estimation.

More Practice. See 17 on page 366.

In each of these problems annex enough zeros to the dividend to avoid a remainder:

6. Miss West used 16 gal. of gasoline to drive her car 292 mi. How many miles did she drive per gallon of gasoline? 18.25

7. In a race a jet bomber flew at an average speed of 494.4 mi. per hour. How many miles is this per minute? 8.24

8. At a school fair 12 lb. of candy were sold for $11.25. What was the average price of 1 lb. of candy? $.9375 or $.93$\frac{3}{4}$

First review the fact that a fraction can indicate division. Then lead a class discussion of ex. 1-4.

Changing Fractions to Decimals

In percentage it is often necessary to change common fractions to decimals. So you need to review this work.

1. Problem Change $\frac{5}{8}$ to a decimal.

Explanation Since the fraction $\frac{5}{8}$ means $5 \div 8$, write 5 as 5.000 and divide by 8. The division gives .625 as the quotient. So $\frac{5}{8} = .625$.

$$8\overline{)5.000}^{\;.625}$$

2. Problem Change $\frac{2}{7}$ to a decimal, carrying the quotient to the nearest thousandth.

Explanation In this example you will continue to have a remainder, no matter how long you divide. It is necessary, therefore, to decide where to stop. One way to do this is to state that the quotient should be found correct to the **nearest hundredth** or the **nearest thousandth.**

$$7\overline{)2.0000}^{\;.2857}$$

In this case the quotient, carried to 4 decimal places, is .2857. If this result is rounded off to the **nearest thousandth,** it becomes .286 because .2857 is closer to .286 (.2860) than to .285 (.2850). To get .286, you dropped the 7 of .2857 and changed 5 to 6. Emphasize that quotient is carried to one more place than needed for answer.

3. If the quotient in ex. 2 had been .2852, it would become .285 when rounded off to the nearest thousandth because Stress. .2852 is closer to .285 than to .286. If the quotient had been .2855, which is halfway between .285 and .286, it would be called .286 to the nearest thousandth.

4. To get a result correct to the nearest hundredth, the quotient is carried to 3 decimal places and then rounded off. If the quotient were .168, it would round off to .17. Why? How would you round off .163? .165?

Emphasize.

In rounding off a decimal, if the last figure of the decimal is less than 5, drop it; if the last figure is 5 or more, drop it and make the figure before it 1 larger.

Practice Exercises. See 18 on page 366.

In ex. 1 be sure the students understand the regrouping involved (5 ones changed to 50 tenths, and so on). Illustrate (use $\frac{1}{3}$, $\frac{4}{11}$, and so on) that when division does not terminate, you always have a repeating decimal. (See the Guide.)

Review division by a decimal (pages 29-30).

More about Division

1. You can multiply a number by 10, 100, or 1000 by moving the decimal point as many places to the right as there are zeros in the multiplier.

▶ Thus $10 \times 4.2 = 42$, and $100 \times 4.2 = 420$. In multiplying 4.2 by 100, it is necessary to write a zero after 4.2 so that you can move the decimal point two places.

2. Multiply each number by 10; by 100; by 1000: 6.3; 4.87; .332; \$6.50; \$27.80; \$.34; .8; .0635. See Guide.

3. You can divide a number by 10, 100, or 1000 by moving the decimal point as many places to the left as there are zeros in the divisor.

▶ Thus $365 \div 10 = 36.5$, and $365 \div 100 = 3.65$. In moving the decimal point two places to the left, it is sometimes necessary to prefix zeros. Thus $5.9 \div 100 = .059$, and $.4 \div 100 = .004$.

4. Divide each number by 10; by 100; by 1000: \$25.00; 2.7; .32; \$126; 5.3; \$2458.00; .7; .05. See Guide.

5.

A. $\begin{array}{r}7\\5\overline{)35}\end{array}$ B. $\begin{array}{r}7\\50\overline{)350}\end{array}$ C. $\begin{array}{r}7\\500\overline{)3500}\end{array}$

In B, both divisor and dividend are 10 times as large as in A; in C, both divisor and dividend are 100 times as large as in A. The quotients are all 7. You can multiply both dividend and divisor by the same number without changing the quotient. This is the same as multiplying both terms of a fraction by the same number. Why?

Stress.

6. If the divisor is a decimal, you can change it to a whole number before dividing. If you multiply the divisor in D by 10 it becomes 53 as shown in E. Sometimes you will need to multiply the divisor by 100 or 1000.

D. $5.3\overline{)47.7}$ E. $53\overline{)477}$

Divide. First change each divisor to a whole number:

7. $\begin{array}{r}342\\.26\overline{)88.92}\end{array}$ $\begin{array}{r}105\\.95\overline{)99.75}\end{array}$ $\begin{array}{r}3.4\\7.9\overline{)26.86}\end{array}$ $\begin{array}{r}55.9\\.34\overline{)19.006}\end{array}$

More Practice. See 19 on page 367.

After discussing ex. 1-4, give more oral practice as needed in multiplying or dividing by powers of 10. Make sure students understand the procedures in ex. 1 and ex. 3. Have students explain how they changed divisors to whole numbers in ex. 7.

Notice that in both methods in ex. 1 the divisor is changed to a whole number by multiplying both the divisor and the dividend by 100. Have the students use whichever method you feel is better for them.

Dividing by a Decimal

1. Problem It is 4.8 mi. from Lone Ranch to the post office. Today Bill walked this distance in 1 hr. 33 min. Find, to the nearest tenth of a mile, the average distance he walked per hour. 3.1

Explanation Show that 1 hr. 33 min. = 1.55 hr. You must divide 4.8 by 1.55. To do this, first change 1.55 to a whole number by multiplying it by 100; this gives 155 as a new divisor. Also multiply 4.8 by 100, which gives 480 as a new dividend. Then divide 480 by 155, as shown at A; the quotient is 3.09, which rounds off to 3.1. Bill walked at the rate of 3.1 mi. per hour. Check the work by going over it carefully.

A

$$\begin{array}{r} 3.09 \\ 155\overline{)480.00} \\ 465 \\ \hline 15\,00 \\ 13\,95 \\ \hline 1\,05 \end{array}$$

In the work above, the original example was $1.55\overline{)4.8}$, which you changed to $155\overline{)480}$ before starting to divide. If you wish to avoid this change, you can work with $1.55\overline{)4.8}$ and use a caret (‸) to indicate the new position of each decimal point as shown at B. Keep the original decimal points in 1.55 and 4.8 to make it easy to see that they are the original numbers, but ignore them when you do the work as shown at B. Stress. Place the decimal point in the quotient directly above the caret in 4.80‸00.

B

$$\begin{array}{r} 3.09 \\ 1.55_{\wedge}\overline{)4.80_{\wedge}00} \\ 4\,65 \\ \hline 15\,00 \\ 13\,95 \\ \hline 1\,05 \end{array}$$

Carry to 3 decimal places and round off to the nearest hundredth:

2.	$1.05\overline{)27.2}$ 25.90	$.65\overline{)38.9}$ 59.85	$7.2\overline{)18.53}$ 2.57	$.8\overline{)17.3}$ 21.63
3.	$9.45\overline{)187}$ 19.79	$.43\overline{)186.5}$ 433.72	$2.4\overline{)365.1}$ 152.13	$.7\overline{)1.08}$ 1.54
4.	$63.2\overline{)4.86}$ .08	$.19\overline{)138.4}$ 728.42	$4.8\overline{)32.73}$ 6.82	$.9\overline{)23.6}$ 26.22

More Practice. See 20 on page 367. Use to increase skill.

To the Teacher. In ex. 1 Methods A and B both change the divisor 1.55 to a whole number. In A, the decimal point in 1.55 is erased and moved mentally 2 places to the right, which gives 155; in B, the decimal point in 1.55 is kept but ignored and the position of the new decimal point is indicated by a caret. Similar changes are made in the dividend 4.8.

In ex. 1, be sure all students understand why 1 hr. 33 min. = 1.55 hr.; then discuss the explanation thoroughly. The caret should not be used until the students have a good understanding of the process.

Present problems involving large numbers and rounding them off. Lead a class discussion of solutions. Be sure that all students can read and work with large numbers.

Petroleum Production

1. In one 5-year period Texas produced these numbers of barrels of petroleum: 744,834,000; 829,874,000; 1,010,270,000; 1,022,139,000; 1,019,164,000. During the same 5 years California produced these numbers of barrels: 332,942,000; 327,607,000; 354,561,000; 359,450,000; 365,085,000. Find the average annual production of petroleum in each state for this 5-year period. 925,256,200 bbl. (Texas); 347,929,000 bbl. (California)

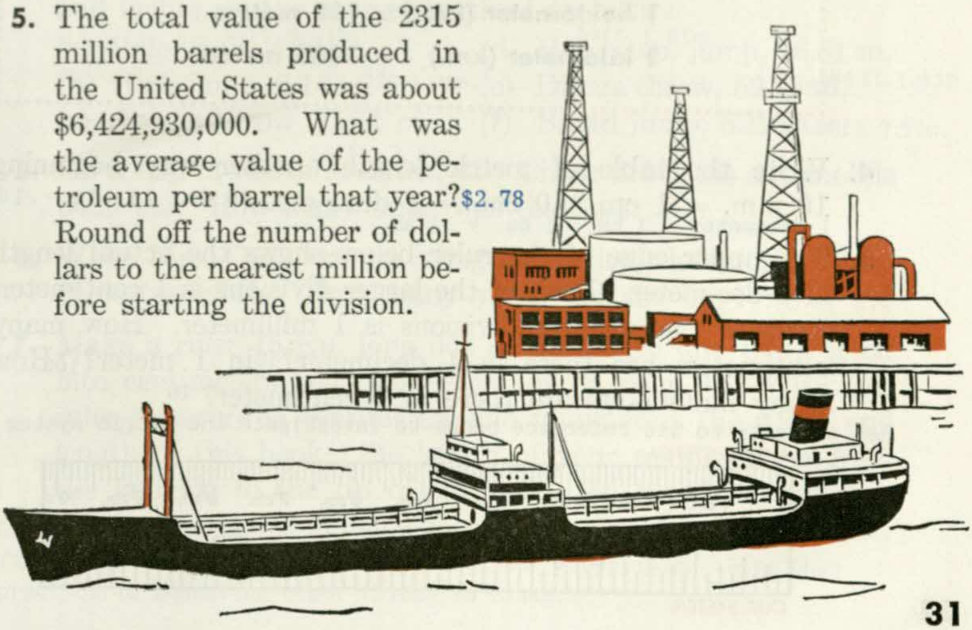
2. In ex. 1 the average annual production in Texas was about how many times as large as that in California? About 3

3. In a recent year Texas produced 974,275,000 barrels of petroleum, California produced 355,865,000 barrels, and Oklahoma, 185,851,000 barrels. If the United States produced a total of 2,314,988,000 barrels, find the production of petroleum in the rest of this country that year. 798,997,000 bbl.

4. In the year mentioned in ex. 3, the world production of petroleum was 5,006,205,000 barrels. What decimal part of the world production was that of the United States? .46

▶ Round off each number to the nearest million before dividing. This will simplify the work and give approximately the same result as would be obtained if the numbers were not rounded off.

5. The total value of the 2315 million barrels produced in the United States was about $6,424,930,000. What was the average value of the petroleum per barrel that year? $2.78 Round off the number of dollars to the nearest million before starting the division.



Present the introduction to the metric system and its uses (pages 32–35). Emphasize that only one basic unit of measurement of length (meter) is used in the metric system.

The Metric System

1. Instead of measuring length by the foot, yard, or mile, most countries of Europe, Asia, and South America use a system of measuring called the **metric system.** The metric system is also used in the United States and Great Britain by scientists, importers, manufacturers, etc.

2. The unit of length in the metric system is the **meter.** The meter equals 39.37 in. A meter may be divided into 10 equal parts called **decimeters,** or into 100 equal parts called **centimeters,** or into 1000 equal parts called **millimeters.**

3. The names of the various units of length are made by joining the word **meter** to the prefixes shown at the right. The table of **metric length** is given below: Have students notice that each unit is 10 times the next smaller one and $\frac{1}{10}$ of the next larger one.

milli $= \frac{1}{1000}$	deka $= 10$
centi $= \frac{1}{100}$	hekto $= 100$
deci $= \frac{1}{10}$	kilo $= 1000$

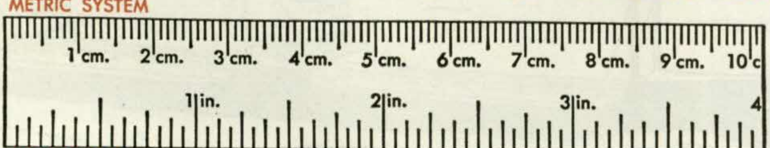
1 millimeter (mm.)	**= $\frac{1}{1000}$ meter (m.)**
1 centimeter (cm.)	**= $\frac{1}{100}$ meter**
1 decimeter (dm.)	**= $\frac{1}{10}$ meter**
1 dekameter	**= 10 meters**
1 hektometer (hm.)	**= 100 meters**
1 kilometer (km.)	**= 1000 meters**

4. Write the table of metric length another way beginning 10 mm. = 1 cm., 10 cm. = 1 dm., etc. 10 dm. = 1 m., 1 m. = .1 dekameter, 1 dekameter = .1 hm., 1 hm. = .1 km.

5. The upper edge of the ruler below shows the actual length of 1 decimeter. Each of the larger divisions is 1 centimeter. Each of the smallest divisions is 1 millimeter. How many centimeters are there in 1 decimeter? 10 in 1 meter? 100 How many millimeters are there in 1 centimeter? 10

Ask students to use reference books to investigate the metric system further.

METRIC SYSTEM



OUR SYSTEM

Using the Metric System

1. In countries that use the metric system, the length of a pencil is measured in centimeters, the length of a piece of cloth in meters, and the distance between two towns in kilometers. Remember these equivalents: If possible, display and contrast a meter stick and a yardstick.

> **1 meter (m.) = 39.37 in. or 1.1 yd.**
> **1 kilometer (km.) = .6 mi.**

Find the answers to the nearest tenth of a unit:

2. Among the events in the Olympic Games are the 110-meter hurdle(1) and the 1600-meter relay.(2) How many yards long is each of these races? (1) 121; (2) 1760

3. Other events in the Olympic Games are the 3000-meter steeplechase,(1) the 10,000-meter run,(2) and the 50,000-meter walk.(3) How many miles long is each event? First change the meters to kilometers and then to miles. (1) 1.8; (2) 6; (3) 30

4. Some of the best performances in the Olympic Games in track and field are given below. Express each distance in feet and inches, to the nearest tenth of an inch.

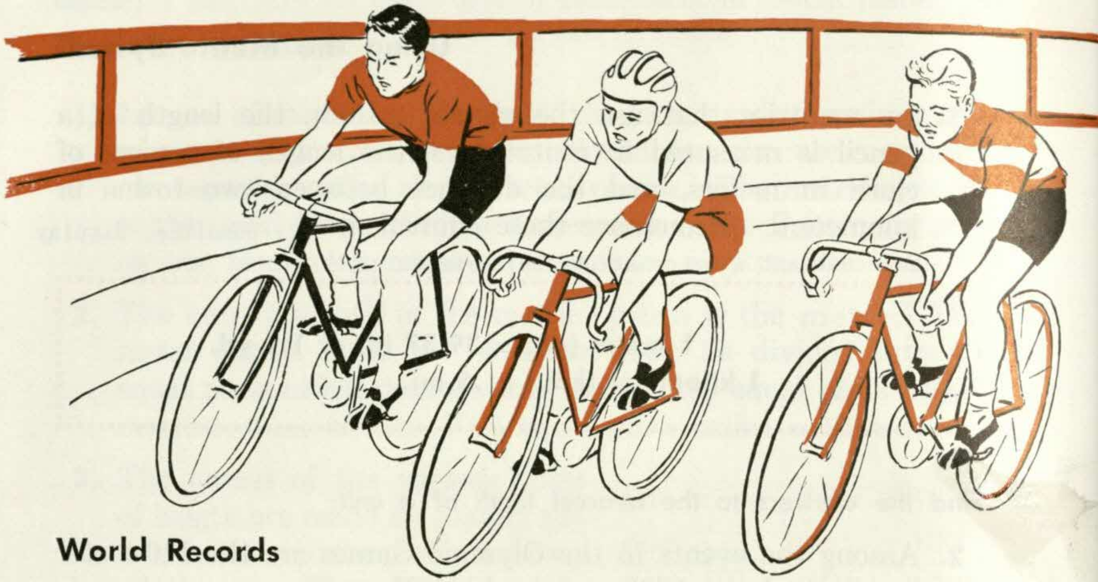
(a) Pole vault, 4.70 m. 15ft. 5.0in.
(b) High jump, 2.16 m. 7ft.1.0in.
(c) Javelin throw, 85.71 m. 281ft. 2.4in.
(d) Hop, step, jump, 16.81 m. 55ft. 1.8in.
(e) Discus throw, 59.18 m. 194 ft.1.9 in.
(f) Broad jump, 8.12 m. 26ft.7.7in.

5. On a trip in South America, Mr. Chase drove his automobile 3420 km. How many miles was that? 2052

6. When Mrs. Cook was traveling in Mexico, she bought 25 m. of cotton cloth. How many yards did she buy? 27.5 or $27\frac{1}{2}$

7. Make a ruler 15 cm. long out of cardboard and mark it off into centimeters and millimeters, as on page 32. With this ruler, measure in centimeters and millimeters the width and length of this book; the length of your middle finger from the knuckle to the tip of the nail; the length of the nail.

Lead a class discussion of the solutions to ex. 2–6. Have the students follow the directions in ex. 7, under your supervision. Give them practice in measuring other things in class.



World Records

1. The world record for the one-hour amateur bicycle race is held by E. Baldini of Italy. If he rode 46,394 m. in this time, how many miles, to the nearest tenth of a mile, did he ride? 27.8

2. The world swimming record of 18 min. 5.9 sec. for 1500 m. was held at one time by George Breen of the United States. How many yards did he swim in this time? 1650 About how many yards did he swim per minute? About 91

3. The world record for the one-hour walk is held by John Mikaelsson of Sweden. If he walked 13,812 m. in this time, how many miles, to the nearest tenth of a mile, did he walk? 8.3

4. A. Egorov of Russia holds the world record for the two-hour walk. If he walked 26,429 m. in that time, how many miles, to the nearest tenth of a mile, did he walk? 15.9 How many miles per hour did he walk? About 7.9

5. Pictures of many races are taken by amateur photographers. Small moving picture cameras use film that is 8 mm. or 16 mm. wide. Many of the projectors used in moving picture theaters use 35-millimeter film. Using the metric ruler on page 32, mark off these widths on paper. Then measure each width in inches, correct to the nearest 16th of an inch.

Additional world records in metric-system measurements may be found in <u>The World Almanac</u>. Use them to supplement the work on the page and have the students make up original problems using the information.

1. In the metric system, the unit used for measuring liquids is the **liter** (l.), which equals about 1 quart in our measure. The unit of weight is the **gram** (g.) though the unit most used is the **kilogram** (kg.), which equals 1000 grams. A kilogram equals about 2.2 lb. Remember these equivalents:

1 kilogram = about 2.2 lb. 1 liter = about 1 qt.

In countries using the metric system, milk is sold by the **liter,** while butter and meat are sold by the **kilogram.** The gram is used to weigh light objects like a letter. In this country the gram is often used to weigh foods.

2. Make a table of the units for measuring liquids by joining the prefixes **milli, centi,** etc., to the word **liter.** This table will be similar to the one for length on page 32. Also make a table of the units of metric weight by writing the usual prefixes before the word **gram.** See Guide.
3. Marie, who lives in France, writes that she weighs 45.4 kg. How many pounds does she weigh? 99.88
4. George, who lives in this country, weighs 121 lb. How many kilograms does he weigh? 55
5. Sue's aunt who lives in France buys 3 l. of milk each day. About how many quarts of milk is that? 3
6. On a motor trip in Mexico, Mr. Young needed about 15 gal. of gasoline. How many liters did he ask for? 60
7. Pedro lives in Brazil. On his shopping list yesterday were $\frac{1}{2}$ l. of olive oil, 700 g. of nuts, and $1\frac{1}{2}$ kg. of coffee. Tell, in our measures, about how much of each he bought.
   1 pt. olive oil; $1\frac{1}{2}$ lb. nuts; $3\frac{1}{3}$ lb. coffee
8. On trips by air to Europe, each first class passenger is allowed, free of charge, to take baggage weighing 30 kg. How many pounds of baggage is that? 66 A tourist class passenger is allowed 20 kg. of baggage. How many pounds is that? 44

Discuss ex. 1-2 with the class. In ex. 2 begin part of the tables on the board and have students complete them. Do problems with the class and be sure all know why they divide or multiply when converting and which process to use.

Present problems on the cost of operating a car. Have volunteers explain the solutions at the board.

Cost of Operating a Car

1. Mr. Hall bought a new car for \$2900. His expenses for operating the car for the first year were as follows: insurance, \$65.75; car license, \$9.00; driver's license, \$3.00; automobile club fee, \$15.00; antifreeze for radiator, \$5.50; repairs, greasing, washing car, etc., \$49.65. What was the total cost of these items for the year? \$147.90

2. During the first year, Mr. Hall paid \$230.95 for 775 gal. of gasoline and \$16.40 for oil. He also paid \$12.50 a month to rent a garage. Find the total of these items. \$397.35

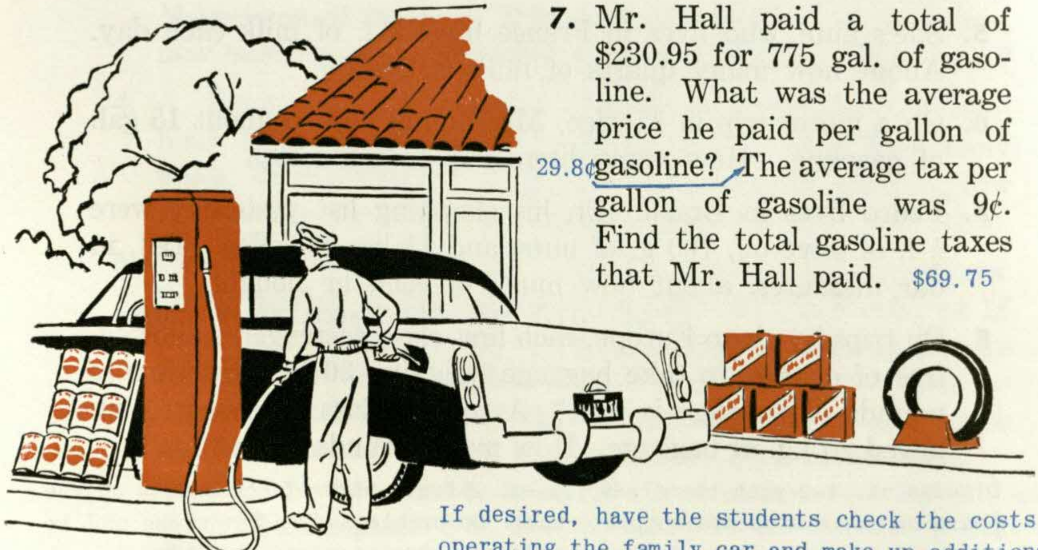
3. At the end of the year, due to the year's wear, the car was worth only \$2300. How much had the car lost in value? This loss in value is called **depreciation** and is one of the expenses of running a car. \$600

4. Find the total cost of operating the car for the first year by adding the expenses in ex. 1 to 3. \$1145.25

5. During the year Mr. Hall drove his car 12,972 mi. Find, to the nearest tenth of a cent, what it cost him per mile to operate his car. See ex. 4. \$.088 or 8.8¢

6. To drive 12,972 mi. Mr. Hall used 775 gal. of gasoline. Find, to the nearest tenth of a mile, how many miles he averaged per gallon of gasoline. 16.7

7. Mr. Hall paid a total of \$230.95 for 775 gal. of gasoline. What was the average price he paid per gallon of gasoline? 29.8¢ The average tax per gallon of gasoline was 9¢. Find the total gasoline taxes that Mr. Hall paid. \$69.75



If desired, have the students check the costs of operating the family car and make up additional problems for the class to solve.

A Problem Test

1. An airplane made a trip of 2500 mi. in 6 hr. 36 min. Find the speed of the airplane to the nearest tenth of a mile per hour. Change 6 hr. 36 min. to hours.
378.8

2. Last week Mr. Rose's cows gave these amounts of milk daily: 426.8 lb., 428.4 lb., 429.1 lb., 431.5 lb., 425.6 lb., 427.8 lb., 430.3 lb. Find the average number of pounds given per day. 428.5

3. How much change should Ann get from $10.00 after buying $6\frac{1}{2}$ lb. of meat at $.79 a pound? $4.86

4. Estimate the cost of $7\frac{7}{8}$ yd. of linen at $.98 a yard. $8.00 Then find the exact answer. $7.72

5. Find the charge for a trip of $2\frac{3}{5}$ mi. in a taxi if it costs 25¢ for the first $\frac{1}{5}$ mi. and 5¢ for each additional $\frac{1}{5}$ mi. 85¢

6. The school nurse measured the height of each pupil. Bill is 4 ft. 7 in. tall and Jim is 5 ft. 2 in. tall. Find the difference in their heights. 7 in.

7. The air distance from Mexico City to Rio de Janeiro is 8015 km. How many miles is this? 4809 1 km. = 0.6 mi.

8. Mr. Hill says it costs 9.8¢ a mile to run his car. How much did it cost him last year to run his car 8945 mi.? $876.61

9. The pupils in our school sold 960 tickets for the school play. 480 of them were 25-cent tickets, 320 were 35-cent tickets, and the rest were 50-cent tickets. The expenses were $78.70 in all. If the money left was divided equally among 5 school clubs, what was each club's share? $46.66

10. Susan read that back in the year 1853 a clipper ship sailed 424 nautical miles in 24 hr. What, to the nearest tenth of a mile per hour, was its speed? 17.7

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Check the papers carefully and analyze any errors. Have the students explain their solutions so you may determine the causes of errors (computational, lack of understanding of problem situations, and so on). Help them to correct their weaknesses.

Present this diagnostic test of the skills reviewed in Chapter 1, with practice-page references. See the Guide for a discussion of the construction of the test.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

						Practice Pages
1. Add. Check by going over your work:						
	$2\frac{5}{6}$ $\underline{6\frac{5}{6}}$ $9\frac{2}{3}$	$3\frac{5}{8}$ $\underline{4\frac{1}{2}}$ $8\frac{1}{8}$	$7\frac{1}{4}$ $\underline{4\frac{2}{3}}$ $11\frac{11}{12}$	$1\frac{5}{6}$ $\underline{8\frac{3}{4}}$ $10\frac{7}{12}$	$6\frac{3}{8}$ $\underline{2\frac{1}{6}}$ $8\frac{13}{24}$	15, 16
2.	2.35 6.04 3.28 4.42 16.09	1.875 6.500 2.750 9.000 20.125	7.333 2.125 0.875 6.375 16.708	$12.49 6.85 9.00 31.43 $59.77		21
Subtract. Check by going over your work:						
3.	9 $\underline{2\frac{1}{3}}$ $6\frac{2}{3}$	$7\frac{5}{6}$ $\underline{2\frac{1}{2}}$ $5\frac{1}{3}$	$8\frac{1}{4}$ $\underline{6\frac{3}{4}}$ $1\frac{1}{2}$	$4\frac{3}{16}$ $\underline{1\frac{7}{8}}$ $2\frac{5}{16}$	$5\frac{2}{3}$ $\underline{3\frac{11}{12}}$ $1\frac{3}{4}$	15, 16
4.	17.6 9.8 7.8	200.0 137.5 62.5	9.375 7.750 1.625	$40.00 18.46 $21.54		21

Multiply or divide and check your work:

				Practice Pages
5.	$\frac{3}{4} \times 18$ $13\frac{1}{2}$	$3\frac{3}{4} \times \frac{3}{10}$ $1\frac{1}{8}$	$6\frac{1}{4} \times 3\frac{3}{5}$ $22\frac{1}{2}$	17
6.	$\frac{4}{5} \times \frac{15}{16}$ $\frac{3}{4}$	$10 \div \frac{5}{16}$ 32	$\frac{7}{8} \div \frac{5}{12}$ $2\frac{1}{10}$	17, 18
7.	$2\frac{1}{2} \div 15$ $\frac{1}{6}$	$4\frac{1}{2} \div \frac{3}{16}$ 24	$4\frac{2}{3} \div 1\frac{1}{6}$ 4	18, 19
8.	$.6 \times 2.8$ 1.68	$.15 \times .25$ .0375	$.07 \times 1.2$ .084	26

Divide. In ex. 11 give the answers to the nearest hundredth:

					Practice Pages
9.	$8\overline{)43}$ 5.375	$9\overline{)78.3}$ 8.7	$5\overline{)29.7}$ 5.94	$36\overline{)1026}$ 28.5	27
10.	$.9\overline{)2.88}$ 3.2	$.5\overline{).178}$ .356	$.4\overline{)8.64}$ 21.6	$.25\overline{)2965}$ 11,860	29
11.	$.6\overline{)1.61}$ 2.68	$.09\overline{)3.47}$ 38.56	$.27\overline{)6.32}$ 23.41	$5.2\overline{)7.748}$ 1.49	30

Correct and analyze the students' papers to determine the kinds of errors. Help the students to analyze the causes of errors. Clear up any difficulties before assigning remedial work.

See the Guide for the specific aims of Chapter 2 and a discussion of teaching percentage.

Chapter 2

Review the meaning of per cent as taught in Grade 7 (pages 39–41).

Buying at a Sale

1. Bob was given $10 for his birthday. He decided to buy a new tennis racket with the money. His father told him that he could buy a better racket for the same money if he waited for a sale. In September the Sport Center had a sale at which the prices of many things were reduced. All tennis equipment was sold at "20% off." Did Bob have enough money to buy a racket whose regular price was $12.00? Yes How much would this racket cost? $9.60

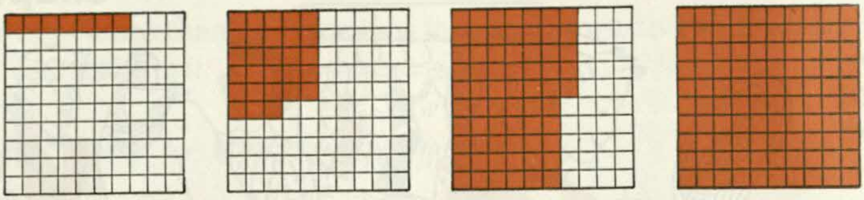
▶ When things are sold at "20% off," they are sold at 20% less than the regular price. 20% of $12 is $2.40. So the price of the racket is $12.00 – $2.40, or $9.60.

Have volunteers explain why 20% of $12 is $2.40.

2. Beach wear was also sold at "20% off" in the sale at the Sport Center. What would Jean pay for a swimsuit with a regular price of $7.50? $6.00 What would Grace pay for a beach coat with a regular price of $6.00? $4.80

Review the fact that per cent tells how many out of 100, or how many hundredths. Discuss the meaning of the % symbol. Ex. 1–2 should be done as a class activity.

What Per Cent Means



A B C D

1. Each large square above is divided into 100 small squares. One small square is $\frac{1}{100}$ of the large square. What per cent of each large square is each small square? 1%

2. In square A 7 small squares are red. What per cent of square A is red? 7% How many small squares in A are white? 93 How can you find the number of white squares without counting them? What per cent of A is white? 93%
Subtract 7 from 100.

3. Look at squares B and C and tell what per cent of each is red and what per cent of each is white. B: 28% red, 72% white; C: 65% red, 35% white.

4. In square D you see that **all** the small squares, or 100 small squares, are red. This means that $\frac{100}{100}$ or 100% of square D is red. You see that 100% of square D means **all** of square D. Emphasize.

5. Draw 3 large squares like squares A, B, and C shown above. Divide each of your squares into 100 small squares. To do this, mark off 10 equal spaces on each side of the large square. By drawing lines you will then have 100 small squares in each large square. Color red 19% of the first square, (1) 35% of the second square, (2) and 60% of the third square. (3) What per cent of each square is not red? (1) 81%; (2) 65%; (3) 40%

6. Make 2 more squares like those in ex. 5. Color red all but 25% of the first square (1) and all but 12% of the second square. (2) What per cent of each square is red? (1) 75%; (2) 88%

7. Draw some squares and divide each into 100 small squares. Draw designs in each square by coloring some of the small squares; then tell what per cent of each square is colored and what per cent is white.

Discuss ex. 1-4 with the class, emphasizing the meaning of per cent as "so many out of 100." Supply the students with graph paper to use for ex. 5-7.

The explanation in ex. 7 may be supplemented as follows: $\frac{1}{3}$ may be changed to a 2-place decimal ($1 \div 3 = .33\frac{1}{3}$); then the decimal may be expressed as a per cent ($.33\frac{1}{3} = 33\frac{1}{3}\%$).

More about Per Cents

1. You have learned that **per cent** is another way of writing **hundredths,** and that the symbol % means **per cent.** Stress.

$$5\% = \frac{5}{100} = .05 \qquad 39\% = \frac{39}{100} = .39$$

Since $5\% = .05$, you see that a per cent can be changed to a decimal. In working problems you frequently change a per cent to a decimal, or a decimal to a per cent.

2. Change each fraction to a per cent: $\frac{16}{100}$ 16%; $\frac{2}{100}$ 2%; $\frac{81}{100}$ 81%; $\frac{100}{100}$ 100%.

3. Write each decimal as a per cent: .61 61%; .03 3%; .29 29%; .93 93%.

4. Write each of the following as a decimal: 25% .25; 72% .72; 4% .04; 19% .19; 7% .07; 1% .01; $\frac{21}{100}$ .21; $\frac{8}{100}$ .08; $\frac{84}{100}$ .84; 23 per cent .23; 6 per cent .06.

5. Sometimes you need to change a per cent like 55% to a common fraction in lowest terms, as shown at the right. Tell what was done to both terms of $\frac{55}{100}$ to get $\frac{11}{20}$.

$$55\% = \frac{55}{100} = \frac{11}{20}$$

6. Change each per cent to a common fraction in lowest terms: 75% $\frac{3}{4}$; 35% $\frac{7}{20}$; 85% $\frac{17}{20}$; 72% $\frac{18}{25}$; 25% $\frac{1}{4}$; 30% $\frac{3}{10}$; 80% $\frac{4}{5}$; 15% $\frac{3}{20}$; 20% $\frac{1}{5}$.

7. Since the whole of anything is represented by 100%, then $\frac{1}{3}$ of anything is $\frac{1}{3}$ of 100%, or $33\frac{1}{3}\%$, as shown at the right. So $33\frac{1}{3}\% = \frac{1}{3}$.

$$3\overline{)100} = 33\tfrac{1}{3}$$

Show, as in ex. 7, that each of the following is true:

8. $\frac{1}{8} = 12\frac{1}{2}\%$ $\qquad \frac{3}{8} = 37\frac{1}{2}\%$ $\qquad \frac{5}{8} = 62\frac{1}{2}\%$ $\qquad \frac{7}{8} = 87\frac{1}{2}\%$

9. $\frac{1}{3} = 33\frac{1}{3}\%$ $\qquad \frac{2}{3} = 66\frac{2}{3}\%$ $\qquad \frac{1}{6} = 16\frac{2}{3}\%$ $\qquad \frac{5}{6} = 83\frac{1}{3}\%$

10. Memorize these unless you already know them:

Have some students make a class chart of the following:

$25\% = \frac{1}{4}$	$20\% = \frac{1}{5}$	$12\frac{1}{2}\% = \frac{1}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$50\% = \frac{1}{2}$	$40\% = \frac{2}{5}$	$37\frac{1}{2}\% = \frac{3}{8}$	$66\frac{2}{3}\% = \frac{2}{3}$
$75\% = \frac{3}{4}$	$60\% = \frac{3}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$
$100\% = 1$	$80\% = \frac{4}{5}$	$87\frac{1}{2}\% = \frac{7}{8}$	$83\frac{1}{3}\% = \frac{5}{6}$

Emphasize that 5% is another way of writing $\frac{5}{100}$ or .05 and that any one of these may be exchanged for another. Ex. 1–7 should be done as a class activity.

Finding Percentages

Reteach finding a per cent of a number.

1. Problem Last year 15% of the graduating class at Redfield School were on the Honor Roll. If 240 pupils were in the class, how many pupils were on the Honor Roll? 36

Explanation To find 15% of 240 pupils, first change 15% to .15. Then multiply 240 by .15, which gives 36.00, or 36. There were 36 pupils on the Honor Roll. Check the work by going over it.

$$\begin{array}{r} 240 \\ \underline{.15} \\ 12\,00 \\ \underline{24\,0} \\ 36.00 \end{array}$$

2. Mr. Bush earned \$380 last month. He saved 25% of his salary. How much did he save? \$95

▶ To find 25% of \$380, you can multiply \$380 by .25 or take $\frac{1}{4}$ of \$380 since 25% = $\frac{1}{4}$. It is shorter to multiply by $\frac{1}{4}$ than by .25.

3. About 71% of the earth's surface is water. If the area of the earth's surface is 196,950,000 sq. mi., find the number of square miles of water surface. 139,834,500

4. Betty saved $33\frac{1}{3}$% of the regular price of a winter coat by buying it in February. If the regular price of the coat was \$45.75, how much money did Betty save? \$15.25

5. Every percentage problem has three parts, the **base,** the **rate,** and the **percentage.** In ex. 1, the base is 240 pupils, the rate is 15%, and the percentage is 36 pupils. What are the base, rate, and percentage in ex. 2–4?

Remind the students to use common-fraction equivalents when possible.

Find the answers correct to the nearest cent: *

6.	38% of \$25	\$9.50	2% of \$15.50	\$.31	$37\frac{1}{2}$% of \$3800	\$1425
7.	75% of \$76	\$57	8% of \$28.63	\$2.29	$33\frac{1}{3}$% of \$3240	\$1080
8.	11% of \$86	\$9.46	4% of \$92.12	\$3.68	$62\frac{1}{2}$% of \$2500	\$1562.50
9.	20% of \$33	\$6.60	1% of \$28.82	\$.29	$16\frac{2}{3}$% of \$9600	\$1600
10.	15% of \$95	\$14.25	5% of \$74.75	\$3.74	$12\frac{1}{2}$% of \$2230	\$278.75
11.	31% of \$65	\$20.15	6% of \$95.98	\$5.76	$66\frac{2}{3}$% of \$1775	\$1183.33

More Practice. See 21 on page 368.

*Review the following with the class: If the figure in the third decimal place is 5 or more, drop it but make the figure before it 1 larger; if it is less than 5, drop it.

Extend the use of per cents to those which include decimals.

What Does 19.3% Mean?

1. Problem In a recent year 28,000 persons were killed by accidents in their homes, and 19.3% of these deaths were caused by burns. How many deaths were caused by burns? 5404

Explanation First change 19.3% to a decimal. Write 19.3% as $\frac{19.3}{100}$ and then divide 19.3 by 100 by moving the decimal point two places to the left. This gives .193. So 19.3% equals .193. To find the number of deaths caused by burns, multiply 28,000 by .193, which gives 5404. There were 5404 deaths caused by burns.

Here is another way to show that 19.3% = .193. You know that 19.3% = $\frac{19.3}{100}$. Multiply both terms of $\frac{19.3}{100}$ by 10, which gives $\frac{193}{1000}$, or .193.

Stress the following after understanding is assured:

To change a per cent to a decimal, drop the sign % and move the decimal point two places to the left.

Write these per cents as decimals:

2. 17.3% .173 24.9% .249 92.5% .925 45.23% .4523 2.6% .026 3.87% .0387

3. Make up a rule for changing a decimal to a per cent.

Students should explain the validity of the rule as is done in ex. 1.

Write these decimals as per cents:

4. .531 53.1% .973 97.3% .021 2.1% .935 93.5% .015 1.5% .0773 7.73%

5. In ex. 1 the ages of 54.3% of the 28,000 persons killed by accidents in the home were 65 or over. How many persons 65 or over were killed in this way? 15,204

Find the answers correct to the nearest whole number:

6. 17.7% of 720 127	8.4% of 972 82	27.5% of 2300 633
7. 11.6% of 385 45	2.9% of 118 3	16.3% of 2485 405
8. 12.5% of 326 41	1.3% of 450 6	22.8% of 1300 296

More Practice. See 22 on page 368. Use to reinforce understanding.

When discussing ex. 1, be sure the students know why 19.3 is written over 100. Also, ask why we can multiply both terms of $\frac{19.3}{100}$ by 10. Give more practice like that in ex. 2 and 4 before assigning ex. 6-8.

Reteach estimating answers in certain types of percentage problems. First give a short drill on frequently used per cents and their fractional equivalents (ex. 10, page 41). A review of rounding off numbers may be needed also.

Estimating Answers

Always make a rough estimate of the answer to a problem before working it; by doing this you avoid foolish mistakes.

1. Problem There are about 5900 families in Grand City and 76% of them own their homes. How many families own their homes in Grand City? 4484

Explanation A rough estimate of the answer can be made by thinking of 5900 as 6000 and of 76% as 75%. Then think of 75% as $\frac{3}{4}$ and find $\frac{3}{4}$ of 6000, which gives 4500. This shows that the exact answer should be somewhere near 4500. The work at the right gives 4484 as the answer. Since 4484 is close to 4500, 4484 is a reasonable answer.

$$\begin{array}{r} 5900 \\ .76 \\ \hline 354\ 00 \\ 4130\ 0 \\ \hline 4484.00 \end{array}$$

2. In estimating a per cent of a number, it often helps to use a common fraction whose value is close to the given per cent. For example, for 32% use $\frac{1}{3}$, which equals $33\frac{1}{3}\%$. Stress. Stress.

3. There are 75 pupils in the graduating class of the high school and 48% $(50\% = \frac{1}{2})$ of them are going to college. Estimate the number who are going to college. 38 pupils

4. Mr. March's salary is \$5100 (\$5000) and he plans to save 12% $(12\frac{1}{2}\% = \frac{1}{8})$ of it. Estimate the amount he plans to save. \$625

5. Out of a total of 133,600,000 (134,000,000) telephones in the world in a recent year, about 57% $(60\% = \frac{3}{5})$ were in North America. How many telephones were there in North America that year? 80,400,000

6. Jack saw a tennis racket that was marked down $33\frac{1}{3}\%$ $(\frac{1}{3})$ from its regular price of \$9.25 (\$9.30). Estimate the amount he could save on this racket. \$3.10

Have the students explain their estimated answers in ex. 7–9.

Estimate the answers. (1) **Then find the exact answers:** (2)

(1) (2) **7.**	21% of \$498 (\$100) \$104.58	65% of \$900 (\$600) \$585	24% of \$199 (\$50) \$47.76
8.	32% of \$903 (\$300) \$288.96	74% of \$399 (\$300) \$295.26	34% of \$270 (\$90) \$91.80
9.	11% of \$196 (\$20) \$21.56	37% of \$800 (\$300) \$296	49% of \$604 (\$300) \$295.96

More Practice. See 23 on page 369. Use for further reteaching.

Ask the students what common fraction they would use in estimating with each of the following per cents: 13%, 19%, 11%, 24%, 34%, 67%, 76%, 85%, 36%.

Introduce per cents less than 1% expressed in fractional or decimal form. Use a diagram similar to the one on page 40 to illustrate the meaning of a fraction of 1%.

Per Cents Less than 1%

1. Judy read that in a recent year only .3% of all the school accidents that happened to eighth grade pupils occurred in science laboratories. Out of 5000 accidents how many, on the average, occurred in science laboratories? 15

▶ Change .3% to a decimal by moving the decimal point two places to the left, which gives .003. Then multiply 5000 by .003.

2. Judy also read that .2% of all school accidents to eighth grade pupils occurred while pupils were playing hockey. Out of 2500 accidents how many, on the average, were of this kind? 5

3. The per cent .3% is often written as 0.3% in order to call attention to the decimal point. Another way to write .3% is $\frac{3}{10}$ of 1%, or $\frac{3}{10}$%. Tell two other ways to write each of the following: $\frac{3}{4}$%; $\frac{1}{2}$ of 1%; 0.7%; $\frac{1}{4}$ of 1%; .9%; $\frac{4}{10}$%. See Guide.

4. Find $\frac{3}{4}$ of 1% of $600. $4.50

▶ First find 1% of $600, which is $6.00. Then find $\frac{3}{4}$ of $6.00.

5. Draw a large square. Divide your square into 100 small squares. Then color red $\frac{1}{2}$ of 1% of your large square.

Have the students show other fractions of 1% ($\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, and so on.)

6. In the Star Store, employees pay $\frac{1}{2}$ of 1% of their salaries into a special activities fund. If Mary Lee earns $308 a month, how much per month does she pay into this fund? $1.54

Have the students use different methods (1% of $600, then $\frac{1}{4}$ of $6.00; or $\frac{1}{4}$ of 1% = .25% = .0025, and so on) to find answers.

Find the answers:

7.	$\frac{1}{4}$ of 1% of $600 $1.50	.6% of $2500 $15	0.4% of $1750 $7
8.	$\frac{3}{4}$ of 1% of $280 $2.10	.2% of $3670 $7.34	0.5% of $2220 $11.10
9.	$\frac{1}{5}$ of 1% of $920 $1.84	.8% of $1850 $14.80	0.3% of $1680 $5.04
10.	$\frac{1}{10}$ of 1% of $400 $.40	$\frac{3}{10}$% of $1200 $3.60	0.1% of $1600 $1.60
11.	$\frac{2}{3}$ of 1% of $750 $5	$\frac{3}{4}$% of $16.00 $.12	0.7% of $1000 $7
12.	$\frac{1}{2}$ of 1% of $324 $1.62	$\frac{4}{5}$% of $35.00 $.28	0.8% of $2500 $20

Have the students explain their solutions to ex. 7-12.

More Practice. See 24 on page 369.

In ex. 1 have the students explain why the decimal point is moved (since per cents are hundredths, .3% = $\frac{.3}{100}$ = .003). Emphasize the difference between 3% and .3%, $\frac{1}{4}$(25%) and $\frac{1}{4}$%, and so on. Be sure the students understand why .3%= $\frac{3}{10}$ of 1%= $\frac{3}{10}$%.

Present applications of percentage in problems.

Our Cotton Crop

1. In a recent year the United States raised 30.5% of the total world production of cotton. If the world production that year was approximately 46,840,000 bales, about how many bales of cotton were raised in the United States? 14,286,200

2. Texas is the leading cotton-growing state of our country. If that state produced 33.2% of the bales of cotton grown in the United States in the year mentioned above, how many bales did it produce? 4,743,018 In working this problem, use the answer you found in ex. 1.

3. One year these eight states produced the following per cents of the cotton crop of the United States: Texas, 28.8%; Mississippi, 11.5%; Arkansas, 9.9%; Alabama, 5.3%; Georgia, 4.5%; South Carolina, 3.7%; North Carolina, 2.7%; Oklahoma, 2.1%. What per cent of the whole crop did these eight states produce? 68.5% What per cent was produced in the rest of the United States? 31.5%

4. One year the United States produced 45.1% of the world production of cotton; India produced 14.7%; China produced 10.8%; Russia, 8.3%; Egypt, 6.0%; and Brazil, 5.7%. What per cent of the total production did these countries together produce? 90.6% What per cent was produced by the rest of the world? 9.4%

5. In the year mentioned in ex. 4, the total world production of cotton was 34,500,000 bales. About how many bales were produced by each of the countries listed in ex. 4? (1) How many bales was this all together? (2) (1) U.S., 15,559,500; India, 5,071,500; China, 3,726,000; Russia, 2,863,500; Egypt, 2,070,000; Brazil, 1,966,500; (2) 31,257,000

6. One year the United States crop of 13,696,000 bales of cotton had a farm value of $2,301,212,000. Find, to the near-

If many students seem weak on a particular skill, class reteaching may be necessary. Have the students explain their work so that you may judge causes of difficulties.

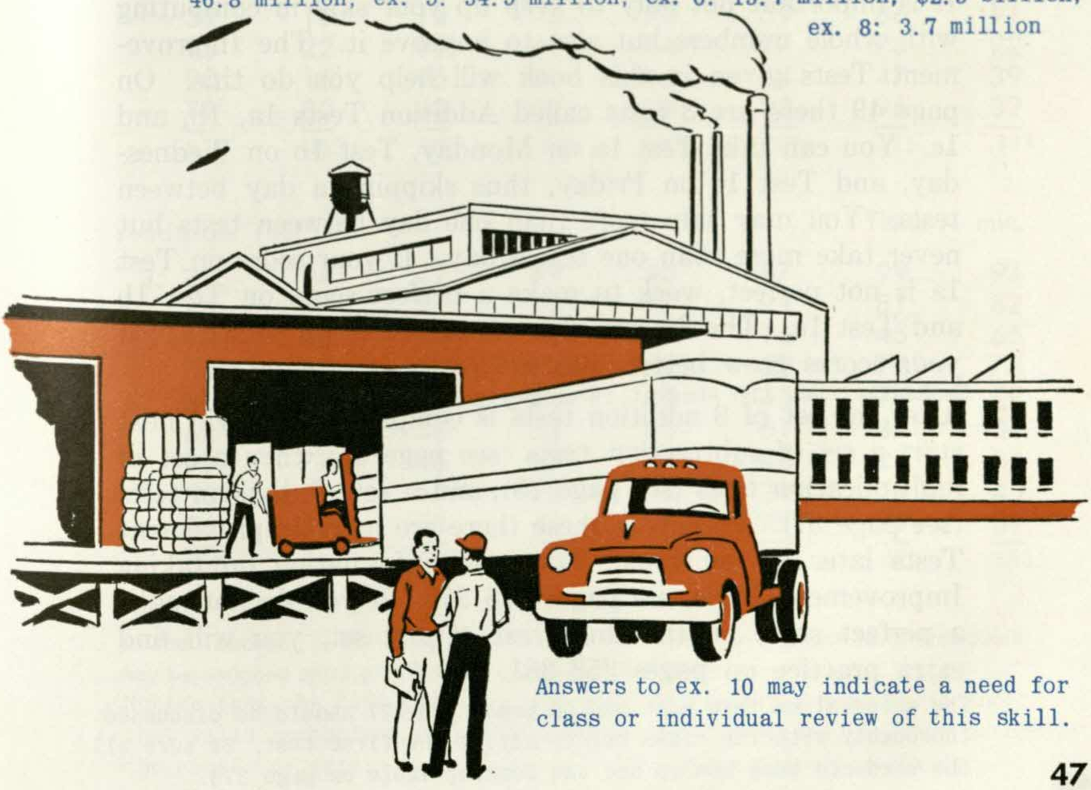
est dollar, the farm value of 1 bale of cotton. Write both numbers to the nearest hundred thousand before dividing. $168

7. If a farmer has 24 acres of land planted with cotton and gets a total of 7844 lb. of cotton from this land, find, to the nearest pound, the average yield of cotton per acre. 327

8. Recently, there were 3,708,000 farms in the United States. Approximately 6.5% of these farms were cotton farms. Find the number of cotton farms to the nearest thousand. 241,000

9. About 19.3% of our cotton farms are in Mississippi and about .8% of them are in Arizona. Using your answer to ex. 8, find the number of cotton farms in each of these states.
Mississippi, 46,513; Arizona, 1928

10. In a recent year 12,971,608,000 yards of woven goods were made in the United States. Of this amount, 10,089,843,000 yards were made of cotton. You have learned to write large numbers in this form: 6.3 million; 4.7 billion. Write the above numbers (1) and also the large numbers in ex. 1, 5, 6, and 8 (2) in this form. (1) 13.0 billion; 10.1 billion; (2) ex. 1: 46.8 million; ex. 5: 34.5 million; ex. 6: 13.7 million, $2.3 billion; ex. 8: 3.7 million



Answers to ex. 10 may indicate a need for class or individual review of this skill.

Show how to take and score improvement tests. See the Guide for a discussion. Have pupils make their own Record Books (see the Guide and page 377).

Practice Makes Perfect

1. It is important not only to keep up your skill in computing with whole numbers but also to improve it. The Improvement Tests given in this book will help you do this. On page 49 there are 3 tests called Addition Tests 1a, 1b, and 1c. You can take Test 1a on Monday, Test 1b on Wednesday, and Test 1c on Friday, thus skipping a day between tests. You may skip more than one day between tests but never take more than one test a day. If your score on Test 1a is not perfect, work to make a perfect score on Test 1b and Test 1c. The 3 tests of a set are of equal difficulty; if your scores grow better, you are really improving.

Emphasize that the student is competing only with himself.

2. After the set of 3 addition tests is completed, you will next start a set of subtraction tests (see page 65), then a set of multiplication tests (see page 75), and a set of division tests (see page 87). Following these there are more Improvement Tests later in the book. For more information on taking Improvement Tests, see pages 375–377. If you do not make a perfect score on the third test of any set, you will find extra practice on pages 358–361.

The material on this page and on pages 375-377 should be discussed thoroughly with the class before giving the first test. Be sure all the students know how to use the Scoring Table on page 376.

Present the first set of improvement tests in addition. Have all the students determine and record their own scores.

Improving by Practice

Addition Test 1a.

Time: 4 min.

1.	76	24	12	56	29	97	93	55
	36	37	31	57	51	75	31	72
	47	28	54	12	97	56	42	26
	63	97	96	83	69	47	78	88
	59	48	58	45	44	99	23	74
	23	14	28	58	11	12	63	87
	57	26	84	77	38	56	40	35
	44	33	62	88	95	91	36	10
	25	99	42	69	65	67	16	83
	430	406	467	545	499	600	422	530

8

Addition Test 1b.

Time: 4 min.

2.	51	94	53	87	13	20	25	67
	74	93	78	82	57	67	60	51
	88	74	20	36	60	51	97	25
	68	25	67	58	88	96	32	46
	42	39	32	21	63	63	53	50
	39	10	15	78	76	39	86	91
	95	22	99	41	84	77	75	92
	90	16	96	80	65	64	47	39
	71	80	49	49	98	28	96	32
	618	453	509	532	604	505	571	493

8

Addition Test 1c.

Time: 4 min.

3.	38	94	42	89	45	87	99	94
	46	57	27	78	37	99	81	82
	13	48	34	86	85	15	45	65
	74	35	95	72	69	88	72	78
	85	84	49	10	12	33	77	99
	64	61	38	62	85	61	85	67
	99	77	46	63	86	45	24	34
	80	19	57	58	60	24	59	45
	56	30	73	43	97	89	29	89
	555	505	461	561	576	541	571	653

8

To the Teacher. If individual pupils get low scores on Improvement Tests, such pupils may be assigned extra practice given on pages 358–361.

Notice that the time for each test must be exactly 4 minutes in order that valid comparison of results may be made. Help the students determine the causes of errors, and clear up difficulties before assigning remedial work.

Reteach the meaning and use of large per cents. Extend the topic to those ending in decimals. First give oral practice in finding 100% of various numbers.

Using Large Per Cents

1. You know that 100% of a number means **all** of the number. 100% of a number also means 1 times the number since $100\% = \frac{100}{100} = 1.00$ or 1. In the same way, $200\% = \frac{200}{100} = 2.00$ or 2; 200% of a number means 2 times the number. What is the meaning of 300%? 3 700%? 7 400%? 4

2. How many pupils are there in your class? What is 100% of the pupils in your class? 200% of them? 500%?

3. If a hotel containing 243 rooms is 100% rented, how many of the rooms are rented? 243

4. To find 175% of \$1800, change 175% to 1.75. Then multiply \$1800 by 1.75. This gives \$3150. Why should the answer be more than \$1800? Bring out the fact that 175% is 100% plus 75%. The answer therefore equals the original number plus another amount.

Write the following per cents as decimals:

5. 300% 3.00 900% 9.00 432% 4.32 348% 3.48 220% 2.20 1200% 12.00

In ex. 5 have the students explain why 300% = 3.00 ($300\% = \frac{300}{100} = 3.00$).

Write these decimals as per cents:

6. 1.93 193% 2.65 265% 15.15 1515% 5.91 591% 4. 400% 1.125 112.5%

7. 3.24 324% 4.13 413% 27.04 2704% 3.87 387% 17. 1700% 2.375 237.5%

8. The summer population of Ocean Village is about 180% of the winter population. If the winter population is 225, about what is the summer population? 405

9. This year the attendance at the outdoor music festival was 210% as large as it was last year. If 1230 people attended last year, how many people were there this year? 2583

Find answers to the nearest cent:

10. 200% of \$378 \$756 160% of \$284 \$454.40 135.5% of \$170 \$230.35

11. 125% of \$250 \$312.50 230% of \$337 \$775.10 216.3% of \$225 \$486.68

12. 350% of \$196 \$686 111% of \$629 \$698.19 $112\frac{1}{2}$% of \$125 \$140.63

More Practice. See 25 on page 369.

If the students have difficulty in visualizing per cents greater than 100%, use graph paper (sheets with 100 squares on each) to illustrate different per cents.

Reteach finding what per cent one number is of another.

Comparing Numbers by Per Cents

1. Problem The Rovers won 13 out of 20 basketball games. What per cent of the games did they win? 65%

Explanation They won $\frac{13}{20}$ of the games. Change $\frac{13}{20}$ to a fraction having 100 for its denominator, by multiplying both terms by 5, which gives $\frac{65}{100}$. Then change $\frac{65}{100}$ to 65%. The Rovers won 65% of the games they played.

2. Problem If your team plays 40 games and wins 29 of them, what per cent of the games do they win? $72\frac{1}{2}$%

Explanation In this problem it is not easy to change $\frac{29}{40}$ to a per cent by multiplying both terms of $\frac{29}{40}$ by the same number. Instead, change $\frac{29}{40}$ to a decimal, carrying the quotient only to 2 decimal places (hundredths) since per cent means hundredths. The remainder of 20 gives $\frac{20}{40}$, or $\frac{1}{2}$, as a fraction in the quotient. The quotient is $.72\frac{1}{2}$, so $\frac{29}{40} = .72\frac{1}{2} = 72\frac{1}{2}\%$. Your team won $72\frac{1}{2}\%$ of the games. Check by finding $72\frac{1}{2}\%$ of 40; it should be 29.

$$\begin{array}{r} .72\frac{1}{2} \\ 40\overline{)29.00} \\ \underline{28\ 0} \\ 1\ 00 \\ \underline{80} \\ 20 \end{array}$$

Emphasize after understanding is assured.

To find what per cent one number is of another, find what fractional part the one number is of the other; change this fraction to hundredths and then to a per cent.

Tell the number to put in each space. Check the work:

3. 12 is . 30 . % of 40 6 is . 2 . % of 300 24 is . $33\frac{1}{3}$ % of 72

4. 11 is . 55 . % of 20 9 is . $7\frac{1}{2}$ % of 120 37 is . $46\frac{1}{4}$ % of 80

5. 26 is . $32\frac{1}{2}$ % of 80 5 is . 4 . % of 125 60 is . $66\frac{2}{3}$ % of 90

Find what per cent the first number is of the second:

6. 18, 90 20% 13, 40 $32\frac{1}{2}$% 11, 25 44% 65, 65 100% 75, 60 125%

7. 51, 68 75% 22, 30 $73\frac{1}{3}$% 13, 80 $16\frac{1}{4}$% 15, 125 12% 450, 300 150%

8. 23, 25 92% 18, 20 90% 15, 48 $31\frac{1}{4}$% 18, 400 $4\frac{1}{2}$% 378, 270 140%

More Practice. See 26 on page 370. Use to reinforce skill.

To the Teacher. Caution pupils never to write in their books to fill in blank spaces.
In ex. 1-2 urge the students to suggest how to change $\frac{13}{20}$ and $\frac{29}{40}$ to per cents. Have them explain why they can do this operation.

Reteach finding a per cent when the result is found to the nearest whole per cent.

Nearest Whole Per Cent

1. Problem In the month of January there were 21 days that were sunny. What per cent of the days of January were sunny? Find the result to the nearest whole per cent. 68%

Explanation The sunny days were $\frac{21}{31}$ of the whole month. Change $\frac{21}{31}$ to a decimal as shown at the right. After carrying the division to two decimal places, you find you still have a remainder, so carry the work to one more decimal place to see whether the result is nearer to .67 or to .68. Since the quotient is .677, the result, to the nearest hundredth, is .68, which means 68%. So 68% of the days were sunny.

$$\begin{array}{r} .677 \\ 31\overline{)21.000} \\ 18\,6 \\ \hline 2\,40 \\ 2\,17 \\ \hline 230 \\ 217 \\ \hline \end{array}$$

To find a result correct to the nearest whole per cent, find the quotient to three decimal places; then round off to two places and change the two-place decimal to a per cent. Emphasize.

2. In February only 13 days out of the 28 days were sunny. What per cent of the days in February were sunny? Find the answer to the nearest whole per cent. 46%

3. At a sale, Jack bought a suit for $5 less than the regular price, which was $37. Find, to the nearest whole per cent, the per cent that Jack saved by this reduction. 14%

Change these decimals to the nearest whole per cent:

4.	.152 15%	.330 33%	.195 20%	.096 10%	.131 13%	1.084 108%	3.218 322%
5.	.467 47%	.059 6%	.291 29%	.008 1%	.245 25%	2.505 251%	7.063 706%

Find, to the nearest whole per cent, what per cent the first number is of the second:

6.	4, 17 24%	72, 79 91%	48, 133 36%	24, 21 114%	109, 73 149%
7.	3, 52 6%	28, 53 53%	19, 107 18%	59, 38 155%	228, 97 235%
8.	7, 29 24%	17, 33 52%	59, 102 58%	109, 135 81%	107, 86 124%
9.	9, 87 10%	19, 49 39%	73, 190 38%	111, 195 57%	106, 77 138%

More Practice. See 27 on page 370.

In discussing ex. 1, compare this solution with the one used on page 51. Emphasize that the quotient is always carried to one more place than is required in the answer, then rounded off.

Extend percentage to finding a per cent correct to the nearest tenth and hundredth (page 55) of 1%.

Which Record Is Better?

1. Problem The pupils in the Adams Junior High School publish a school magazine. There are 973 pupils in this school and 568 of them subscribe to the magazine. What per cent of the pupils subscribe to the magazine? Find the result to the nearest tenth of 1%. 58.4%

Explanation In this problem you are asked to find the result correct to the **nearest tenth** of 1%. To do this, carry the quotient
Stress why to 4 decimal places; do you get .5837? Change .5837 to a per cent by moving the decimal point 2 places to the right; this gives 58.37%. Round off 58.37% to the nearest tenth, which gives 58.4%. You see that 58.4% of the pupils subscribe to the magazine. To the nearest whole per cent, the result is 58%. You can see that 58.4%
Stress is a more accurate result.

2. The pupils in the Ford Junior High School also publish a school magazine. Of the 742 pupils in that school, 487 subscribe to the magazine. Find, to the nearest tenth of 1%, what per cent of the pupils subscribe to the magazine. 65.6%

3. In ex. 1 and 2, which school has the better record for getting subscriptions to its magazine? Bill says the Adams School shows the better record because it has more subscribers than the Ford School. Lucy says the Ford School has the better record because a larger per cent of its pupils are subscribers than in the Adams School. She says that 487 subscribers out of 742 pupils is better than 568 out of 973 pupils. Who is right, Bill or Lucy? Lucy Explain.

Change each decimal to a per cent correct to the nearest tenth of 1%:

4. .8953 89.5% .7169 71.7% .1034 10.3% .2231 22.3% .2085 20.9%

5. .0009 0.1% .5808 58.1% .2999 30.0% .8362 83.6% .2345 23.5%

Find, to the nearest tenth of 1%, which record is better. "96 from 189" means 96 subscriptions from 189 pupils:

6. 96 from 189 (50.8%) or 99 from 278 (35.6%) 745 from 888 (83.9%) or 856 from 915 (93.6%)

7. 88 from 173 (50.9%) or 93 from 206 (45.1%) 587 from 950 (61.8%) or 556 from 910 (61.1%)

Ex. 1 should be discussed carefully with the students. Be sure all understand the meaning of "to the nearest tenth of 1%."

Apply percentage to computing baseball standings.

Baseball Standings

1. Here is the record of the number of games won and lost one season by the teams of the American League. The table shows that New York won 96 games and lost 66 games, which gives New York the best record that season of any team in its league. A team's record, or **standing,** is best shown by finding what per cent the number of games it has won is of the number it has played. Since New York won 96 games and lost 66, it played 162 games in all. If you find, to the nearest tenth of 1%, what per cent 96 is of 162, you get 59.3%. So New York won 59.3% of all the games it played. This 59.3%, which is New York's **standing,** is written in the last column of the table as .593 and not as 59.3%.

AMERICAN LEAGUE

Team	Won	Lost	P. C.
New York	96	66	.593
Minnesota	91	71	.562
Los Angeles	86	76	.531
Detroit	85	76	.528
Chicago	85	77	.525
Cleveland	80	82	.494
Baltimore	77	85	.475
Boston	76	84	.475
Kansas City	72	90	.444
Washington	60	101	.373

▶ You might have expected that 59.3% would be written in the last column as 59.3 instead of .593 since that column is marked P. C. at the top; P. C. means per cent. But in all baseball records that give standings, batting averages, and pitching records per cents are written in the form of equivalent 3-place decimals. Stress.

2. In the above table, find the standing of each of the other teams. Carry the division to 4 decimal places and round off to 3 places, correct to the nearest thousandth. See the table.

In discussing ex. 1, emphasize the fact that the number of games won is compared with the total number of games played. Also point out in ex. 2 that the standing, when expressed as per cent, is correct to the nearest tenth of 1%.

In ex. 2 be sure the students understand why computations must be carried to an extra decimal place and that the resulting numbers, when rounded off, are correct to the nearest hundredth of 1%.

Baseball Standings

1. At one time during the season Team A had won 38 games and lost 57 games, while Team B had won 37 games and lost 56 games. Find the standing of each team only to the nearest whole per cent. 40 Can you tell which team is ahead? No Explain why the standings have to be found to the nearest tenth of 1%. What is the standing of each team? A, .400; B, .398
See the Guide.

2. Suppose that Team A had won 50 games and lost 48 when Team B had won 51 games and lost 49. Which team was ahead? A, .5102; B, .5100

▶ Sometimes you cannot tell which team is ahead if you find their standings to the nearest tenth of 1% (to the nearest thousandth). In such cases, carry the division to five decimal places and round off to the nearest hundredth of 1% (to the nearest ten-thousandth).

3. One season New York had won 58 games and lost 54 when Chicago had won 59 games and lost 55. Which team was ahead? New York, .5179; Chicago, .5175

4. Pittsburgh had won 65 games and lost 47 when St. Louis had won 69 games and lost 50. Which team was ahead?
Pittsburgh, .5804; St. Louis, .5798

5. Batting averages are also given in the form of 3-place decimals. One year Roger Maris had 159 hits out of 590 times at bat. Find his batting average. .269

6. A pitcher's record is given as a 3-place decimal. One year Whitey Ford was credited with winning 25 games out of 29 he pitched. Find his pitching record. .862

7. One year Mickey Mantle had 188 hits out of 533 times at bat. Find his batting average. .353

8. One year Skowron had 129 hits out of 478 times at bat, (1) and the same year Green had 168 hits out of 619 times at bat. (2) Find their batting averages. Which was the better record? Green's
(1) .270; (2) .271

9. Another year Aaron had 198 hits out of 615 times at bat, (1) and the same year Mays had 195 hits out of 585 times at bat. (2) Find their batting averages. Which was the better record? Mays's
(1) .322; (2) .333

Have the students check the standings of teams, as given in newspapers, by their own computations. They might compute also the standings of various school teams.

Point out that the amount of increase or decrease is always divided by the original number since that is the one increased or decreased.

Per Cent of Increase and Decrease

1. Problem During the past year the number of Boy Scouts in Weston increased from 155 members to 225 members. Find, to the nearest whole per cent, the per cent of increase in the membership of the Boy Scouts in Weston. 45%

Explanation The amount of increase is 70 members, which you get by subtracting 155 from 225. You must find what per cent 70 is of 155. To do this, divide 70 by 155, which gives 45% to the nearest whole per cent. So the membership increased 45%.

Stress. Stress. In ex. 1 you compared the **amount of increase,** 70, with the **original number,** 155, from which the increase started.

2. Recently the Boy Scouts of Weston have helped to direct traffic before school and after school. Serious accidents to school children have been reduced from 15 accidents last month to 6 this month. Find the per cent of decrease. 60%

▶ The amount of decrease is 9 accidents, which you find by subtracting 6 from 15. Find what per cent 9 is of 15. What per cent is it? This is the per cent of decrease in accidents to school children.

Stress. Stress. In ex. 2 you compared the **amount of decrease,** 9, with the **original number,** 15, from which the decrease started.

3. Last summer Don went to Scout Camp. When he left home he weighed 97 lb.; when he returned he weighed 109 lb. Find, to the nearest whole per cent, the per cent of increase. 12%

Find, to the nearest whole per cent, the per cent of increase (inc.) or decrease (dec.) that the second number shows over the first number:

4. 624, 702 13% inc. 3520, 3696 5% inc. $40.88, $76.65 88% inc.

5. 270, 138 49% dec. 1833, 2444 33% inc. $64.50, $60.85 6% dec.

More Practice. See 28 on page 370.

Stress the fact that the students must first find the amount of increase or decrease. Emphasize that this amount is then compared with (divided by) the original number.

Extend per cents of increase and decrease to large per cents and apply this work in problem solving (pages 57-58).

Large Increases and Decreases

1. Dick's salary was \$20 a week. A year later his salary was increased 100%. What was the amount of increase in his salary? \$20 What was his salary after the increase? \$40

2. In ex. 1, suppose Dick's salary of \$20 had been increased 150%. What would be the amount of the increase? \$30 What would be his salary after the increase? \$50

3. Problem Three years ago when Tom began to work he earned \$25 a week. Now he earns \$60 a week. What per cent has his salary increased during the 3-year period? 140%

Explanation The amount of increase in his salary is \$35 a week. The original salary is \$25. You must find what per cent the amount of increase, \$35, is of the original salary, \$25. Dividing \$35 by \$25, you get 1.40, which equals 140%. So Tom's salary was increased 140%. Emphasize that you must first find the amount of increase, and then compare this with the original number.

4. In ex. 3 the **amount of increase** in Tom's salary was \$35, which is **more** than his original salary of \$25. When the amount of increase is larger than the original number, is the per cent of increase more than or less than 100%? More If Tom's original salary of \$25 had increased 100%, what would the amount of the increase have been? \$25

5. Paul had \$60 in the bank. Now he has \$50 in the bank. What per cent have Paul's savings decreased? $16\frac{2}{3}$%

6. Suppose Paul's savings of \$60 had decreased 100%. What would his savings be then? \$0 Could his savings decrease more than 100%? No Could Paul's savings of \$60 increase more than 100%? Yes

7. Ex. 3 shows that a number can increase more than 100%; in fact, it can increase 200%, 500%, or even 1000%. But a number can never decrease more than 100%. Stress.

8. Mr. Page's salary has increased from \$50 a week to \$115 a week. What per cent has it increased? 130% Suppose his salary had decreased from \$50 a week to \$42 a week. What per cent would it have decreased? 16%

Be sure the students understand that when a number is increased by a per cent, the new value is the original value plus the increase (% of original value), not just the increase alone. The concepts in ex. 6-7 should be discussed thoroughly.

Problems

1. When the town of Mill Creek was settled in 1720, the population was only 75 people. Today the population is 900. What per cent has the population increased since 1720? 1100%

2. In June an automobile factory produced 7500 cars; in July the production dropped to 6375 cars. What per cent smaller was the July production than that in June? 15%

3. Shoes that sold for \$8.00 a pair 10 yr. ago are now selling at \$12.00 a pair. At what per cent more are the shoes selling now than 10 yr. ago? 50%

4. The population of New City is 12,500, while that of Bedford is 28,000. The population of Bedford is what per cent larger than that of New City? 124%

5. The rent of Mr. Clark's house was raised from \$95 a month to \$110 a month. What per cent, to the nearest whole per cent, was the rent raised? 16%

6. Pine Street School has an enrollment of 403 this year. Last year there were 369 pupils in the school. What per cent greater is the enrollment this year than last year? Find the answer to the nearest whole per cent. 9%

What per cent of increase (inc.) or decrease (dec.) does the second number show over the first number?

7. 270, 756 180% inc. 3640, 8372 130% inc. \$26.00, \$133.90 415% inc.

8. 238, 833 250% inc. 1400, 4368 212% inc. \$47.50, \$104.50 120% inc.

9. 243, 162 $33\frac{1}{3}\%$ dec. 1980, 5445 175% inc. \$195.25, \$117.15 40% dec.

More Practice. See 29 on page 370. Use for individual assistance. Have a class discussion of the solutions. Try to clear up difficulties now.

Reteach discount in the form in which it is used in retail stores.

Buying at a Discount

1. When merchandise is sold at less than the regular price, the reduction in price is called the **discount.** The regular price is called the **marked price;** the per cent taken off is called the **rate of discount;** and the price paid after subtracting the discount is called the **net price.** Discuss and illustrate the meaning of new words.

2. Today Mr. Swift bought a new motion picture camera. The camera usually sold for $147, but the salesman told Mr. Swift that it had been marked down to $98. How much was the discount on the camera? $49 What was the rate of discount? $33\frac{1}{3}\%$

3. Mr. Swift also bought a roll of color movie film marked $3.76 for $3.29. What rate of discount was that? 12%

4. At the end of the summer Mr. Gold, who runs the store, marked 60-dollar cameras down to $40. What was the rate of discount? $33\frac{1}{3}\%$ At that rate, what would he charge for a camera that usually sold for $45? $30

5. Mr. Gold also put signs in the window advertising these articles. Find the rate of discount on each: Have volunteers explain their solutions.

	Formerly	Now	
Exposure Meter	$21.00	$10.50	50%
Movie Projector	$150.00	$90.00	40%
Movie Screen	$20.00	$16.00	20%
Camera Stand	$10.00	$7.00	30%

Emphasize that the amount of discount and the original price (marked price) are compared to find the rate of discount. Have students make up similar problems based on advertisements of sales in newspapers.

Emphasize the difference between the list price and the net price.

Discounts in Business

1. When you do your shopping, you usually go to a **retail** store to buy the things you need. The retail merchant buys these things from a **wholesale** merchant or a manufacturer.

When the retailer orders merchandise, he looks up the prices in a catalogue that the wholesaler sends out. The wholesaler usually makes these **list prices** high. Then when he wants to change his prices, he sends out **discount sheets** to the retailers, stating what discounts he will allow on the list prices. These discounts are **trade** discounts. The list price less the discount is called the **net price.**

2. Mr. Brooks ordered supplies listed at \$250. He was given a trade discount of 26%. What was the net price? \$185

▶ First find the discount, which is 26% of \$250, or \$65. Then find the net price, which is \$250 − \$65, or \$185.

3. The Field School ordered textbooks listed at \$185, with a 25% discount. What net price did the school pay? \$138.75

4. When Coach Mason ordered uniforms for the football team, he received a discount of 25% for ordering a large number. Find the net price if the coach ordered 25 uniforms at \$24.80 each. \$465

5. Which cost less, fountain pens listed at \$5.00 with a 40% discount or pens listed at \$3.50 with a 20% discount?
\$3.50 pens cost \$.20 less for each one.

Find the discount (1) and the net price (2) when the list price and the rate of discount are as shown below: Have students do ex. 6–10 independently.

		(1)	(2)		(1)	(2)		(1)	(2)
6.	\$745, 30%	\$223.50	\$521.50	\$693, 20%	\$138.60	\$554.40	\$2776, $12\frac{1}{2}\%$	\$347	\$2429
7.	\$618, 40%	\$247.20	\$370.80	\$245, 10%	\$24.50	\$220.50	\$1620, 35%	\$567	\$1053
8.	\$320, 60%	\$192	\$128	\$723, $33\frac{1}{3}\%$	\$241	\$482	\$4816, $37\frac{1}{2}\%$	\$1806	\$3010
9.	\$496, 15%	\$74.40	\$421.60	\$456, 25%	\$114	\$342	\$1750, 20%	\$350	\$1400
10.	\$580, 25%	\$145	\$435	\$552, $16\frac{2}{3}\%$	\$92	\$460	\$2320, 15%	\$348	\$1972

More Practice. See 30 on page 371.

Let volunteers explain the solutions. Urge the students to ask questions now if they are not sure of the work.

A New Way to Find Net Prices

1. Problem A sport coat which has a list price of $40 is sold at a discount of 30%. Find the net price. $28

Explanation You know that one way to work this problem is to find 30% of $40, which is $12, and then to subtract $12 from $40, which gives $28 as the net price.

Here is another way to work the problem: The list price, which is $40, equals 100% of itself. If you subtract 30% of $40 from $40, you have 70% of $40 left. So the net price equals 70% of $40. This shows that you can work this problem a new way as follows: Subtract 30% from 100%, which gives 70%. Then find 70% of $40, which gives $28 as the net price.

$$\begin{array}{r} 100\% \\ \underline{30\%} \\ 70\% \end{array} \qquad \begin{array}{r} \$40 \\ \underline{.70} \\ \$28.00 \end{array}$$

2. If the list price is $75 and the discount is 20%, what per cent of $75 is the net price? 80% Find the net price the new way, as shown in ex. 1. .80 x $75 = $60

3. A book listed at $3.00 is sold at a discount of 25%. To find the net price in this problem, which of the above methods is easier to use? Why? It is easier to find $\frac{1}{4}$ of $3.00 and subtract. $2.25

4. Which method is easier for you to use if the list price is $50 and the discount is 10%? Take 10% of $50 and subtract. ($45) if the list price is $45 and the discount is 15%? Subtract 15% from 100%; then find 85% of $45 ($38.25).

Find the net prices, using these list prices and discounts. Work each problem both ways. Tell which method you prefer in each case: See the Guide.

5.	$10, 5% $9.50	$360, 30% $252	$632, 40% $379.20	$255.00, 5% $242.25
6.	$25, 7% $23.25	$800, 55% $360	$720, $12\frac{1}{2}$% $630	$304.50, 2% $298.41
7.	$90, 20% $72	$410, 12% $360.80	$300, $7\frac{1}{2}$% $277.50	$621.96, $33\frac{1}{3}$% $414.64
8.	$80, 25% $60	$924, $16\frac{2}{3}$% $770	$860, 25% $645	$864.20, 25% $648.15
9.	$60, 15% $51	$243, $33\frac{1}{3}$% $162	$240, $37\frac{1}{2}$% $150	$976.00, 85% $146.40
10.	$75, 12% $66	$160, $12\frac{1}{2}$% $140	$156, $16\frac{2}{3}$% $130	$250.00, 10% $225

More Practice. See 31 on page 371.

The new method in ex. 1 needs careful explanation and discussion. Be sure the students understand why the list price equals 100% of itself.

Reteach the work on finding amount and rate of commission.

The students should become familiar with the terms in ex. 1. Illustrate further if necessary.

Selling a House on Commission

1. Mr. Wood wished to sell his house, so he asked Mr. Hunt, a real estate agent, to sell it for him. Mr. Hunt found a customer who would buy the house for \$15,000, so it was sold for that amount. For his services, Mr. Hunt was paid 5% of the \$15,000. What amount did Mr. Hunt get? \$750 Mr. Wood did not get the entire \$15,000 for the house since he had to pay Mr. Hunt out of this amount. What amount did Mr. Wood actually get for the house? \$14, 250

In this problem Mr. Hunt is the **agent.** The amount he was paid for his services, which was 5% of \$15,000, is his **commission.** The amount Mr. Wood received for the house after the commission was deducted is the **net proceeds.**

2. Mr. Hunt charges a commission of 5% when he sells a house. Find the commission and the net proceeds if he sells a house at \$9500; \$475, \$9025 at \$12,900; \$645, \$12, 255 at \$17,500; \$875, \$16, 625 at \$24,000. \$1200, \$22, 800

3. At 5%, find the commission and net proceeds if Mr. Hunt sells a store at \$45,000; \$2250, \$42, 750 at \$73,500; \$3675, \$69, 825 at \$94,000. \$4700, \$89, 300

Do ex. 1–2 as a class activity, letting volunteers explain the solutions. If there are no apparent difficulties, have the students complete ex. 3 independently.

1. Mr. Walker asked an agent to sell his house for him. It was agreed that the commission would be 5% of the first \$20,000 of the amount received for the house and 3% of the amount received in excess of \$20,000. What will be the agent's commission if he sells the house for \$21,000? \$1030 if he sells it for \$22,500? \$1075 if he sells it for only \$18,500? \$925 What net proceeds will Mr. Walker receive in each case? \$19,970, \$21,425, \$17,575

2. Miss Drew works in a department store. She receives \$35 a week and in addition a commission of 3% on her weekly sales. Find her total earnings in a week when her sales amount to \$300; \$44 to \$425; \$47.75 to \$550; \$51.50 to \$785. \$58.55

3. Jim sells magazine subscriptions on a commission of 25%. Last month he got 40 subscriptions at \$3.00 each. Find his total commission. \$30 What amount did he send to the publisher of the magazine? \$90

4. An agent sold a house for Mr. Harris. After the sale was made, the agent kept \$875 and sent \$16,625 to Mr. Harris. At what price was the house sold? \$17,500 What rate of commission did the agent receive? 5%

Find the commission (1) and the net proceeds: (2)

(1), (2)

5.	\$300, 6% \$18, \$282	\$805, 40% \$322, \$483	\$4000, 5% \$200, \$3800	\$82.65, 2% \$1.65, \$81
6.	\$916, 5% \$45.80, \$870.20	\$242, 30% \$72.60, \$169.40	\$6500, 6% \$390, \$6110	\$63.50, 4% \$2.54, \$60.96
7.	\$233, 20% \$46.60, \$186.40	\$580, 25% \$145, \$435	\$7800, 4% \$312, \$7488	\$37.85, 6% \$2.27, \$35.58

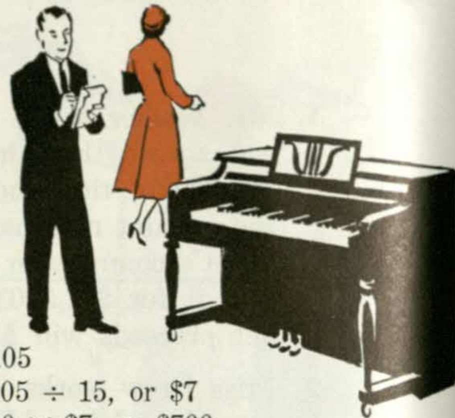
The first amount represents the total sales; the second amount is the commission. Find the rate of commission:

8.	\$50, \$8 16%	\$600, \$72 12%	\$740, \$185 25%	\$8000, \$400 5%
9.	\$90, \$30 $33\frac{1}{3}$%	\$800, \$60 $7\frac{1}{2}$%	\$700, \$105 15%	\$7500, \$300 4%
10.	\$84, \$21 25%	\$150, \$12 8%	\$895, \$179 20%	\$9500, \$570 6%

More Practice. See 32 and 33 on page 371.

A quick review of the work on page 52 may be necessary before the students do ex. 4.

Teach the finding of a whole amount when a per cent of it is given.

Finding the Whole Amount

1. Problem When Mr. Baker sold a piano, he received a commission of \$105. If this was 15% of the selling price, at what price did he sell the piano? \$700

Explanation

$$15\% \text{ of the selling price} = \$105$$
$$1\% \text{ of the selling price} = \$105 \div 15, \text{ or } \$7$$
$$100\% \text{ of the selling price} = 100 \times \$7, \text{ or } \$700$$

Since 100% of the selling price is the entire selling price, Mr. Baker sold the piano for \$700.

To check the work, see if 15% of \$700 equals \$105.

Emphasize after understanding is assured.

To find a number when a per cent of it is given, first find 1% of the number; then multiply the result by 100 to find 100% of the number.

In ex. 4-7 have students show work as in ex. 1. Let volunteers explain solutions at the board.

Check the answers in the problems below:

2. Miss Bell saved \$360 last year, which was 8% of her annual salary. How much was her salary last year? \$4500

3. Miss French is a clerk in a store. She does not receive a regular salary; instead, she is paid a commission of 7% on the total amount of her sales. Last week her total commission was \$59.50. How much were her sales last week? \$850

▶ 1% of her sales = $\frac{1}{7}$ of \$59.50, or \$8.50. Then 100% of her sales = 100 × \$8.50. What amount is it? In cases like this, you must watch the decimal points carefully. \$850

Be sure students understand why 1% of sales = $\frac{1}{7}$ of \$59.50.

4. In ex. 3, at a commission of 7%, what are Miss French's weekly sales if her weekly commission is \$38.50? \$550 if her commission is \$42.00? \$600 \$52.50? \$750 \$63.00? \$900 \$66.50? \$950 \$61.25? \$875

5. 15 is 6% of what number? 250 9% of what amount is \$2.88? \$32

6. 32 is 8% of what number? 400 5% of what amount is \$2.20? \$44

7. 18 is 8% of what number? 225 4% of what amount is \$2.72? \$68

More Practice. See 34 on page 372.

Follow the development as given in ex. 1. Lead the students to suggest how to find 1% of the selling price and then 100% of it. Do ex. 2-3 orally also.

Present the first set of improvement tests in subtraction.

Improving by Practice

Subtraction Test 1a.

Time: 4 min.

1.	87050	39905	90008	49159	87492	
	32493	29623	76104	25371	50738	
	54,557	10,282	13,904	23,788	36,754	
2.	70000	54826	81670	92312	30210	
	47853	34950	63294	83256	11147	
	22,147	19,876	18,376	9056	19,063	
3.	61246	74134	60000	67438	26128	
	39377	22159	18058	43972	23849	15
	21,869	51,975	41,942	23,466	2279	

Subtraction Test 1b.

Time: 4 min.

4.	30000	67342	90012	84311	68239	
	12582	64878	39155	22588	52305	
	17,418	2464	50,857	61,723	15,934	
5.	24726	40000	54177	60108	35538	
	16259	16813	14358	17445	16279	
	8467	23,187	39,819	42,663	19,259	
6.	64953	90016	46162	94519	50000	
	19285	57398	42585	45728	16958	15
	45,668	32,618	3577	48,791	33,042	

Subtraction Test 1c.

Time: 4 min.

7.	20000	92071	78738	72533	60101	
	19822	72487	38954	46643	16406	
	178	19,584	39,784	25,890	43,695	
8.	61962	94043	80000	45297	90602	
	44284	67464	19175	17928	36724	
	17,678	26,579	60,825	27,369	53,878	
9.	49834	60107	95525	10000	63613	
	32259	23149	69257	1844	36679	15
	17,575	36,958	26,268	8156	26,934	

To the Teacher. The Improvement Tests above are to be given to the pupils on three different days. See page 375 for the procedure to follow.

Check papers carefully and note types of errors (facts, regrouping, zero difficulties, and so on). Have the students determine and record their own scores in Record Books (page 48).

Problems and Practice

1. At the Fairview School the graduating class took a vote to decide whether to have a class picnic. There were 85 votes in favor of the picnic, which was 68% of all the votes cast. How many votes were cast in all? 125

▶ To find 1% of the votes you have to divide 85 by 68. Before dividing, write 85 as 85.00. 85.00 ÷ 68 = 1.25, which is 1% of the votes. How do you find 100%, or all, of the votes?

2. The class also voted to decide whether to have the picnic at Blue Lake or at some other place. There were 92 votes in favor of Blue Lake, which was 80% of all the votes cast. How many votes in all were cast this time? 115

3. Mr. Black was offered $624 for his old car. He said that was only 24% of what the car had cost him when it was new. What did he pay for the car? $2600

4. Last season our baseball team won 9 games, which was 60% of all the games played. How many games were played? 15

5. In ex. 3 on page 64, Miss French is paid a commission of 7% on the total amount of her sales each week. What must be the amount of her weekly sales if she wishes to earn $50 a week? $714.29 $55 a week? $785.71 $65 a week? $928.57 $70 a week? $1000

▶ To earn $50 a week, 1% of her sales must equal $\frac{1}{7}$ of $50. When you find $\frac{1}{7}$ of $50, the result is not exact; in this case, carry the result to the nearest hundredth of a cent, which gives $7.1429. Then find 100% of her sales. Stress.

6. If $17.49 is 12% of some amount, what is the amount? $145.75

Tell what numbers belong in the spaces. Check the work: Stress.

7. 27 is 9% of ... 300	24% of ... 350 is 84	204 is 48% of ... 425
8. 31 is 5% of ... 620	16% of ... 275 is 44	156 is 65% of ... 240
9. 22 is 8% of ... 275	25% of ... 68 is 17	533 is 52% of ... 1025
10. 12 is 15% of ... 80	28% of ... 225 is 63	171 is 18% of ... 950
11. 15 is 12% of ... 125	35% of ... 160 is 56	127 is 25% of ... 508

Discuss the solutions and clear up any difficulties. Ex. 5 should be done as a class activity to be sure all students understand "nearest hundredth of a cent."

Show the relationship of the three types of percentage problems by means of a formula.

The Percentage Formula

1. You have learned that every percentage problem has three parts: the base, the rate, and the percentage. You have also learned that:

Percentage = Rate × Base

If you let p represent the percentage, r the rate, and b the base, the above relationship can be expressed as a formula:

$$p = rb$$

Remember that rb means $r \times b$.

2. When you find 23% of 400, what is the base? 400 the rate? 23% The formula gives p, the percentage:

$$p = .23 \times 400 = 92$$

Use the formula to find: 30% of 1250; 375 17% of 800; 136 89% of 1400; 1246 47% of 650; 305.5 73% of 2280; 1664.4 29% of 117. 33.93

3. You can use the formula to find the rate when you know the base and the percentage. When you find what per cent 16 is of 64, you know that the base is 64 and the percentage is 16. Using the formula, you have:

$$16 = r \times 64$$

The product of r and 64 equals 16. To find the unknown factor r, you can divide 16 by 64, which gives .25. Therefore the rate is .25 or 25%. So 16 is 25% of 64.

Use the formula to find what per cent the first number is of the second: 8, 40; 20% 24, 25; 96% 136, 850; 16% 518.4, 1200; 43.2% 36, 24. 150%

4. You can also use the formula to find the base when it is unknown. If 15% of a number is 600, and you wish to find this number, you know that the rate is 15% and the percentage is 600. Using the formula, you have:

$$600 = .15 \times b$$

The product of .15 and b equals 600. To find the unknown factor b, you can divide 600 by .15, which gives 4000. Therefore the base is 4000. Is 15% of 4000 equal to 600? Yes

Use the formula to find a number if you know that 35% of it is 70; 200 27% of it is 1944; 7200 53% of it is 901. 1700

Have the students identify the factors and the product in the percentage formula. Ask the students to solve some examples and problems on preceding pages by using the formula.

Reteach the relationship of profit and loss to selling price, cost, and expenses (pages 68–69).

Profit and Loss

1. Mr. Green has a shoe store. Last month he sold $4000 worth of shoes. These shoes had cost Mr. Green $2600; so he sold them for $1400 more than they cost him. This Stress. $1400 is not all profit because that month Mr. Green had to pay $1200 for light, heat, rent, a clerk's salary, his own salary, and other expenses. If you subtract the $1200 from the $1400, you get $200, which is his profit for the month.

If you add the cost of the shoes ($2600), the expenses ($1200), and the profit ($200), you get $4000. This $4000 represents the selling price of the shoes. So you see that

Selling Price = Cost + Expenses + Profit

2. In ex. 1, what would have been Mr. Green's profit for the month if his total expenses had been $1000 instead of $1200? $400 if his expenses had been $1400 instead of $1200? $0 What would have happened if his total expenses had been $1600 for the month? $200 loss

3. In ex. 2, you see that the expenses are a very important item in running a store. If the expenses are high, the profit is small. If the cost plus the expenses are greater than the selling price, there is a loss instead of a profit. Illustrate and emphasize.

Stress meaning.

4. The word **cost** means the net price (after deducting discounts) that is paid for merchandise plus freight and delivery charges paid to get the merchandise into the store. Stress.

Emphasize that selling price is price received for merchandise when it is sold. Stress that profit means actual gain.

5. Another month Mr. Green sold $5000 worth of shoes. These shoes cost him $3100. His expenses that month were $1500 because he had an extra clerk. What was his profit? $400

In each exercise, find the missing item which is marked with a star. Remember that Selling Price = Cost + Expenses + Profit:

	Selling Price	Cost	Expenses	Profit	Loss
6.	$1000	$600	$300	* $100	
7.	$1400	$770	* $504	$126	
8.	$2000	* $1000	$760	$240	
9.	* $1600	$992	$528	$80	
10.	$1200	$780	$516		* $96
11.	$1800	$1134	* $720		$54
12.	$6500	$4030	$1885	* $585	
13.	* $8200	$5166	$3690		$656

More Practice. See 35 on page 372. Use for individual assistance. After discussing ex. 5, have the students complete ex. 6–13. Let volunteers explain their solutions. Urge the students to ask questions if they are unsure of this work.

Extend the study of profit, loss, cost, and expenses as per cents of the selling price (pages 70–72).

Finding the Per Cent of Profit

1. Mr. King keeps a clothing store. Last year his sales for the entire year were $30,000. This $30,000 is the selling price of the goods he sold. The cost of the goods sold was $18,000 and the total expenses were $9000. Find his profit. $3000

2. In order to carry on his business profitably, Mr. King finds it necessary to know what per cent his yearly expenses are of his total sales. Last year his sales were $30,000 and his expenses were $9000. What per cent is $9000 of $30,000? 30% He also wants to know what per cent his profit of $3000 is of the sales of $30,000. What per cent is it? 10% The cost of the goods he sold was $18,000. What per cent is $18,000 of $30,000? 60%

3. In ex. 2, you see that last year Mr. King's profit was 10% of his sales, his expenses were 30% of his sales, and the cost of the goods sold was 60% of his sales. These per cents give Mr. King helpful information about his business. For example, if the selling price of a hat is $10.00, he knows that his profit on the hat is 10% of $10.00, or $1.00. He also knows that 30% of $10.00, which is $3.00, has to be used toward paying the expenses of the store, while 60% of $10.00, which is $6.00, is the amount the hat cost him.

Profit =	10% of $10
Expenses =	30% of $10
Cost =	60% of $10
Selling Price =	100% of $10

4. The per cents of the selling price allowed for expenses, profit,
Stress. and cost are not the same in all stores. If a store has high rent and many clerks, it may have to allow 40% of the selling price for expenses, 5% for the profit, and 55% for the cost. In each case, the sum of the three per cents is
Stress. 100%, since the selling price represents 100%. If one of Stress. these per cents is made larger, then one or both of the other per cents must be made smaller to keep the sum 100%.

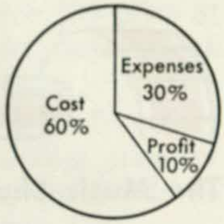
5. In ex. 6 to 13 on page 69, find what per cent the cost, the expenses, and the profit each are of the selling price. See Guide.

In discussing ex. 1–4 emphasize that per cents of profit, loss, cost, and expenses are found by comparing these with the selling price. Emphasize that the sum of the per cents must equal 100% and explain why this is true.

Review of the use and drawing of circle graphs may be necessary before beginning this work.

The Sales Dollar

1. Problem Every dollar that is received from sales is called a **sales dollar.** In ex. 3 on page 70, Mr. King found that each of his sales dollars was divided as follows: 10% of each dollar, or 10¢, was profit; 30% of each dollar, or 30¢, was used for expenses; and 60% of each dollar, or 60¢, was used for the cost. Make a copy of Mr. King's sales dollar, which is shown at the right.



The Sales Dollar

Explanation To find the parts of the circle, proceed as follows:

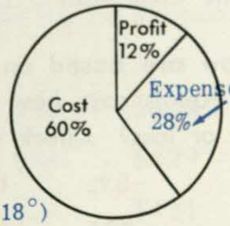
The whole circle, which equals 360°, represents a sales dollar. Since the profit is 10% of the dollar, it is represented by 10% of the circle. 10% of 360° = 36°. With your protractor, draw an angle of 36° with its vertex at the center of the circle. If you have forgotten how to use a protractor, see pages 146 and 147.

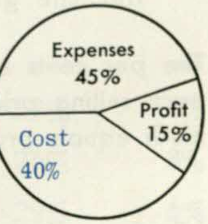
The expenses are represented by 30% of the circle. 30% of 360° = 108°. Draw an angle of 108° next to the first angle.

30% + 10% = 40%, 100% − 40% = 60%

The rest of the circle represents the cost, which is 60% of the sales dollar. Why? While you do not have to draw this angle, you should measure it as a check on the accuracy of your work. How many degrees should this angle contain? 216°

2. In each sales dollar shown at the right, tell what should be written in the blank part of each circle.





3. Mr. West allows 5% (18°) of his sales dollar for (119°) profit and 33% of it for expenses. What per cent of his sales dollar does he allow for cost? Draw his sales dollar. See Guide.

62% (223°)

▶ 33% of 360° equals 118.8°, which rounds off to 119°.

4. Draw a sales dollar showing an allowance of 5% (18°) for profit, (90°) 25% for expenses, and (252°) 70% for cost. See Guide for graph.

In ex. 1 emphasize that the sales dollar represents the selling price. Point out that all per cents on circle graphs of a sales dollar are computed with the selling price as the basis. Do this work slowly to assure good understanding.

The Music Shop

1. Mr. Young has a music store. He finds by experience that he has to allow 25% of the selling price of an article for expenses, 65% for the cost, and 10% for the profit. If he sells a record player for $40, how much of this $40 is profit? $4 How much is used toward paying the expenses of the store? $10 How much represents the cost of the record player? $26

2. In ex. 1 check the work by adding the amounts used for profit, expenses, and cost. What should this sum be? $40

Have students explain why the sum should be $40.

3. In ex. 1 there are two different ways that you can find how much of the $40 represents the cost. What are they? (1) 65% of $40; (2) find amounts representing profit and expenses; subtract their sum from $40.

4. Mr. Smith also has a music store. Last year he ran his store at a loss equal to 5% of his total sales. His expenses were 45% of the sales and his costs were 60% of the sales. When he sold a guitar for $50, what was his loss? $2.50 What did the guitar cost him? $30 How much were his expenses? $22.50

The per cents below are based on the selling price. How much of each selling price equals cost, how much equals expenses, and how much equals profit or loss? Check the work as in ex. 2: Stress.

	S.P.	Cost	Expenses	Profit	Loss
5. Snare Drum	$25	60% $15	33% $8.25	7% $1.75	
6. Violin	$95	65% $61.75	45% $42.75		10% $9.50
7. Trumpet	$115	55% $63.25	35% $40.25	10% $11.50	
8. Tape Recorder	$125	63% $78.75	32% $40	5% $6.25	
9. Saxophone	$220	60% $132	42% $92.40		2% $4.40

Have the students who make mistakes do their computations aloud so you can determine causes of errors.

Teach how to find the selling price when the profit and expenses are given as per cents of the selling price.

Finding the Selling Price

1. Problem Mr. Ware has a furniture store. When new goods come into the store, Mr. Ware must determine the price at which to sell them. The selling price must be large enough to cover the expenses, the cost, and the profit needed to run the business successfully. By past experience Mr. Ware has found that he must allow 34% of the selling price for expenses and 12% of it for profit. At what price should he sell a dining room table that cost him $72.90? $135

Explanation Since the expenses and profit total 46% of the selling price, the remaining 54% of the selling price equals the cost of the table, which is $72.90. The selling price can be found as follows:

54% of the selling price = $72.90
Then 1% of the selling price = $72.90 ÷ 54, or $1.35
and 100% of the selling price = 100 × $1.35, or $135

So the selling price of the table should be $135.

2. Show how to use the percentage formula in the work of ex. 1.

3. Miss Long bought some dresses to sell in her shop. If she allows 15% of her sales for profit and 30% for expenses, at what price should she sell a dress that cost $11.55? $21 a coat that cost $29.15? $53 a sweater that cost $3.74? $6.80

Find the selling price. The per cents for expenses and profit are based on the selling price: Have volunteers explain solutions.

	Cost	Expenses	Profit		Cost	Expenses	Profit	
4.	$31.00	25%	13%	$ 50	$522	29%	13%	$900
5.	$30.24	30%	7%	$ 48	$429	26%	8%	$650
6.	$96.60	19%	11%	$138	$481	29%	6%	$740
7.	$55.25	23%	12%	$ 85	$850	27%	5%	$1250
8.	$28.98	22%	9%	$ 42	$224	26%	10%	$350
9.	$66.08	32%	12%	$118	$536	35%	8%	$940

More Practice. See 36 on page 372.

Have the students explain why 54% of the selling price equals the cost. Then ask the students how they can find a number when a per cent of it is given. Have the students use the percentage formula to solve some problems on this page.

Successive Discounts

1. Problem On an order of 200 books at \$1.50 each, Mr. White received a trade discount of 20% and a second discount of 10% because the order was large. Find the net price. \$216

Explanation First find the list price of the books by multiplying \$1.50 by 200, which gives \$300. Then find 20% of \$300, which gives \$60. Subtracting \$60 from \$300, you get \$240.

The second discount is taken on \$240. 10% of \$240 is \$24. Subtracting \$24 from \$240, you get \$216 as the net price.

$$\begin{array}{r} \$\,300 \\ .20 \\ \hline \$60.00 \end{array} \qquad \begin{array}{r} \$300 \\ 60 \\ \hline \$240 \\ 24 \\ \hline \$216 \end{array}$$

Discuss the meaning of the terms successive and chain in ex. 2.

2. When two or more discounts are given, they are called **successive** or **chain** discounts. In computing successive discounts, compute the first discount on the list price; then compute the second discount on the amount left after the first discount is deducted. Do not add the discounts together. Show by computation that a discount of 20%, 10% on \$100 is not the same as a discount of 30% on \$100. Explain why this is so. .20 x \$100 = \$20, \$100–\$20 = \$80, .10 x \$80 = \$8, \$80 – \$8 = \$72; .30 x \$100 = \$30, \$100 – \$30 = \$70

3. Occasionally a third discount is allowed for paying cash. Mr. White might have received an extra discount of 2% besides the discount of 20%, 10%. Then, to find the net price, he would take 2% of \$216, which is \$4.32; subtracting \$4.32 from \$216 he would get \$211.68 as the net price.

4. On a shipment of toys listed at \$500, a chain discount of 30%, 10%, 4% was allowed. Find the net price of the toys. \$302.40

5. In ex. 4, show that you get the same net price if you deduct the discounts in this order: 10%, 30%, 4%. \$302.40

Find the net prices, using the following list prices and discounts:

6.	\$400, less 30%, 10% \$252	\$800, less 20%, 10%, 5% \$547.20
7.	\$600, less 20%, 15% \$408	\$480, less 37½%, 5%, 2% \$279.30
8.	\$900, less 25%, 20% \$540	\$260, less 30%, 20%, 5% \$138.32
9.	\$280, less 60%, 12½% \$98	\$300, less 33⅓%, 10%, 2½% \$175.50

Emphasize and show that a chain discount (20%, 10%) is not the same as a single discount (30%). In ex. 5 stress the fact that the order of the discounts does not affect the net price.

Present the first set of improvement tests in multiplication. The method of scoring and recording the tests is the same as for the addition tests (see pages 375–377). Have the students determine and record their own scores.

Improving by Practice

Multiplication Test 1a. Time: $3\frac{1}{2}$ min. after copying.

1.	981 × 348	528 × 806	146 × 724	457 × 852	304 × 193	537 × 930 (6)
	341,388	425,568	105,704	389,364	58,672	499,410

Multiplication Test 1b. Time: $3\frac{1}{2}$ min. after copying.

2.	217 × 709	239 × 852	407 × 146	316 × 685	758 × 579	285 × 320 (6)
	153,853	203,628	59,422	216,460	438,882	91,200

Multiplication Test 1c. Time: $3\frac{1}{2}$ min. after copying.

3.	704 × 682	269 × 193	975 × 734	639 × 706	834 × 273	184 × 590 (6)
	480,128	51,917	715,650	451,134	227,682	108,560

The Language of Arithmetic

Present a review of important arithmetical terms.

Read the statements below and tell the correct word to put in each space. Do not write in the spaces:

4. The selling price of an article should equal the cost plus the expenses ------- plus the profit -------.
5. The list price of an article less the discount ------- is called the net ------- price -------.
6. In South America the unit of measure used for measuring length is called the meter -------.
7. If the cost of an article plus the expenses of selling it is more than the selling price, the article is sold at a loss -------.
8. When you write 6,287,325 as 6.3 million, you are writing it as a round ------- number -------.
9. The money that Jack is paid for selling magazine subscriptions is called his commission -------.
10. The man who sells a house and is paid a commission for selling it is called the agent -------.

The students should copy test examples (leaving room for working each) before the test is begun. Check work carefully and note kinds of errors. Through conferences with students, determine causes of errors. Clear up difficulties before assigning remedial work.

Present a review of problems that involve percentage.

Percentage Problems

1. Texas, Ohio, and Illinois have about 14.8% of the 1,793,500 mi. of rural free delivery mail routes in this country. How many miles of rural delivery routes are there in these 3 states? Give the answer to the nearest mile. 265,438

2. Don is a 4-H Club member and owns a cow that weighs 1200 lb. He has read that the daily feed for a cow should be hay equal to 1% of the cow's weight and silage equal to 3% of its weight. How many pounds of hay (1) and how many pounds of silage (2) should Don's cow eat a day? (1) 12; (2) 36

3. From experience Mr. Lee has learned that in selling clothing in his store he should allow about 30% of the selling price for expenses, 10% of the selling price for profit, and the rest of the selling price for the cost. If he pays $45 for a man's overcoat, for how much should he sell it? What will be his profit on the coat? $7.50 $75

4. A large cake of sweet chocolate used to sell for 25¢. Now the price has been increased to 35¢. Find the per cent of increase in the price. 40%

5. On the first science test Ray had 30 correct answers out of 35. (1) On the second test he had 40 correct out of 45. (2) Find, to the nearest whole per cent, his score on each test. (1) 86%; (2) 89%

6. Nancy's commission for selling Christmas cards was $16.50. This was 15% of the total amount Nancy took in. What was the entire amount she took in? $110

7. Find the net cost to the dealer of 12 radios listed at $35 each if he received a discount of 10%, 10%. Also find the cost of each radio. $28.35 $340.20

8. If the dealer in ex. 7 figures that his cost is 60% of his selling price, for how much should he sell each radio? $47.25

9. The list price of a chair is $44. Which is better, a discount of 20% or a discount of 10%, 10%? What is the net price in each case? $35.20; $35.64 20% discount

Assign ex. 1-9 as independent work. Have volunteers explain the solutions at the board. Group students who make mistakes to help discover causes and the need for reteaching.

Present this project, which requires the use of percentage.

Our Rubber Supply

1. In a recent year the world production of natural rubber was 2,040,000 long tons. A **long ton** equals 2240 lb. and is sometimes used as a unit of measure instead of an ordinary ton. How many pounds of natural rubber were produced that year? 4,569,600 000

2. How many pounds is an ordinary ton? 2000 A long ton is what per cent larger than an ordinary ton? 12%

3. Of the total production of 2,040,000 long tons of natural rubber, 1,870,900 tons were produced in the Far East, (1) 28,100 tons were produced in Tropical America, (2) and 141,000 tons were produced in Africa. (3) What per cent of this natural rubber, to the nearest tenth of 1%, was produced in each of these areas? (1) 91.7%; (2) 1.4%; (3) 6.9%

4. One year in the United States we used 634,800 long tons of natural rubber, 894,899 long tons of synthetic rubber, and 312,781 long tons of reclaimed rubber. How much rubber was used in all that year? 1,842,480 Round off these numbers to the nearest thousand. See Guide. Find, to the nearest whole per cent, what per cent of the rubber was natural rubber; 34% was synthetic rubber; 49% was reclaimed rubber. 17%

5. One year our factories in the United States made 97,223,000 passenger car tires and 14,955,000 truck and bus tires. Round off these numbers to the nearest million. 97,000,000; 15,000,000 What per cent of the total tires made were for passenger cars? 87% for trucks and busses? 13% Find per cents to the nearest whole per cent.

Point out that the long ton is the standard ton in Great Britain and is the unit of measure for production of rubber. As the students complete ex. 1–5, notice evidence of difficulties with particular problems. Further review may be needed.

Use review as preparation for the diagnostic test.

Chapter Review

(inc.) (dec.)

Find, to the nearest whole per cent, the per cent of increase or decrease that the second number shows over the first number:

1. 9, 14 56% inc. 57, 23 60% dec. 217, 226 4% inc. 6381, 4215 34% dec.
2. 15, 11 27% dec. 42, 53 26% inc. 150, 416 177% inc. 2925, 5992 105% inc.

Find, to the nearest cent, the commission and net proceeds:

	Amount	Rate	Amount	Rate	Amount	Rate
	(1), (2) $253.00, $12,397.00		$2.17, $84.53		$104.56, $313.69	
3.	$12,650	2%	$86.70	$2\frac{1}{2}\%$	$418.25	25%
	$561.25, $10,663.75		$4.87, $43.82		$20.64, $254.61	
4.	$11,225	5%	$48.69	10%	$275.25	$7\frac{1}{2}\%$

Using these list prices and rates of discount, find the discount to the nearest cent. Also find the net price:

	List	Discount	List	Discount	List	Discount
	(1), (2) $1.43, $12.82		$1.06, $7.44		$33.00, $242.00	
5.	$14.25	10%	$8.50	$12\frac{1}{2}\%$	$275	12%
	$3.99, $11.96		$3.25, $6.50		$57.90, $135.10	
6.	$15.95	25%	$9.75	$33\frac{1}{3}\%$	$193	30%

Find the amount of profit. S. P. means selling price:

	Cost	Expenses	S. P.	Cost	Expenses	S. P.
7.	$4.50	$2.25	$7.50 $.75	$124	$62	$200 $14
8.	$3.78	$1.80	$6.00 $.42	$210	$75	$300 $15
9.	$5.20	$2.40	$8.00 $.40	$183	$99	$300 $18

10. In ex. 7–9, what per cent is the profit of the selling price? Ex. 7: 10%, 7%; ex. 8: 7%, 5%; ex. 9: 5%, 6%

In ex. 11–13, the per cents allowed for expenses and profit are computed on the selling price. Find the selling price:

	Cost	Expenses	Profit	Cost	Expenses	Profit
11.	$8.40	25%	5% $12	$7.68	30%	6% $12
12.	$5.67	31%	6% $9	$9.60	29%	7% $15
13.	$4.34	31%	7% $7	$9.10	27%	8% $14

After checking papers and noting kinds of errors, return them to the students so they may find and correct mistakes. If many students seem weak in a particular part of the work, reteaching of the whole class may be needed.

A Problem Test

1. Tony read that in a recent year our average yield of peanuts per acre was 936.3 lb. At this rate, how many pounds of peanuts could be raised on 50 acres? 46,815

2. In one year the number of workers in a factory increased from 2683 to 4778. Find, to the nearest tenth of 1%, the per cent of increase in the number of workers. 78.1%

3. Find the entire cost of sending a telegram of 21 words if the rate is \$1.45 for the first 15 words and 6.5¢ for each extra word. There is a tax of 10% on the cost of sending the telegram. \$2.02

4. Jane saved \$2.40 when she bought a pair of skates at a discount of 25%. What was the original price of the skates? \$9.60

5. Buttermilk is composed of 91% water, 3% protein, 4.8% carbohydrates, 0.5% fat, and 0.7% ash. How many pounds of each substance are there in 500 lb. of buttermilk? Water, 455 lb.; protein, 15 lb.; carbohydrates, 24 lb.; fat, 2.5 lb.; ash, 3.5 lb.

6. In South America a man traveled 750 kilometers by plane. His baggage weighed 20 kilograms. How many miles did he travel? 450 How many pounds did his baggage weigh? 44 Remember that 1 km. = .6 mi.; 1 kg. = 2.2 lb.

7. Find the profit on a chair selling at \$95 if the profit is $12\frac{1}{2}$% of the selling price. \$11.88

8. How many weeks will it take Sam to earn \$37.50 if he works $1\frac{1}{4}$ hr. each school day at 60¢ an hour? 10

9. How much change should Ben receive from \$5.00 after buying 3 radio tubes at \$1.17 each? \$1.49

10. On an airplane, baggage over 40 lb. is charged for at the rate per pound of $\frac{1}{2}$ of 1% of the one-way fare. Find the charge for a bag weighing 47 lb. where the fare is \$124.00. \$4.34

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Instruct the students to read the problems carefully before beginning the work. Try to determine causes of errors from students' explanations of solutions. Notice if vocabulary, processes, or problem situations cause difficulty and plan remedial work accordingly.

Present this diagnostic test of skills taught or reviewed in Chapter 2, with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Page for that row.

Find the answers to the nearest cent: — Practice Pages

1. 4% of \$9 \$.36 — 32% of \$12 \$3.84 — $4\frac{1}{2}$% of \$38 \$1.71 — 42

2. 75% of \$5 \$3.75 — $12\frac{1}{2}$% of \$15 \$1.88 — $66\frac{2}{3}$% of \$53 \$35.33 — 42

3. 1.4% of \$7 \$.10 — 11.5% of \$24 \$2.76 — 33.8% of \$90 \$30.42 — 43

4. .2% of \$8 \$.02 — $\frac{3}{4}$% of \$80 \$.60 — $\frac{1}{2}$ of 1% of \$65 \$.33 — 45

5. 200% of \$3 \$6 — 125% of \$16 \$20 — 350% of \$84 \$294 — 50

Find the missing numbers. In ex. 7 find the nearest whole per cent. In ex. 8 find the nearest tenth of 1%:

6. 42 is . . . % of 56 75 — 53 is . . . % of 40 $132\frac{1}{2}$ — 51

7. 81 is . . . % of 95 85 — 46 is . . . % of 67 69 — 52

8. 19 is . . . % of 52 36.5 — 71 is . . . % of 88 80.7 — 53

The first number is the amount of commission; the second is the rate of commission. Find the amount of the sales:

9. \$344, 5% \$6880 — \$96.60, 15% \$644 — \$18.50, 20% \$92.50 — 64

The first number is the profit; the second, the selling price. What per cent is the profit of the selling price?

10. \$1.50, \$25 6% — \$2.80, \$40 7% — \$5.20, \$80 $6\frac{1}{2}$% — 70

11. \$2.70, \$45 6% — \$6.00, \$75 8% — \$4.55, \$65 7% — 70

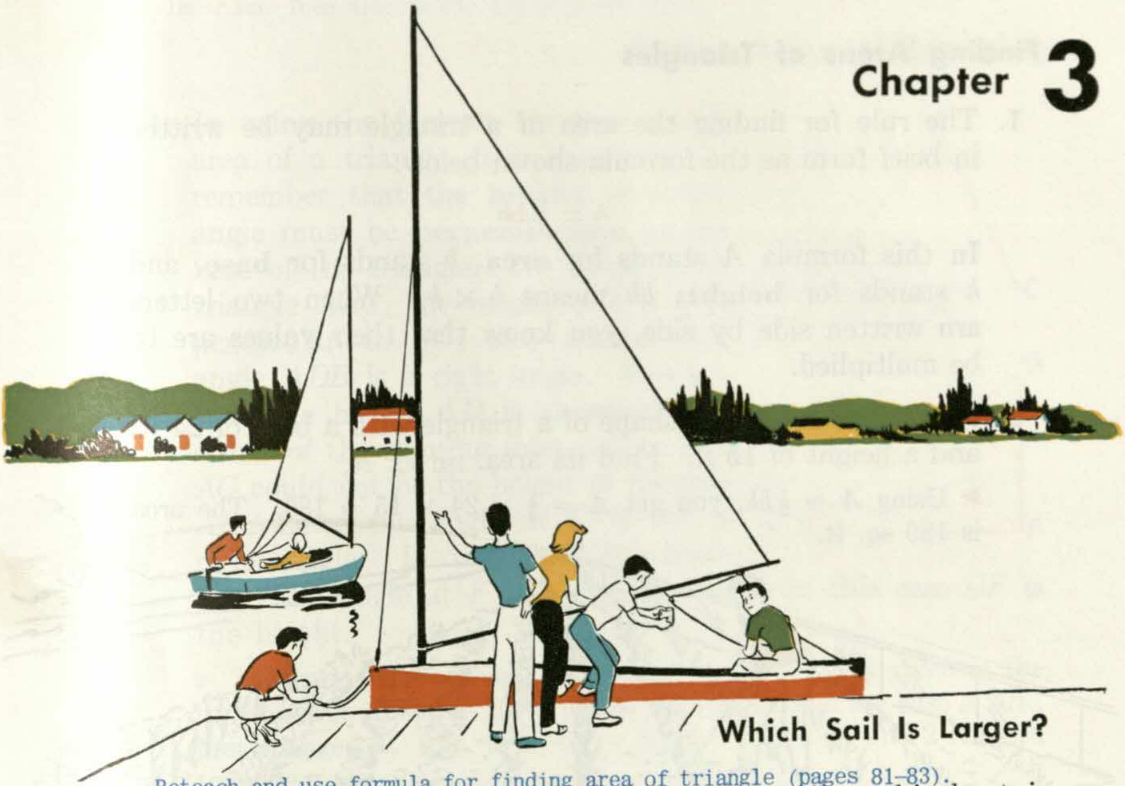
Find the selling price; the expenses and profit are given as per cents of the selling price:

	Cost	Exp.	Profit		Cost	Exp.	Profit		Practice Pages
12.	\$32	30%	6%	\$50	\$262	28%	$6\frac{1}{2}$%	\$400	73
13.	\$51	24%	8%	\$75	\$130	25%	10%	\$200	73

Have students do computations aloud so you can determine causes of errors (computational, lack of understanding, and so on). Reteach before assigning remedial work.

See the Guide for the specific aims of Chapter 3.

Chapter 3

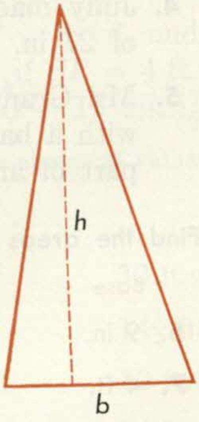


Which Sail Is Larger?

Reteach and use formula for finding area of triangle (pages 81–83).

1. Fred wanted to know if the triangular sail on his boat is as large as the one on Jim's boat. Fred was told that the way to compare two sails is to compare their areas, so Fred found the area of his sail as shown below.

2. Fred's sail is a triangle. The base, *b*, of the sail is 9 ft. long. The height, *h*, is 18 ft. long and is drawn perpendicular to the base. Fred remembered the rule for finding the area of a triangle, which is



Area of triangle $= \frac{1}{2} \times$ *base* $\times$ *height*

Since the base = 9 ft. and the height = 18 ft., he found that

$$\text{Area of sail} = \tfrac{1}{2} \times 9 \times 18 = 81$$

So Fred's sail has an area of 81 sq. ft.

3. Jim's sail has a base of 10 ft. and a height of 16 ft. What is its area? 80 sq. ft. Which sail is larger, Fred's or Jim's? Fred's

First review the meaning of area (amount of space covered by a flat surface), definition of triangle, meaning of perpendicular, and meaning of base and height.

Finding Areas of Triangles

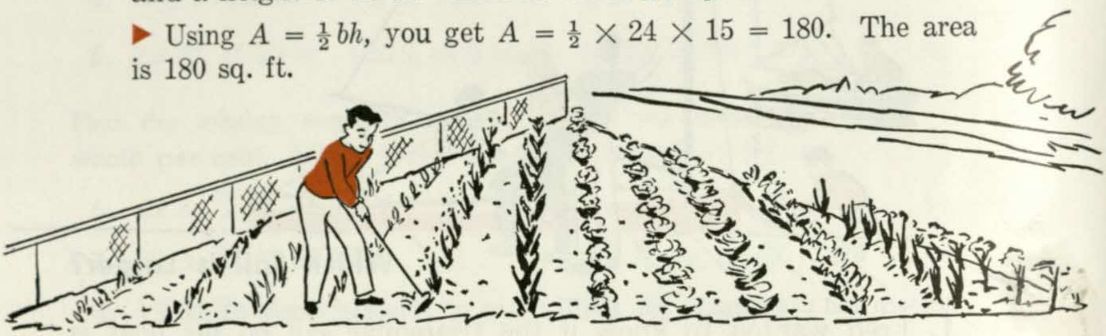
1. The rule for finding the area of a triangle may be written in brief form as the formula shown below:

$$A = \frac{1}{2} bh$$

In this formula A stands for **area,** b stands for **base,** and h stands for **height;** bh means $b \times h$. When two letters are written side by side, you know that their values are to be multiplied.

2. Ned's garden has the shape of a triangle with a base of 24 ft. and a height of 15 ft. Find its area. 180 sq. ft.

▶ Using $A = \frac{1}{2} bh$, you get $A = \frac{1}{2} \times 24 \times 15 = 180$. The area is 180 sq. ft.



3. Mark's boat has a triangular sail with a base of 9 ft. and a height of 10 ft. Find its area. 45 sq. ft.

4. Judy made a scarf in the shape of a triangle with a base of 27 in. and a height of 10 in. Find its area. 135 sq. in.

5. Mr. Grant bought a piece of land in the shape of a triangle with a base of 110 ft. and a height of 200 ft. About what part of an acre did he buy? $\frac{1}{4}$ 1 acre = 43,560 sq. ft.

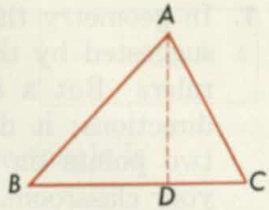
Find the areas of the following triangles:

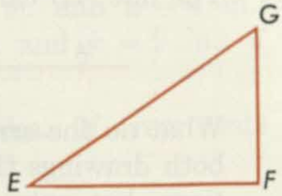
	Base	Height			Base	Height	
6.	9 in.	14 in.	63 sq. in.	**10.**	25 ft.	18 ft.	225 sq. ft.
7.	4 ft.	17 ft.	34 sq. ft.	**11.**	19 in.	13 in.	$123\frac{1}{2}$ sq. in.
8.	8 in.	17 in.	68 sq. in.	**12.**	4.2 in.	6.7 in.	14.07 sq. in.
9.	6 yd.	10 yd.	30 sq. yd.	**13.**	7.3 ft.	6.2 ft.	22.63 sq. ft.

The formula in ex. 1 should be memorized by the students. Emphasize that when two letters are written side by side, their values are to be multiplied.

Review the meaning of right angles and have students point some out in the classroom. Then discuss ex. 1–3 with the class.

Finding Areas of Triangles





1. In using the formula for finding the area of a triangle, it is important to remember that the **height** of a triangle must be **perpendicular** to the base of the triangle. For example, in triangle ABC, the height AD is perpendicular to BC; this means that angle ADB is a right angle. You see that the height AD is **shorter** than either of the slanting sides AB or AC. AC could not be the height of triangle ABC since the angle at C is not a right angle. In triangle EFG, however, the angle at F is a right angle, so in this case GF is the height.

▶ The base and the height of a triangle must both be given in the same unit of measure. If the height is 1 ft. 6 in. and the base is 2 ft., first change 1 ft. 6 in. to $1\frac{1}{2}$ ft. or change 1 ft. 6 in. and 2 ft. both to inches.

2. In ex. 1, find the area of triangle ABC if $AD = 9$ ft. and $BC = 12$ ft.; 54 sq. ft. if $AD = 15$ in. and $BC = 20$ in.; 150 sq. in. if $AD = 6$ yd. and $BC = 8$ yd.; 24 sq. yd. if $AD = 1$ ft. 6 in. and $BC = 2$ ft. $1\frac{1}{2}$ sq. ft.

3. In ex. 1, find the area of triangle EFG if $EF = 3$ yd. and $FG = 6$ ft.; 3 sq. yd. if $EF = 4.5$ ft. and $FG = 3.2$ ft.; 7.2 sq. ft. if $EF = 4$ ft. and $FG = 2$ ft. 6 in.; 5 sq. ft. if $EF = 2\frac{1}{2}$ mi. and $FG = 1\frac{1}{2}$ mi. $1\frac{7}{8}$ sq. mi.

In ex. 4–8 have pupils write formula first, then substitute numerical values in it.

Find the areas of the following triangles:

	Base	Height		Base	Height
4.	12 in.	7 in. 42 sq. in.	**9.**	$15\frac{1}{2}$ ft.	30 ft. $232\frac{1}{2}$ sq. ft.
5.	14 ft.	9.6 ft. 67.2 sq. ft.	**10.**	$9\frac{1}{2}$ ft.	$14\frac{1}{2}$ ft. $68\frac{7}{8}$ sq. ft.
6.	23 in.	9 in. $103\frac{1}{2}$ sq. in.	**11.**	4 yd. 2 ft.	10 ft. 70 sq. ft.
7.	35 ft.	42 ft. 735 sq. ft.	**12.**	6 ft. 3 in.	14 ft. $43\frac{3}{4}$ sq. ft.
8.	3.2 yd.	1.6 yd. 2.56 sq. yd.	**13.**	8 ft. 8 in.	15 ft. 65 sq. ft.

Emphasize that the height of a triangle is the length of the line segment drawn from the vertex perpendicular to the base. Stress also that base and height must be given in, or changed to, the same unit of measure.

Draw several examples of lines, line segments, and rays on separate cards or sheets for class identification.

Lines, Rays, and Segments

1. In geometry the term **line** means a straight line. A line is suggested by the edge of a sheet of paper or the edge of a ruler. But a line extends without ending in each of two directions; it does not stop at a point. The line through two points on the floor of your classroom extends outside your classroom. A line is named by giving two points on it. The line AB can be drawn these two ways:

What do the arrowheads in the second drawing suggest? In both drawings the line AB extends indefinitely in two directions, but only part of the line is actually drawn.

2. A **ray** is a part of a line. It has one **endpoint** and extends indefinitely in one direction from this endpoint. You may have used the word "ray" in the phrase "a ray of light." In geometry the term "ray" has a similar meaning. A ray is named by giving its endpoint and one other point on it. The ray AB can be drawn this way:

The point A is the endpoint of this ray.

3. The part of a line between two points on it, including the two **endpoints,** is called a **line segment** or a **segment.** The sides of a rectangle are segments. A segment is named by giving its endpoints. The segment AB is drawn below:

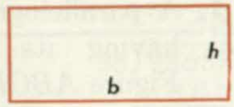
4. Give illustrations of lines, rays, and segments.

5. Sometimes the term "line" is used for any one of these three: line, ray, and segment. It is better to use the correct term for a ray and for a segment. However, when you encounter the term "line" you can usually tell from the situation in which it is used whether it means line, or ray, or segment.

After the class can correctly identify lines, rays, and segments, have students reproduce them with proper labels. Reproduction is usually more difficult than identification. Both are necessary.

Review the formulas for the area of a rectangle and the area of a square, and present problems which involve areas of triangles and rectangles.

Area of a Rectangle



1. A rectangle is a 4-sided figure containing 4 right angles. The **base,** b, of a rectangle is the side on which it stands. The **height,** h, is one of the sides **perpendicular** to the base. The formula for the area of a rectangle is

$$A = bh$$

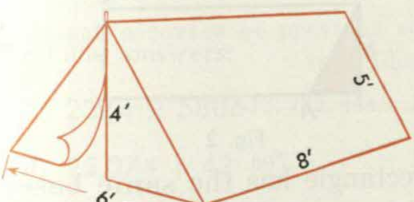
2. Find the area of a rectangle if $b = 6$ in. and $h = 4$ in.; 24sq. in. if $b = 9$ ft. and $h = 7$ ft.; if $b = 1$ ft. and $h = 9$ in.; if $\frac{3}{4}$sq. ft. $b = 1$ yd. and $h = 2$ ft. 6 sq. ft. 63 sq. ft.

3. A square is a rectangle that has 4 equal sides. If s represents each side of a square, its area is

$$A = s^2$$

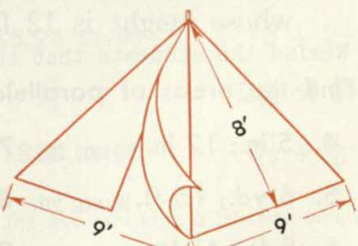
You know that s^2 means $s \times s$. s^2 can be read "s squared."

4. What is the area of a square if one of its sides is 13 ft.? 169 sq. ft. 23 in.? 529 sq. in. 94 ft.? 115 ft.? 14 yd.? 75 ft.? 5625 sq. ft. 8836 sq. ft. 13, 225 sq. ft. 196 sq. yd.



5. Bill has made a canvas tent like the one shown here. Each side of the tent is a rectangle 8 ft. long and 5 ft. wide. Each end of the tent is a triangle with a base of 6 ft. and a height of 4 ft. Find the total area of the sides and ends of the tent. 104 sq. ft. Bill wants to paint the tent with waterproof liquid to make the tent waterproof. How many quarts of this liquid should he buy if 1 qt. will cover 25 sq. ft. of canvas? 5 What will it cost at $1.09 a quart? $5.45

6. Each side of Jerry's tent is a triangle having a base of 9 ft. and a height of 8 ft. How many quarts of waterproof liquid will he need to cover his tent if 1 qt. covers 25 sq. ft.? 6 Find the cost at $1.17 a quart. $7.02



In ex. 5–6 remind the students that the total area is the sum of the areas of all the sides.

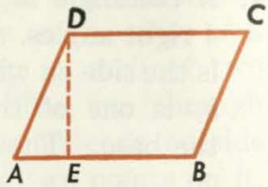
Reteach and use the formula for the area of a parallelogram. Show experimentally that it is correct. First review and illustrate the meaning of parallel lines.

Area of a Parallelogram

Stress.

1. A parallelogram is a four-sided figure having its opposite sides parallel. Figure $ABCD$ is a parallelogram; sides AB and DC are parallel, and sides AD and BC are parallel. The side AB, on which the parallelogram stands, is called its **base;** the segment DE, which is perpendicular to the base, is its **height.** The height of a parallelogram is always measured on a line which is **perpendicular** to the base. The slanting side AD is **not** the height.

Stress.



2. On a sheet of paper draw a parallelogram $ABCD$ as shown below in Fig. 1. Then draw CX perpendicular to AB; CX is the **height,** h, of the parallelogram.* With scissors, cut out the parallelogram $ABCD$; then cut off the colored triangle CXB and place it at the left of AD as shown in Fig. 2. You see that Fig. 2 is a rectangle which has been built

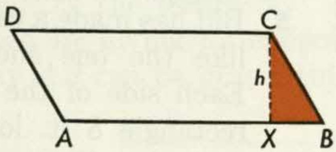


Fig. 1

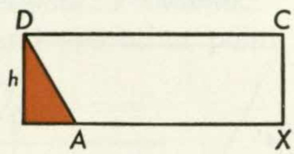


Fig. 2

from the parallelogram. This rectangle has the **same base, height,** and **area** as the parallelogram. The area of a rectangle equals its base times its height, so you have:

*Be sure the students see why this is true.

$$\text{Area of parallelogram} = \text{base} \times \text{height}$$

or

$$A = bh$$

3. Find the area of a parallelogram whose base is 20 ft. and whose height is 12 ft. Use the formula $A = bh$. 240 sq. ft.

Remind the students that the base and the height must be in the same unit.

Find the areas of parallelograms with these bases and heights:

4. 5 in.; 12 in. 60 sq. in.
5. 6 yd.; 15 ft. 30 sq. yd.
6. 8 ft.; $4\frac{1}{2}$ ft. 36 sq. ft.
7. 14 ft.; $12\frac{1}{2}$ ft. 175 sq. ft.
8. 11 in.; 71 in. 781 sq. in.
9. 10 in.; 1 ft. $\frac{5}{6}$ sq. ft.
10. 32 in.; 2 ft. $5\frac{1}{3}$ sq. ft.
11. 2 yd.; $4\frac{1}{2}$ ft. 27 sq. ft.
12. $3\frac{1}{3}$ yd.; 6 ft. $6\frac{2}{3}$ sq. yd.

Emphasize that the height of a parallelogram is the length of the line segment perpendicular to the base, not the length of the slanting side.

Present the first set of improvement tests in division. The method of scoring and recording the tests is the same as for addition tests (see pages 375-377). Have students determine and record their own scores (page 48).

Improving by Practice

Division Test 1a. Time: 5 min. after copying.

1. $58\overline{)20126}$ 347 $27\overline{)79272}$ 2936 $36\overline{)98657}$ $2740\frac{17}{36}$

2. $64\overline{)34472}$ $538\frac{5}{8}$ $52\overline{)17472}$ 336 $21\overline{)26313}$ 1253 6

Division Test 1b. Time: 5 min. after copying.

3. $96\overline{)53664}$ 559 $23\overline{)49381}$ 2147 $32\overline{)17993}$ $562\frac{9}{32}$

4. $44\overline{)95728}$ $2175\frac{7}{11}$ $57\overline{)16359}$ 287 $19\overline{)61883}$ 3257 6

Division Test 1c. Time: 5 min. after copying.

5. $98\overline{)33529}$ $342\frac{13}{98}$ $71\overline{)89744}$ 1264 $67\overline{)65861}$ 983

6. $25\overline{)79475}$ 3179 $84\overline{)79548}$ 947 $32\overline{)44136}$ $1379\frac{1}{4}$ 6

Mixed Practice

Present a review of previous work. Use it to spot-check for class or individual weaknesses.

Find the answers:

7. 2374×5806 13,783,444	8 is .32. % of 25	$\frac{4}{5} \times 12$ $9\frac{3}{5}$
8. $25,074 \div 42$ 597	\$15.00 − \$9.52 \$5.48	$40\frac{1}{2} \div 3\frac{3}{8}$ 12
9. $4203 - 2899$ 1304	$12\frac{1}{2}\%$ of \$2760 \$345	$20 - 6\frac{2}{3}$ $13\frac{1}{3}$
10. 24% of \$675 \$162	$\frac{3}{16} + \frac{1}{4} + \frac{7}{8}$ $1\frac{5}{16}$	$6\frac{2}{3} \times 2\frac{1}{2}$ $16\frac{2}{3}$
11. \$2.75 + \$.88 \$3.63	9 is 6% of . . . 150	$1\frac{2}{3} + 3\frac{1}{2}$ $5\frac{1}{6}$
12. $16,512 \div 192$ 86	6.4% of \$2355 \$150.72	$4\frac{1}{2} - 2\frac{7}{8}$ $1\frac{5}{8}$
13. .5% of \$250 \$1.25	\$10.00 − \$5.94 \$4.06	$15 \div 2\frac{1}{4}$ $6\frac{2}{3}$
14. $85 \times 17,823$ 1,514,955	$2\frac{5}{6} + \frac{1}{2} + 1\frac{1}{3}$ $4\frac{2}{3}$	2% of 5 .1
15. 75% of \$257 \$192.75	225% of \$1824 \$4104	$5\frac{1}{2} + \frac{9}{10}$ $6\frac{2}{5}$

To the Teacher. In ex. 1–6, instruct pupils to write the quotient with a fraction when there is a remainder.

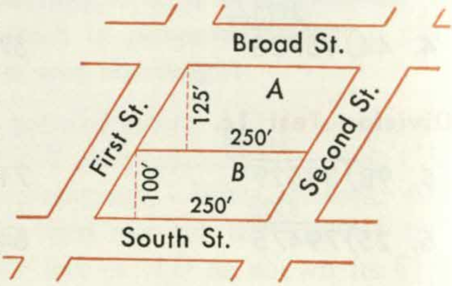
For division tests, always tell the students how you wish remainders to be handled. Help the students to analyze the causes of their errors. Provide remedial work based on specific difficulties of students.

Present problems that involve finding areas of rectangles and parallelograms. Have the students do ex. 1-4 independently to see how well they apply formulas to problem situations.

Finding Areas

1. A rectangular floor is 42 ft. long and 27 ft. wide. Find its area in square feet. 1134 Also find its area in square yards, doing the work in two different ways. 1134 ÷ 9 = 126 sq. yd.; 42 ft. = 14 yd., 27 ft. = 9 yd., 14 x 9 = 126 sq. yd.

2. How many gallons of varnish will it take to varnish the floor in ex. 1 if 1 gallon of varnish covers 600 sq. ft. and if the varnish can be purchased only in 1-gallon cans? 2

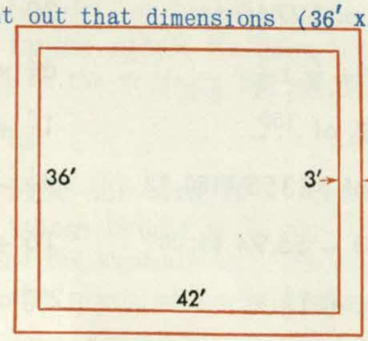


3. Mr. Brooks owns 2 lots, A and B, which cover a whole block as shown on the map at the right. Broad Street and South Street are parallel; First Street and Second Street are also parallel. What kinds of figures are lots A and B? Parallelograms Lot A has a base of 250 ft. and a height of 125 ft. Lot B has a base of 250 ft. and a height of 100 ft. Find the area of each lot in square feet.(1) The 2 lots together equal how many acres?(2) Find the answer to the nearest hundredth of an acre. 1 acre = 43,560 sq. ft.

(1) A: 31,250 sq. ft., B: 25,000 sq. ft.; (2) 1.29

4. A hard-surfaced driveway 10 ft. wide and 45 ft. long (50 sq. yd.) is to be built. What will it cost at $2.25 a square yard? $112.50

Point out that dimensions (36′ x 42′) apply to the inside rectangle.



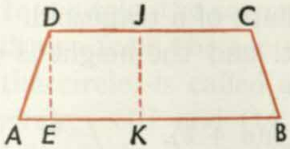
5. Peter's father is building a concrete walk 3 ft. wide around the outside edges of a garden measuring 42 ft. by 36 ft. Find the total area of this walk. 56 sq. yd. What will it cost to build this walk at $4.75 a square yard? $266.00

6. Today Mr. Thomas sold 3 rugs each measuring 9 ft. by 12 ft. and 2 rugs each measuring 12 ft. by 18 ft. If they sold at $9.75 a square yard, how much in all did he receive for the rugs? $819.00

Have the students find out the cost per square yard of different kinds of hard-surface driveways and redo ex. 4. Ex. 5 may be solved in two ways: subtract the area of the inside rectangle from that of the outside one; find the area of the walk directly (4 rectangles).

Develop and apply the formula for finding the area of a trapezoid.

The Area of a Trapezoid

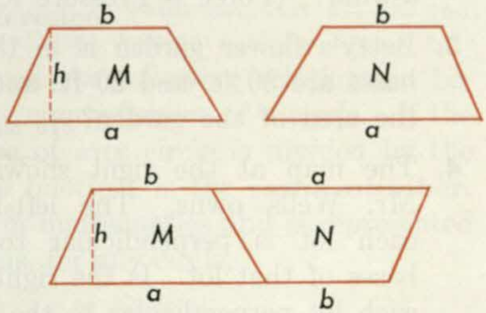


1. A **trapezoid** is a four-sided figure which has two opposite sides parallel. The other two sides are not parallel. In trapezoid $ABCD$, sides AB and DC are parallel, but sides AD and BC are not parallel. The parallel sides AB and DC are called the **bases**, and the segment DE is the **height.** JK is also the height. The height of a trapezoid is always measured perpendicular to the base. Draw five trapezoids of different shapes.

Stress.

Stress.

Stress.



2. You can make a formula for finding the area of a trapezoid by doing this experiment. Cut out from cardboard two trapezoids exactly alike in size and shape, like M and N. Call the lower base of each trapezoid a and the upper base b; draw segment h in trapezoid M to represent the height.

Turn N upside down and place it beside M so that you make a long parallelogram as shown above. The base of this parallelogram equals $a + b$, or the **sum** of the two bases; and its height is h. Thus the area of this parallelogram equals *height* $\times$ *sum of the bases* or $h \times (a + b)$. A shorter way to write $h \times (a + b)$ is $h(a + b)$.

Each trapezoid is only half as large as the long parallelogram, so the area of each trapezoid is $\frac{1}{2}$ the area of the parallelogram, or

$$\textit{Area of trapezoid} = \frac{\textit{height} \times \textit{sum of bases}}{2}$$

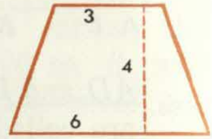
or $$A = \frac{h(a + b)}{2}, \quad \text{or} \quad A = \frac{1}{2}h(a + b)$$

Have the students label the bases and the height in trapezoids they draw. Have them perform the experiment under your direction and, through questions, lead them to discover the formula.

Problems about Trapezoids

1. Problem Joe has a sail made in the shape of a trapezoid. The two parallel sides are 6 ft. and 3 ft. and the height is 4 ft. Find the area of the sail. 18 sq. ft.

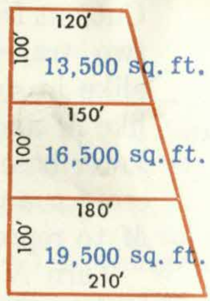
Explanation Use the formula $A = \frac{1}{2}\,h(a + b)$. Since $a = 6$, $b = 3$, and $h = 4$, the area is $\frac{1}{2} \times 4 \times 9$. How do you get the number 9? What is the area of the sail?



2. If a brisk wind pushes against a sail with a pressure of $2\frac{1}{2}$ lb. per square foot, what is the total force on Joe's sail in such a wind? (Force = Pressure × Area.) 45 See ex. 1.

3. Betty's flower garden is in the shape of a trapezoid. The bases are 30 ft. and 20 ft. and the height is 15 ft. What is the area of the garden? 375 sq. ft.

4. The map at the right shows 3 lots that Mr. Wells owns. The left-hand side of each lot is perpendicular to the parallel bases of that lot. Is the right-hand side of each lot perpendicular to the bases? No What kind of figure is each lot? Trapezoid What is the height of each lot? 100 ft. Find the area of each lot. See map. The 3 lots together equal how many acres? 1.1 Find the answer to the nearest tenth of an acre.



5. A playground which is in the shape of a trapezoid has bases of 300 ft. and 400 ft. and a height of 425 ft. Find its area to the nearest tenth of an acre. 3.4

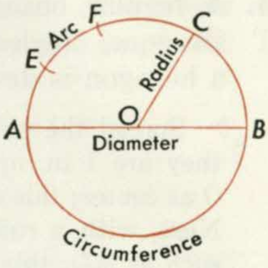
Find the area of these trapezoids:

	Bases	Height			Bases	Height	
6.	35 in., 23 in.	40 in.	1160 sq. in.	**10.**	$4\frac{1}{2}$ ft., $6\frac{1}{2}$ ft.	10 ft.	55 sq. ft.
7.	17 yd., 19 yd.	8 yd.	144 sq. yd.	**11.**	70 in., 30 in.	15 in.	750 sq. in.
8.	90 ft., 70 ft.	85 ft.	6800 sq. ft.	**12.**	5.6 ft., 9.4 ft.	18 ft.	135 sq. ft.
9.	44 yd., 28 yd.	30 yd.	1080 sq. yd.	**13.**	10 yd., 8 yd.	11 ft.	297 sq. ft.

Ex. 1 should be discussed with the class. Emphasize that the height is the length of the line segment perpendicular to the base.

Reteach the formula for finding the circumference of a circle.

Circumference of a Circle



1. In a circle, the segment *OC*, which is drawn from the center to a point on the circle, is called a **radius** of the circle. *OB* and *OA* are also **radii** (plural of radius). The segment *AB*, which passes through the center, with both its ends on the circle, is called a **diameter.** You see that a diameter is twice as long as a radius. A part of the circle, such as *EF*, is an **arc** of the circle. The distance around a circle is called its **circumference.** Point out that all the radii of the same circle are equal and all the diameters of the same circle are equal.

2. Mathematicians have proved the following relationship between the diameter and the circumference of a circle. If the length of the circumference of **any** circle is divided by the length of its diameter, the quotient is the **same number.** This number is important in mathematics and is represented by the Greek letter π. Thus, for any circle,

$$\text{Circumference} = \pi \times \text{diameter}$$

or briefly,

$$C = \pi d$$

The letter π is spelled **pi** and pronounced "pie." π is approximately equal to $3\frac{1}{7}$. As a decimal π is equal to 3.14, correct to the nearest hundredth. A more exact value of π is 3.14159, but for most of your work you can use either 3.14 or $3\frac{1}{7}$ for the value of π.

3. Since a diameter is twice as long as a radius, you can replace d with $2r$ in the above formula and get the formula

$$C = 2\pi r$$

These formulas make it possible to find the circumference of a circle without measuring it directly.

Remind the students to notice whether the diameter or the radius is given in ex. 4–5.

Using $\pi = 3.14$, find the circumferences of these circles:

4. $d = 25$ ft. 78.5 ft. $d = 30$ in. 94.2 in. $r = 5$ mi. 31.4 mi. $r = 17.5$ in. 109.9 in.

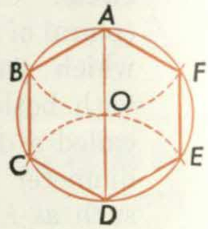
5. $r = 62.5$ ft. 392.5 ft. $d = 16$ in. 50.24 in. $d = 200$ ft. 628 ft. $r = 2\frac{1}{2}$ yd. 15.7 yd.

Draw a circle on the board. Then draw a diameter and emphasize that it always passes through the center of the circle. Show by measuring that the diameter is the longest distance across a circle.

Reteach the drawing of designs based on a hexagon inscribed in a circle.

Drawing a Hexagon

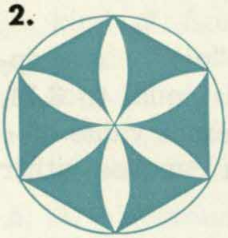
1. A regular hexagon is a figure having six equal sides and six equal angles. A quick and accurate way to construct a hexagon is described below.



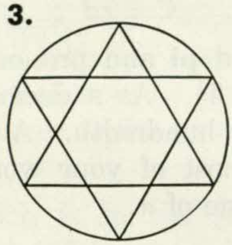
▶ Spread the points of your compasses so that they are 1 in. apart. Then draw a circle with O as center; this circle will have a radius of 1 in. Next, with a ruler, draw lightly any diameter, such as AD; this diameter passes through point O. Using point A as center, and a radius of 1 in., draw a long arc BOF cutting the circle at B and at F. Then with the same radius, use D as center and draw a long arc COE cutting the circle at C and at E; these long arcs each pass through the center O. Draw segments connecting A, B, C, D, E, and F and you will have a regular hexagon $ABCDEF$.

Copy each design below. First draw each circle with a radius of $1\frac{1}{2}$ in.; then divide each circle into 6 equal parts by drawing a hexagon. Design 3 shows how to start designs 4 to 7:

2.

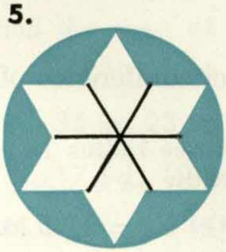


3.

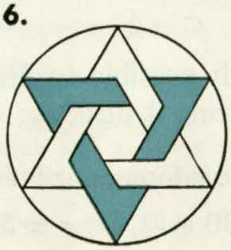


4.

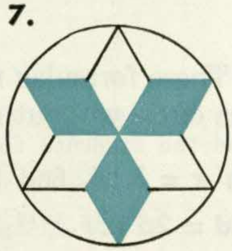
5.



6.



7.



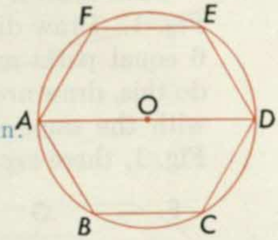
Emphasize the meaning of regular hexagon as given in ex. 1. After the students complete ex. 2–7, encourage them to make original designs for the bulletin board or scrapbooks.

Show experimentally that the formula for the circumference of a circle is a reasonable one.

A Reasonable Formula

1. You have learned that the circumference of a circle is a little longer than 3 times the diameter of the circle. The experiment below shows that this is true.

▶ Draw a circle with a radius of 1 in. Then draw a hexagon in this circle as you did on page 92. In your drawing, how many inches long is OD? 1 How many inches long is DE? 1 in. How long is EF? FA? 1 in. AB? 1 in. BC? 1 in. CD? 1 in. What is the total distance in inches around the hexagon? 6 in. How many inches long is the diameter AD of the circle? 2 The distance around the hexagon is how many times as long as the diameter AD? 3 times



2. Which is longer, the distance around the circle (which is the circumference) or the distance around the hexagon? Distance around circle

3. In ex. 1, you found that the distance around the hexagon is exactly 3 times as long as the diameter AD. In ex. 2, you see that the circumference of the circle is a little longer than the distance around the hexagon. Thus the circumference of the circle is a little longer than 3 times the diameter. In ex. 2, page 91, you are given this relationship:

Circumference = π × diameter

Since π is approximately equal to 3.14, which is a little more than 3, you see that this relationship is a reasonable one.

4. Each wheel on Ted's bicycle has a diameter of 28 in. Find the circumference of each wheel in inches. 88 How many feet does each wheel of the bicycle travel in one complete turn? $7\frac{1}{3}$ Use $\pi = 3\frac{1}{7}$.

5. To find the distance from his home to school, Ted tied a piece of cloth on the rim of the front wheel of his bicycle so he could count the number of revolutions the front wheel made. On the trip from his home to school, the front wheel made 182 revolutions. How many feet is it to the school? $1334\frac{2}{3}$ Is it about $\frac{1}{4}$ mi. or $\frac{1}{2}$ mi.? 1 mi. = 5280 ft. About $\frac{1}{4}$

Teach the drawing of designs based on a circle divided into 12 equal parts (pages 94–95).

Drawing a Dodecagon

1. A **regular dodecagon** is a figure having 12 equal sides and 12 equal angles. To draw a dodecagon, follow the directions given below:

▶ First draw a circle with a radius of 1 in., using *O* as center; see Fig. 1. Draw diameter *AB*. Next, in Fig. 1, divide the circle into 6 equal parts as you did on page 92 in drawing a hexagon. To do this, draw arc *EOF* with *A* as center and a radius of 1 in. Then, with the same radius and with *B* as center, draw arc *GOH*. In Fig. 1, there are now 6 points on the circle, *A, F, H, B, G,* and *E*.

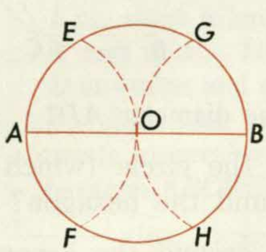


Fig. 1

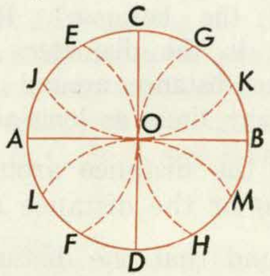


Fig. 2

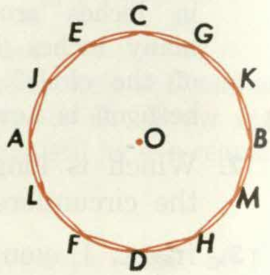
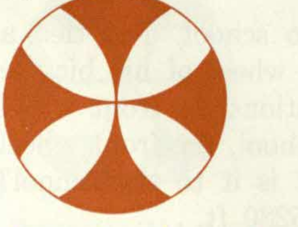


Fig. 3

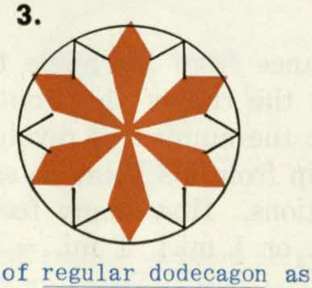
Now draw diameter *CD* perpendicular to *AB* at O, as shown in Fig. 2, and proceed as if you were drawing another hexagon. With *C* as center, and a radius of 1 in., draw arc *JOK* as in Fig. 2; with *D* as center, and the same radius, draw arc *LOM*. In Fig. 2 there are now 12 points on the circle which divide it into 12 equal parts. Connect the 12 points in order with segments, as in Fig. 3, and you will have a regular dodecagon. Erase the diameters and arcs which you drew.

Copy these designs. First divide the circle into 12 equal parts as shown above:

2.

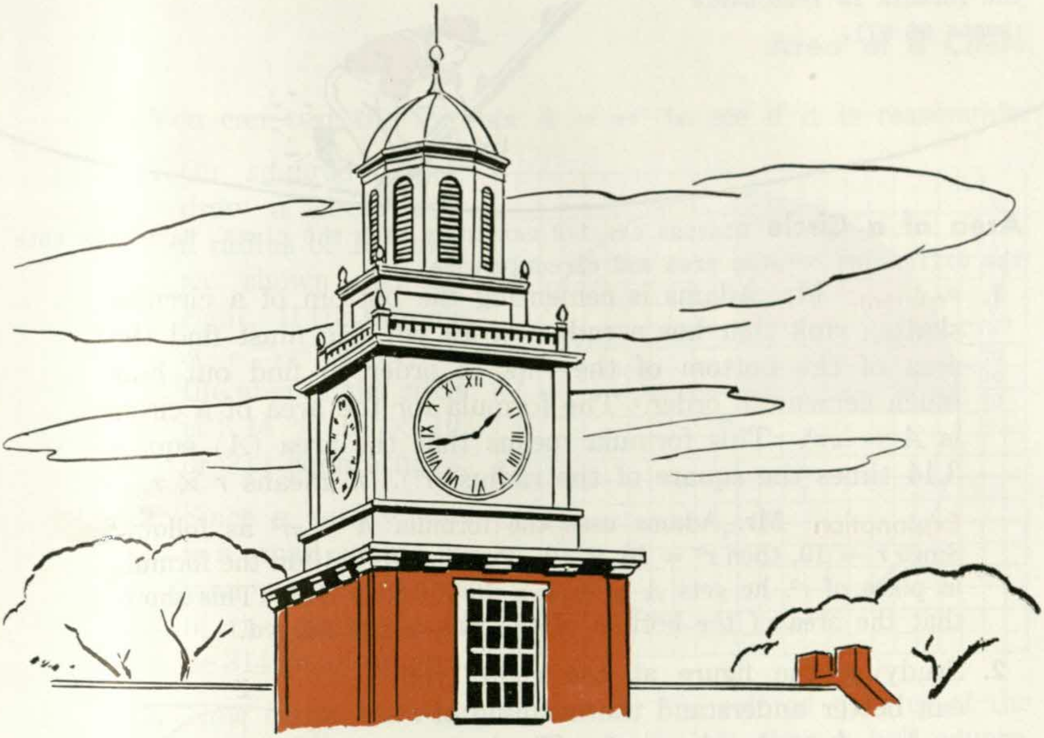


3.



4.

Emphasize the meaning of regular dodecagon as given in ex. 1. Have the students do ex. 1 under your supervision. Then have them copy the designs in ex. 2–4.



5. The building in the above picture has a large clock as a part of its design. To make a clock it is necessary to divide a circle into 12 equal parts. Why?

6. Draw a circle with a radius of 2 in. and divide this circle into 12 equal parts, using the method given on page 94. Then make a clock face in the circle and draw the hands of the clock to show 10:30.

7. Draw a circle with a radius of $1\frac{1}{2}$ in. and divide it into 12 equal parts. Then make a design of your own similar to those on page 94.

8. The windows of a cathedral often have designs in which the circle is divided into 12 equal parts. Try to find some pictures of cathedrals which have this kind of window. Post them on the bulletin board.

9. Tell the class about any other designs in architecture that you have seen where a circle is divided into 12 equal parts.

For ex. 7 urge the students to make several designs of their own for the bulletin board or scrapbooks.

Present the formula for the area of a circle and show experimentally that the formula is reasonable
(pages 96-97).

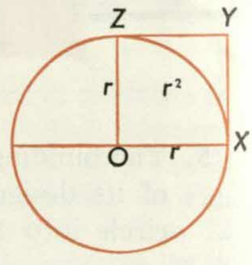
Area of a Circle

Discuss ex. 1–2 carefully with the class. Have them note the difference between area and circumference.

1. **Problem** Mr. Adams is cementing the bottom of a circular skating rink that has a radius of 10 yd. He must find the area of the bottom of the rink in order to find out how much cement to order. The formula for the area of a circle is $A = \pi r^2$. This formula means that the area (A) equals 3.14 times the square of the radius (r^2). r^2 means $r \times r$.

   Explanation Mr. Adams uses the formula $A = \pi r^2$ as follows: Since $r = 10$, then $r^2 = 10 \times 10$. Putting 10×10 in the formula in place of r^2, he gets $A = 3.14 \times 10 \times 10$, or 314. This shows that the area of the bottom of the rink is 314 sq. yd.

2. Studying the figure at the right, you can better understand the meaning of r^2 in the formula $A = \pi r^2$. The square $OXYZ$ has each side equal to a radius (r) of the circle; therefore its area is $r \times r$, or r^2. The area of the circle is 3.14 times as large as the area of this square.



3. Using 3.14 for π, find the area of the bottom of a circular skating rink having a radius of 60 ft. 11,304 sq. ft.

Have students make representative diagrams (showing radii) for ex. 4.

4. Measure, to the nearest eighth of an inch, the diameters of at least four of the following: a round table, a large round plate, a small round plate, the top of a round tin can, a phonograph record, the bottom of a pail. Find the radius of each and then find the area. Remind students to take the greatest length across the circle as the diameter.

5. Using $3\frac{1}{7}$ for π, find the areas of circles with these radii: 63 in., 49 yd., 56 in., 70 ft., 77 in., 35 yd. 3850 sq. yd.

12,474 sq. in. 7546 sq. yd. 9856 sq. in. 15,400 sq. ft. 18,634 sq. in.

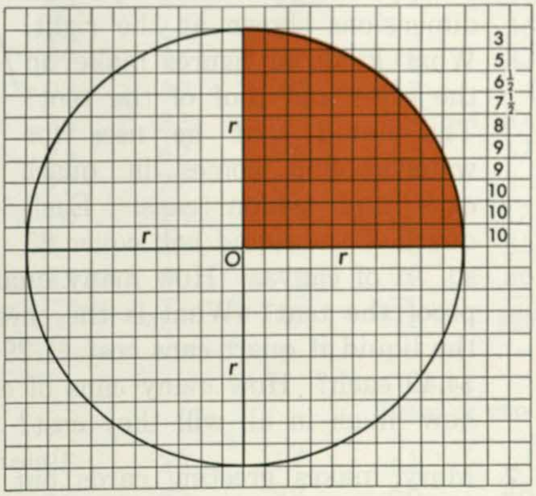
6. Using 3.14 for π, find the areas of circles with these radii: 70 ft., 50 yd., 20 in., 80 yd., 40 ft., 100 ft. 31,400 sq. ft.

15,386 sq. ft. 7850 sq. yd. 1256 sq. in. 20,096 sq. yd. 5024 sq. ft.

Make clear and emphasize that $A = \pi r^2$ means $\pi \times r \times r$. Stress the fact that the area is measured in square units.

You can test the formula $A = \pi r^2$ to see if it is reasonable.

1. On squared paper, draw a circle with a radius of 10 units, as shown at the right. If the formula $A = \pi r^2$ is correct, the area of the circle is $3.14 \times 10 \times 10$, or 314 square units.



2. Since a square unit is a small square on your paper, the area of the circle should be 314 small squares.
3. Now count the small squares in the colored quarter of the circle. If any part of a square is less than a half square, drop it; if it is larger than a half square, count it as a whole square; if it is a half square, count it as a half square.

Stress. Stress. Stress.

4. In the top row of the colored part, beginning at the left, the first square is a whole square, the second and third squares count as whole squares, the fourth square is dropped because it is less than a half square. Thus the top row counts as 3 squares in all. Write 3 in the column at the right. Count the squares in each of the other colored rows in the same way. Adding the numbers in the column, you get 78 squares in this quarter of the circle.
5. Since the colored quarter of the circle contains about 78 squares and since all quarters of the circle are equal, the entire circle contains about 4×78 squares, or 312 squares. This result is only approximately correct because small parts of squares were dropped as you counted. By using the formula, you found that the area should be 314 squares. Since 312 is so close to 314, you see that the formula is reasonable.

Stress.

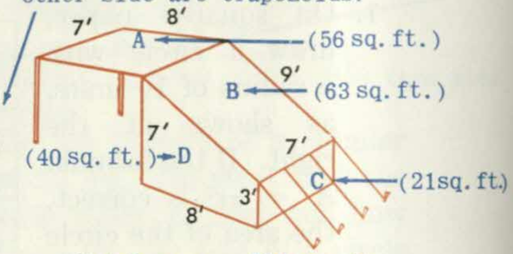
Do this experiment as a class activity first, using the diagram shown above. Have the students count the squares as explained in ex. 3-4. Then discuss ex. 5. Each student should then perform his own experiment using squared paper.

Present a review of problem solving connected with areas. In ex. 3 refer the students to the table of square measure on page 373.

Problems

1. Mr. Lane wishes to waterproof a tent that has the shape and dimensions shown at the right. What kinds of figures make up the sides and roof of the tent? The liquid that he uses for waterproofing comes in quart cans and gallon cans. Each quart of the liquid will cover 25 sq. ft. of canvas. How many quarts does he need to waterproof the tent? 9 What is the cheapest way for him to buy the liquid if quart cans cost $1.29 each and gallon cans cost $4.49 each? How many cans of each size does he need and how much in all will they cost? He should buy 2 gallon cans and 1 quart can, at a cost of $10.27.

A,B,C are rectangles; D and other side are trapezoids.



Total area = 220 sq. ft.

2. Mary makes greeting cards out of large colored blotters. How many cards each measuring 3 in. by 5 in. can she cut out of a blotter that is 25 in. long and 18 in. wide? 30 Make a scale drawing and show how the cards can be cut without wasting any material. (15sq. in. (450 sq. in.)

3. Today Mr. Gray sold a field shaped like a parallelogram with a base of 605 ft. and a height of 180 ft. How many acres did this field contain? 2.5 If Mr. Gray received $425 for the field, at what price per acre did he sell it? $170 (108,900 sq. ft.)

4. Mrs. Reed wants to cover her kitchen floor with linoleum. It measures 12 ft. by 14 ft. She can buy linoleum in strips either 6 ft. wide or 9 ft. wide. Which of these could she use without waste? (1) Draw a plan of the floor and show how to lay the strips of linoleum. How many running feet of linoleum should Mrs. Reed order? (2) What will the cost be if the 6-foot linoleum sells for 69¢ a running foot and the 9-foot linoleum sells for $1.03 a running foot? (3) (1) Two strips that are 6 ft. wide and 14 ft. long; (2) 28; (3) $19.32

5. Jane Dale is buying tulip bulbs for a square garden that measures 9 ft. on a side. (81 sq. ft.) If each bulb needs $\frac{1}{2}$ sq. ft. of space, how many bulbs should Jane buy? 162

A class review of scale drawings may be necessary if many students evidence difficulties.

Present the second set of improvement tests in addition. Have the students determine and record their own scores in Record Books (page 48).

Addition Test 2a.

Time: 4 min.

1.

53	83	85	75	39	94	12	99
42	34	78	24	68	51	71	36
86	99	83	59	81	37	78	83
65	28	67	93	69	49	63	14
21	44	56	86	83	56	19	84
37	35	72	25	62	96	65	57
43	97	70	22	77	67	20	34
58	52	53	16	25	10	67	17
49	71	49	71	41	98	48	85
49	68	24	35	80	76	99	56
503	611	637	506	625	634	542	565

8

Addition Test 2b.

Time: 4 min.

2.

75	18	72	85	38	17	41	14
82	21	53	67	45	99	66	48
37	83	76	24	97	67	26	57
14	39	60	65	34	84	83	26
71	53	52	88	50	57	59	98
80	47	66	99	43	87	67	95
42	78	95	21	89	59	53	34
55	80	25	74	66	60	16	64
29	47	39	50	88	56	60	43
69	40	92	82	66	46	78	26
554	506	630	655	616	632	549	505

8

Addition Test 2c.

Time: 4 min.

3.

47	20	57	85	99	33	12	37
79	98	49	86	26	70	51	46
87	55	59	58	47	92	23	85
99	67	32	71	85	63	45	98
63	86	97	98	57	13	11	77
37	18	76	49	32	52	64	66
64	37	99	87	86	63	24	23
82	87	27	79	58	40	69	83
50	69	18	86	44	52	52	70
46	39	99	98	69	68	59	55
654	576	613	797	603	546	410	640

8

Check papers carefully and note the kinds of errors. Have the students compare scores with previous ones to note improvements or weaknesses. Set up remedial activities as needed.

Provide more problems involving areas of circles.

Finding Areas of Circles

1. Susan has a circular flower garden. The radius of the garden is 6 ft. Find the area of the garden. Use $\pi = 3.14$. 113.04 sq. ft.

2. Jean also has a circular garden. Her garden has a diameter of 8 ft. What is the area of her garden? 50.24 sq. ft. First find the radius of the garden. Stress.

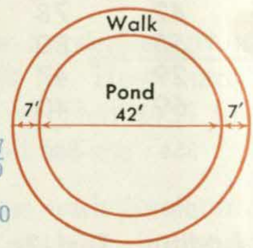
3. John has two gardens. One garden has the shape of a square which measures 28 ft. on each side; (784 sq. ft.) the other has the shape of a circle with a diameter of 28 ft. (616 sq. ft.) Which garden has the larger area? The square garden How many square feet larger is it? 168 Use $\pi = 3\frac{1}{7}$.

Find the areas of these circles. Let $\pi = 3.14$:

4. $r = 15$ ft. 706.5 sq. ft. $d = 20$ in. 314 sq. in. $d = 50$ ft. 1962.5 sq. ft. $r = 12$ mi. 452.16 sq. mi.

5. $d = 60$ ft. 2826 sq. ft. $r = 8$ yd. 200.96 sq. yd. $d = 7$ in. 38.465 sq. in. $r = 18$ in. 1017.36 sq. in.

6. At the Pine Tree Playground, there is a circular wading pond that is 42 ft. in diameter. A concrete walk, 7 ft. wide, is to be laid around the edge of the pond. What is the total area of this walk in square feet? 1078 in square yards? $119\frac{7}{9}$ How much will it cost at \$4.95 a square yard? \$594.00



▶ To find the area of the walk, subtract the area of the small circle from the area of the large circle. In finding the cost, call a fraction of a square yard another square yard. Use $\pi = 3\frac{1}{7}$.

7. The new park has a large circular pond that has a radius of 93 ft. In winter it is used for skating. If Dick skates around the pond, keeping 2 ft. in from the outside edge, about how many times must he skate around the pond to cover 1 mile? $9\frac{1}{4}$ Next summer a new cement bottom will be put in the pond. Find its area. $27,182\frac{4}{7}$ sq. ft.

Remind the students that when the diameter is given they must first find the radius. For ex. 7, have the students make a diagram similar to the one in ex. 6.

Teach the rule for choosing the correct value of π for use under certain conditions.

What Value of π Should You Use?

1. In your work with circles, you have been using $3\frac{1}{7}$ or 3.14 as the value of π. The values $3\frac{1}{7}$ and 3.14 are both approximate values of π. A more exact value is 3.14159, but even this value is not exact.

2. Round off 3.14159 correct to the nearest ten-thousandth and show that the result is 3.1416. Next round off 3.14159 correct to the nearest thousandth; what result do you get? 3.142 Then round off 3.14159 to the nearest hundredth; what is the result? 3.14

Approximate Values of π

3.14159
3.1416
3.142
3.14

3. In ex. 2, you have 4 different approximate values of π, which are listed at the right. In working a problem, which value of π should you use?

4. Mathematicians say that if the number to be multiplied by π represents a measurement that has been carefully made, your results will be most sensible if you follow these rules:
   (a) If you multiply a 2-figure number like 2.8 by π, use the 3-figure value of π, which is 3.14.
   (b) If you multiply a 3-figure number like 278 by π, use the 4-figure value of π, which is 3.142.
   (c) If you multiply a 4-figure number like 602.5 by π, use the 5-figure value of π, which is 3.1416.

Emphasize *Use the value of π that contains 1 more figure than there are in the number that is to be multiplied by π.*

In the following, use the value of π suggested in ex. 4:

5. A circular steel plate is manufactured with a diameter of 8.65 in. Find its circumference. Round off your answer to the nearest hundredth of an inch. 27.18

6. Find the circumference of a circle if $d = 2.5$ ft.; 7.85 ft. if $d = 325$ ft.; 1021.15 ft. if $d = 45.25$ in.; 142.16 in. if $r = 145.27$ ft. 912.76 ft.

7. Find the area of a circle whose radius is 23 in. 1661.1 sq. in.

▶ Since the radius is a 2-figure number, use the 3-figure value of π.

Teach how to make tables from formulas and how to make and read graphs of formulas (pages 102–104). Supply the students with squared paper for use in making graphs on these pages.

Making Tables from Formulas

1. Problem At the rate of 3 mi. per hour, how far can Jack walk in 2 hr.? 6 mi. in 3 hr.? 9 mi. in 4 hr.? 12 mi. in 5 hr.? 15 mi.

Explanation You find the distance Jack walks by multiplying his rate in miles per hour by the number of hours he walks. Since Jack's rate is 3 mi. per hour, you can express the distance he walks by the following formula, in which D stands for **distance** in miles and t stands for the **time** in hours.

$$D = 3t$$

To find the distance Jack walks in 2 hr., substitute 2 for t in the formula. This gives $D = 3 \times 2$, or $D = 6$. So Jack walks 6 mi. in 2 hr. To find the distance he walks in 3 hr., substitute 3 for t in the formula, which gives $D = 3 \times 3$, or 9. Find the value of D when $t = 4$; when $t = 5$.

2. In the work above, each time you put a new value of t in the formula, you get a new value of D. These values are shown in the table at the right. Continue this table to show the distance Jack walks in 6 hr.; 18 mi. in 7 hr.; 21 mi. in 8 hr. 24 mi.

Hours t	Miles D
0	0
1	3
2	6
3	9
4	12
5	15

3. Ann rides her bicycle at the rate of 8 mi. per hour. Write a formula $D = 8t$ from which you can find the distance she rides in 2 hr.; 16 mi. in 3 hr.; 24 mi. in 4 hr.; 32 mi. in 5 hr. 40 mi. In your formula, let t stand for the number of hours she rides. Then put the values of D and t in a table as you did in ex. 2. See Guide.

4. An automobile travels at the rate of 40 mi. per hour. Write a formula showing the distance it travels in t hours. $D = 40t$ Make a table showing these distances, using values of t from 0 hr. to 9 hr. See Guide.

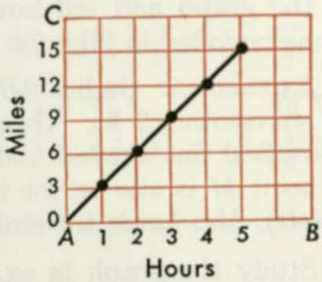
5. Write a formula from which you can find the distance a plane travels in t hours if its speed is 300 mi. per hour. $D = 300t$ Then make a table, using values of t from 0 hr. to 6 hr. See Guide.

6. Write a formula from which you can find the distance a train travels in t hours if its speed is 55 mi. per hour. $D = 55t$ Then make a table, using values of t from 0 hr. to 8 hr. See Guide.

Review the reading and interpreting of line graphs taken from newspapers. Discuss ex. 1–2 with the class. Be sure all understand how the formula is derived and how to use it to make a table.

1. The table below is the one given in ex. 2 on page 102. This table shows the values of D that you get by substituting different values of t in the formula $D = 3t$. By using these values you can make a graph of the formula $D = 3t$.

Hours t	Miles D
0	0
1	3
2	6
3	9
4	12
5	15



▶ This is the way to make the graph. On squared paper, first draw a horizontal line AB and mark on it the hours that are shown in the table; AB is called the **horizontal scale.** You see that each space on AB stands for 1 hr. Next draw vertical line AC and mark the miles on it; AC is called the **vertical scale.** Each space on AC stands for 3 mi. Then locate each of the points shown on the graph as follows:

In the table you see that Jack walked 3 mi. in 1 hr. To show this fact on the graph, place a point directly above 1 hr. and directly to the right of 3 mi. To show that Jack walked 6 mi. in 2 hr., place a point directly above 2 hr. and directly to the right of 6 mi. Where will you place a point to show that Jack walked 9 mi. in 3 hr.? 12 mi. in 4 hr.? 15 mi. in 5 hr.? Connect the points you have just placed and you will find that they all lie on the same straight line. This straight line is the **graph** of the formula $D = 3t$.

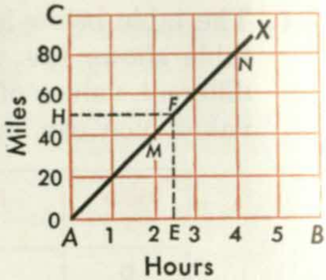
2. Using the table you made in ex. 3 on page 102, draw a graph showing the distances covered by Ann at the end of each hour. Let each space on the horizontal scale represent 1 hr.; let each space on the vertical scale represent 4 mi.

3. On the graph you made in ex. 2, prolong the graph line that you drew by locating points and show that this line will pass through the point representing 6 hr. and 48 mi. and also through the point representing 7 hr. and 56 mi.

When discussing ex. 1, emphasize that each point on the graph represents two values at the same time. Have the students construct the graph in ex. as each point is determined. Then have them do ex. 2-3 under your supervision.

Reproduce the graph in ex. 1 on the board. Give the students an opportunity to locate points on it as called for in ex. 1–5.

Reading Graphs



1. Problem Mr. Lewis travels at the rate of 20 mi. per hour in his motorboat. The graph at the right shows a trip that Mr. Lewis took. Read the graph and tell how many miles he traveled in 2 hr. 40

Explanation On line *AB*, find the point representing 2 hr. Then move up the vertical line above 2 until this line cuts *AX* at point *M*. Since point *M* is also on the horizontal line that is marked 40 (at the left), Mr. Lewis traveled 40 mi. in 2 hr.

2. Study the graph in ex. 1. How far had Mr. Lewis traveled at the end of 3 hr.? 60 mi. at the end of 4 hr.? 80 mi.

3. By reading the graph above, tell how many hours it took Mr. Lewis to travel 80 mi., 4 60 mi., 3 40 mi. 2

▶ On line *AC*, find 80; then move to the right on the horizontal line marked 80 until this line cuts the graph line *AX* at *N*. You see that *N* is also on the vertical line that is marked 4 (at the bottom); so it took 4 hr. to cover 80 mi.

4. On the graph above, find the distance traveled in $2\frac{1}{2}$ hr. 50 mi.

▶ First, on line *AB*, find the point *E*, representing $2\frac{1}{2}$, which is halfway between 2 and 3. Then draw a vertical line *EF* upward from *E* until it cuts line *AX* at *F*. From *F* draw a horizontal line *FH* to the left until it cuts *AC* at *H*. Since *H* is halfway between 40 and 60, it represents 50 mi. Mr. Lewis had traveled 50 mi. at the end of $2\frac{1}{2}$ hr.

5. How far did Mr. Lewis travel in $1\frac{1}{2}$ hr.? 30 mi. in $3\frac{1}{2}$ hr.? 70 mi. in $\frac{1}{2}$ hr.? 10 mi.

6. On squared paper, make a copy of the graph in ex. 1. Then extend the horizontal scale to 8 hr., the vertical scale to 160 mi., and the graph line *AX* until it cuts the vertical line representing 8 hr. From this graph, read the distance traveled in 6 hr.; 120 mi. in 7 hr.; 140 mi. in $5\frac{1}{2}$ hr.; 110 mi. in $6\frac{1}{2}$ hr.; 130 mi. in $4\frac{1}{2}$ hr. 90 mi.

7. From your drawing, tell how many hours it took to travel 120 mi.; 6 140 mi.; 7 150 mi.; $7\frac{1}{2}$ 70 mi.; $3\frac{1}{2}$ 130 mi.; $6\frac{1}{2}$ 90 mi. $4\frac{1}{2}$

Most of the work on this page should be done as a class activity. Be sure all the students can read information from the graph, using the vertical or horizontal scale. Have the students bring to class and interpret other line graphs found in newspapers.

Present a review of computation with fractions, decimals, and percentage. Plan remedial activities or reteaching on a class basis or an individual basis as needed.

Mixed Practice

Find the answers:

1. $4\frac{3}{16} + 2\frac{7}{8} + 9 + 5\frac{1}{4} + 1\frac{5}{8}$ $22\frac{15}{16}$ $\quad$ $2\frac{5}{6} + 3\frac{7}{12} + 7\frac{1}{2} + 1\frac{3}{4}$ $15\frac{2}{3}$
2. $8\frac{7}{10} + 4\frac{1}{2} + \frac{4}{5} + 12 + 6\frac{3}{5}$ $32\frac{3}{5}$ $\quad$ $9\frac{3}{4} + 4\frac{5}{6} + 5\frac{2}{3} + 6\frac{1}{2}$ $26\frac{3}{4}$
3. $2\frac{5}{8} \div 1\frac{3}{4}$ $1\frac{1}{2}$ $\quad$ $15 \div \frac{2}{3}$ $22\frac{1}{2}$ $\quad$ $12 \div 1\frac{1}{8}$ $10\frac{2}{3}$ $\quad$ $\frac{5}{6} \div 1\frac{1}{4}$ $\frac{2}{3}$
4. $24 \times 6\frac{7}{8}$ 165 $\quad$ $\frac{9}{10} \times \frac{5}{6}$ $\frac{3}{4}$ $\quad$ $1\frac{3}{5} \times 3\frac{3}{4}$ 6 $\quad$ $\frac{9}{10} \times 4\frac{1}{2}$ $4\frac{1}{20}$
5. $9\frac{7}{8} - 3\frac{3}{8}$ $6\frac{1}{2}$ $\quad$ $12 - \frac{7}{16}$ $11\frac{9}{16}$ $\quad$ $4\frac{1}{5} - 2\frac{7}{10}$ $1\frac{1}{2}$ $\quad$ $8\frac{3}{8} - 4\frac{5}{6}$ $3\frac{13}{24}$

Divide. If you continue to have a remainder, find the answer correct to the nearest hundredth: Emphasize.

6. $25\overline{)28.5}$ 1.14 $\quad$ $79\overline{)83.5}$ 1.06 $\quad$ $46\overline{)2.51}$ .05 $\quad$ $33\overline{)117.4}$ 3.56 $\quad$ $58\overline{)21.66}$ .37
7. $.19\overline{)133}$ 700 $\quad$ $6.4\overline{)203}$ 31.72 $\quad$ $.08\overline{)131}$ 1637.5 $\quad$ $11.6\overline{)667}$ 57.5 $\quad$ $4.72\overline{)117}$ 24.79
8. $3.6\overline{)28.3}$ 7.86 $\quad$ $.05\overline{)1.45}$ 29 $\quad$ $.56\overline{)8.94}$ 15.96 $\quad$ $.004\overline{)15.4}$ 3850 $\quad$ $.128\overline{).047}$ .37

Estimate the answers.(1) Then find the exact answers:(2)

(1),(2)
9. 48% of \$10.25 \$5, \$4.92 $\quad$ 12.4% of \$80 \$10, \$9.92 $\quad$ 151% of \$79 \$120, \$119.29
10. 11% of \$49.00 \$5, \$5.39 $\quad$ 19.6% of \$35 \$7, \$6.86 $\quad$ 198% of \$81 \$160, \$160.38
11. 24% of \$28.75 \$7, \$6.90 $\quad$ 33.2% of \$75 \$25, \$24.90 $\quad$ 74% of \$60 \$45, \$44.40
12. 19% of \$26.00 \$5, \$4.94 $\quad$ 37.4% of \$40 \$15, \$14.96 $\quad$ $49\frac{1}{2}$% of \$80 \$40, \$39.60

Find the number that belongs in each space. In ex. 14, find the per cent to the nearest whole per cent; in ex. 15, find the per cent to the nearest tenth of 1%:

13. 31 is ... % of 40 $77\frac{1}{2}$ $\quad$ 51 is ... % of 85 60 $\quad$ 40 is ... % of 125 32
14. 15 is ... % of 78 19 $\quad$ 86 is ... % of 99 87 $\quad$ 79 is ... % of 384 21
15. 29 is ... % of 63 46.0 $\quad$ 49 is ... % of 58 84.5 $\quad$ 82 is ... % of 217 37.8
16. 28 is 8% of ... 350 $\quad$ 36 is 15% of ... 240 $\quad$ 48 is 20% of ... 240
17. 82 is 2% of ... 4100 $\quad$ 70 is 56% of ... 125 $\quad$ 60 is 16% of ... 375

Note the kinds of errors made by students. Have volunteers explain the solutions and encourage a class discussion of them.

Review the formulas presented in Chapter 3 through problems. Notice that the students must use formulas to find unknown dimensions, rather than the area or the circumference (except ex. 4–5).

More Problems

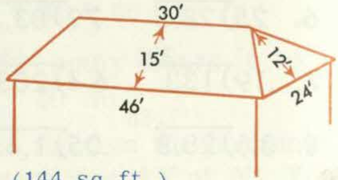
1. The circumference of a circular pond is 198 ft. long. What is the diameter of the pond? 63 ft. Use $\pi = 3\frac{1}{7}$.

▶ The formula for the circumference is $C = \pi d$. Substituting 198 for C in the formula, you get $198 = 3\frac{1}{7}\,d$. How do you find d?

Do ex. 1 with the class before assigning ex. 2–8.

2. Mrs. Allen has a kitchen floor that is 5 yd. long. When she had the floor covered with linoleum, it took 20 sq. yd. of linoleum to cover it, so the area of the floor is 20 sq. yd. How many yards wide is the floor? 4

3. For his new boat Carl wants to make a triangular sail that will have an area of 15 sq. ft. If he makes the base of the sail 6 ft. long, how many feet high should he make it? 5



4. This picture shows the roof of Mr. Beach's house. It consists of two trapezoids and two triangles. Each (570 sq. ft.) trapezoid has bases of 46 ft. and 30 ft. and a height of 15 ft.; each (144 sq. ft.) 1428 sq. ft. triangle has a base of 24 ft. and a height of 12 ft. What is the total area of the roof? How many bundles of shingles must Mr. Beach buy to cover the roof if 1 bundle will cover 33 sq. ft.? 44 What will the shingles cost at \$3.20 a bundle? \$140.80

5. Bob's garden is a trapezoid; the bases are 175 ft. and 125 ft. and the height is 100 ft. What is the area of this garden? 15,000 sq. ft. How many pounds of fertilizer will Bob need for the garden if he allows 40 lb. of fertilizer per 1000 sq. ft.? 600 If the fertilizer is sold in 100-pound bags, how many bags in all must Bob buy? 6

6. A field shaped like a parallelogram has an area of 75,000 sq. ft. If the height of the parallelogram is 250 ft., how many feet long is the base? 300

7. A circle has a circumference of 628 ft. How long is its diameter? 200 ft.

8. A trapezoid has an area of 200 sq. ft. The bases are 10 ft. and 15 ft. Find the height of the trapezoid. 16 ft.

Encourage the students to make diagrams (as in ex. 4) for problems, showing the facts given. Remind them to write the formula first, before substituting values in it. Review finding an unknown factor of a product.

1. Which record is better, 17 games won out of 21 (1) played or 38 games won out of 45 played? (2) In each case find, to the nearest tenth of 1%, the per cent of games won. (1) 81.0%; (2) 84.4%

2. The triangular sail on Paul's boat has a base of 8 ft. and a height of 9 ft. What is the total force on Paul's sail in a high wind if its pressure is 5 lb. per sq. ft.? 180

3. Andy's canoe has a sail shaped like a trapezoid. The bases of the sail are 5 ft. and 3 ft., and its height is 6 ft. Find the area of this sail. 24 sq. ft.

4. The list price of a stove is \$240. The trade discount is 15%. Find the net price in two different ways. 15 x \$240 = \$36, \$240 − \$36 = \$204; 100% − 15% = 85%, .85 x \$240 = \$204

5. Mr. Woodman was paid a commission of 5% for selling a lot. At what price did he sell the lot if his commission was \$375.00? \$7500

6. Peter is paid \$.02$\frac{1}{4}$ for each newspaper he sells. Last week his earnings were \$12.42. How many papers did Peter sell all together? 552

7. In 10 years the population of Northfield increased from 11,243 to 20,437. Find, to the nearest whole per cent, the per cent of increase in the population of that town. 82%

8. At an average speed of 3$\frac{1}{2}$ mi. per minute, how far can an airplane fly in 2$\frac{1}{2}$ hr.? 525 mi.

9. Mr. Smith sells shoes. If a pair of shoes cost him \$6.90, at what price must he sell them if he finds that the cost of the shoes must be 60% of the selling price? \$11.50

10. Miss Mason bought \$300 worth of travelers checks. The charge for issuing these checks is 1% of their value. How much in all did the checks cost her? \$303

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Through conferences with the students, try to determine whether problem situations, processes, poor judgment, and so on, cause errors. If many students seem weak in a particular area, reteaching of the topic may be necessary.

Present a diagnostic test on the formulas taught in Chapter 3, with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Remind the students that dimensions must be given in, or changed to, the same unit of measure.

Find the areas of these triangles:

	Base	Height		Base	Height		Practice Pages
1.	12 ft.	$9\frac{1}{2}$ ft.	57 sq. ft.	6 ft. 9 in.	10 ft.	$33\frac{3}{4}$ sq. ft	82, 83

Find the areas of rectangles with these dimensions:

2. 8 ft. × 12 ft. 96 sq. ft. 9 ft. 4 in. × 6 ft. 56 sq. ft. 85

Find the areas of these parallelograms:

	Base	Height		Base	Height		
3.	17 ft.	15 ft.	255 sq. ft.	10 in.	2 ft.	$1\frac{2}{3}$ sq. ft.	86

Find the areas of squares having these sides:

4. 6 ft. 36 sq. ft 8 in. 64 sq. in $\frac{1}{2}$ yd. $\frac{1}{4}$ sq. yd 9.5 ft. 90.25 sq. ft. 85

Find the areas of these trapezoids:

	Bases	Height		Bases	Height		
5.	6′, 8′	5′	35 sq. ft.	11″, 16″	1′	162 sq. in.	90
6.	$2\frac{1}{2}$′, 4′	8′	26 sq. ft.	25″, 18″	$2\frac{1}{2}$′	645 sq. in.	90

Find the circumferences of circles with these diameters. Use $\pi = 3.14$:

7. 20 yd. 62.8 yd 45 ft. 141.3 ft. 11 in. 35.54 in. 300 ft. 942 ft. 91, 93

Find the areas of circles with these radii. Use $\pi = 3.14$:

8. 14 in. 615.4 sq. in. 10 ft. 314 sq. ft. 2.5 yd. 19.6 sq. yd. 21 in. 1384.7 sq. in. 100

Find the areas of circles with these diameters. Use $\pi = 3\frac{1}{7}$:

9. 56 ft. 2464 sq. ft. 21 ft. $346\frac{1}{2}$ sq. ft. 40 in. $1257\frac{1}{7}$ sq. in. 14 yd. 154 sq. yd. 100

After checking and returning the papers, have volunteers explain the work at the board so that the students can find and correct their mistakes. Be sure all difficulties are cleared up before assigning practice pages.

See the Guide for the specific aims of Chapter 4.

Chapter 4

Banks

Discuss the common services of a commercial bank.

1. Almost everyone in this country makes use of a bank in one way or another. A bank offers a safe place to keep money and valuables. It permits a person who keeps his money in a checking account to pay it out by check. It also cashes the checks its depositors receive from other individuals. A bank may also permit a person to deposit his savings in a savings account and receive interest on the amount saved. Banks also rent safe deposit boxes in which valuable papers may be kept.

Stress.

2. Banks must earn money in order to pay the interest on savings accounts. The services that a bank provides for its depositors all cost the bank money. To earn money banks lend, at interest, a large part of the money that is deposited with them.

Discuss why a bank is a safe place to keep money and valuables, the importance of a checking account, and so on. Point out that interest is a charge for the use of borrowed money.

Reteach the procedure for opening a checking account and making out deposit slips.

Checking Accounts

DEPOSITED IN THE

COMMERCIAL BANK

By Henry Chester

DATE Nov. 12 19—

COIN		
BILLS	35	00
CHECKS		
Peoples Bank	28	25
Newtown	73	54
TOTAL	136	79

1. When Mr. Chester opened a checking account with the Commercial Bank of Woodside he had to give his name, address, and occupation. He also had to leave a copy of his signature on a card. Mr. Chester wanted to deposit \$35 in bills and also 2 checks, so he filled out a deposit slip like the one shown here. He wrote the total amount of the bills and the amount of each separate check. For each check he also wrote the name of the bank if it was in the same city and the name of the city if the bank was in another city. What was the total amount that he deposited? \$136.79

Make deposit slips like the one above. Then fill out these slips for the deposits below. Supply dates and find the total amount of each deposit:

2. Alice R. Case of Los Angeles deposits: coin, \$3.44; bills, \$74; check on Empire Bank, \$18.75; check on New York bank, \$25.34; check on Philadelphia bank, \$57.30. Total: \$178.83

3. Charles S. King of Dayton deposits: coin, \$11.63; bills, \$381; check on Peoples Bank, \$17.31; check on New York bank, \$64.55; check on Detroit bank, \$108.47; check on Omaha bank, \$1276.09. Total: \$1859.05

4. James F. Worth of Boston deposits: coin, \$18.97; bills, \$417; check on Exchange Bank, \$207.12; check on Chicago bank, \$95.38; check on St. Louis bank, \$113.86; check on Harrisburg bank, \$73.49. Total: \$925.82

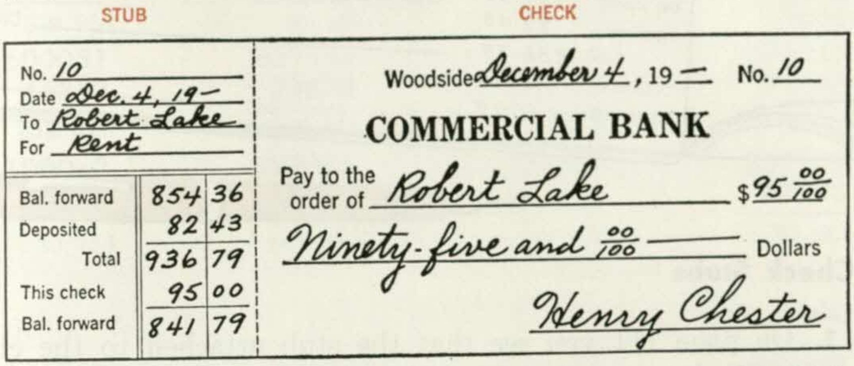
5. Lucy M. Peters of Fort Worth deposits: bills, \$89; check on First National Bank, \$207.14; check on Newtown bank, \$29.85; check on Los Angeles bank, \$179.38; check on Chicago bank, \$53.75. Total: \$559.12

Try to have deposit slips available for the students to use in ex. 2–5.

Reteach writing checks and keeping a record of bank balances on check-book stubs (pages 111-112). If possible, display a checkbook so that pupils may see actual checks and stubs.

Writing a Check

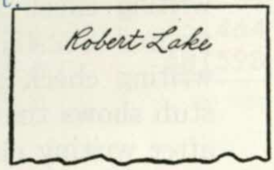
1. At the beginning of the next month after Mr. Chester had opened his checking account, he wrote check No. 10 for $95, as shown below, to pay the rent for his store:



To the left of the dotted line is the **stub** of the check, which remains in the checkbook; to the right of the dotted line is the check Mr. Chester sent to Mr. Lake.

Emphasize that the stub remains to serve as record.

2. What advantage is there in paying the rent by check rather than in cash? The check serves as a receipt.

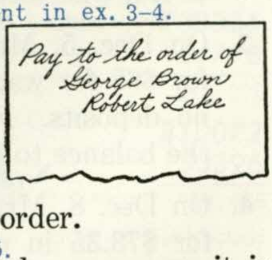


3. When Mr. Lake receives the above check, he may take it to the bank where he has a checking account and deposit it to the credit of his account. He must first endorse the check by writing his name across the back of the left end of the check. This is called an **endorsement in blank.** When Mr. Lake endorses the check he shows that he has received the money.

Stress.

Emphasize meaning.

Emphasize difference between types of endorsement in ex. 3-4.



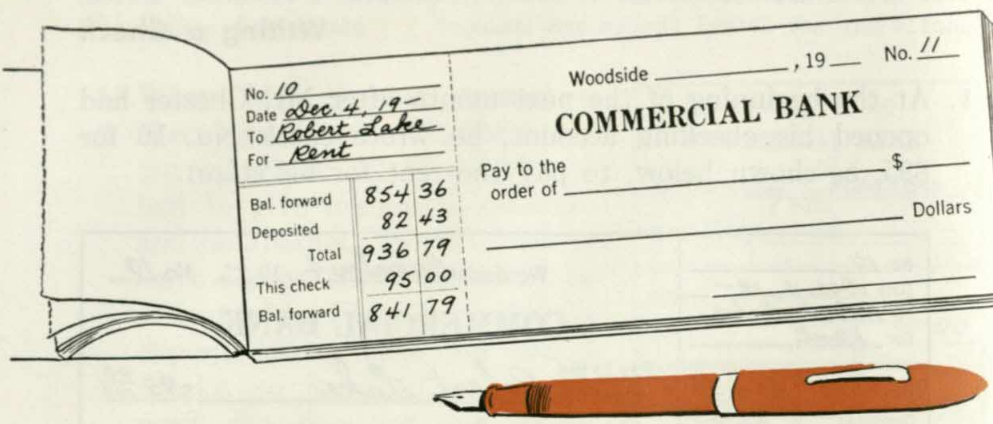
4. Instead of depositing the check in his bank, Mr. Lake may give it to Mr. Brown in payment for his services. In this case, Mr. Lake endorses the check **in full,** as shown here, making it payable only to Mr. Brown, or to his order.

Emphasize meaning.

Have students give reasons for answers in ex. 5.

5. Do you think it is wise to endorse a check as soon as it is received? No Which kind of endorsement is safer? Endorsement in full

Explain and discuss the purpose of each blank on both the check and the stub, and the correct way of writing amounts (see the Guide). If possible, display canceled checks so that students may see endorsements. Point out the date stamped on each and explain its use.



Check Stubs

1. On page 111 you see that the stub attached to the check is filled out so that Mr. Chester knows to whom the check was written and the balance he has in the bank. After writing the check for $95 to Mr. Lake, the stub remained in the checkbook. The first item labeled "Bal. forward" shows that Mr. Chester had $854.36 in his account after writing check No. 9. Since writing that check he has deposited $82.43, making a total balance of $936.79 before writing check No. 10 to Mr. Lake. The last item on the stub shows that Mr. Chester had $841.79 left in his account after writing check No. 10. This balance is carried forward to the stub of the next check.

2. When should the stub of a check be filled out? (1) Why is it important to record the bank balance on the stubs of checks? (2) Is it a wise idea to number checks? (3) (1) When check is written; (2) to keep record of balance so as not to overdraw account; (3) yes, to keep track of item.

3. On Dec. 5, Mr. Chester wrote check No. 11 to Paul Baker for $68 for wages. Since writing check No. 10 he had made no deposits. Fill out the stub of this check and then find the balance to be carried forward to the stub of check No. 12. ($773.79) See Guide.

4. On Dec. 8, Mr. Chester wrote check No. 12 to Hall Brothers for $73.35 in payment for two chairs. Since writing check No. 11 he had deposited $56.75 in his account. Write the stub and find the balance. ($757.19) See Guide.

Make clear the meaning of "Bal. forward." Use ex. 2 to discuss the meaning of overdrawn and to emphasize why stubs should be made out immediately.

Present a second set of improvement tests in subtraction.

Subtraction Test 2a.

Time: 4 min.

1.	897321 − 617467 = 279,854	752500 − 723640 = 28,860	900064 − 94786 = 805,278	831427 − 682696 = 148,731
2.	600051 − 46352 = 553,699	859554 − 24876 = 834,678	950097 − 405099 = 544,998	596243 − 78696 = 517,547
3.	100042 − 79288 = 20,754	922563 − 258569 = 663,994	650303 − 613448 = 36,855	331179 − 67596 = 263,583 (12)

Subtraction Test 2b.

Time: 4 min.

4.	151868 − 55589 = 96,279	604029 − 434535 = 169,494	897218 − 452575 = 444,643	300032 − 78595 = 221,437
5.	905048 − 617467 = 287,581	177932 − 96134 = 81,798	793155 − 576427 = 216,728	200003 − 61908 = 138,095
6.	940034 − 524287 = 415,747	573196 − 35357 = 537,839	400007 − 31581 = 368,426	751464 − 681598 = 69,866 (12)

Subtraction Test 2c.

Time: 4 min.

7.	853826 − 196268 = 657,558	500073 − 398622 = 101,451	545932 − 197249 = 348,683	104027 − 16879 = 87,148
8.	541724 − 49526 = 492,198	600943 − 307454 = 293,489	169617 − 83848 = 85,769	700054 − 124778 = 575,276
9.	800001 − 689710 = 110,291	638125 − 70146 = 567,979	641265 − 13679 = 627,586	810072 − 50396 = 759,676 (12)

To the Teacher. Have each pupil compare the graph of his scores on Subtraction Tests 2a, 2b, and 2c above with the graph he made of his scores on Subtraction Tests 1a, 1b, and 1c. In this way he can see whether he is improving in subtraction.

Have the students determine and record their own scores. Group students who had mistakes to help them determine causes.

Explain service charges on checking accounts and the reasons for them.

Service Charges on Bank Accounts

1. Some checking accounts require more work on the part of the bank than others. For example, the checking account of a business man who draws many checks and makes many deposits each month requires much bookkeeping by the bank. On the other hand, the account of a private individual takes much less time; such a person may make only 2 or 3 deposits a month and draw only a few checks.

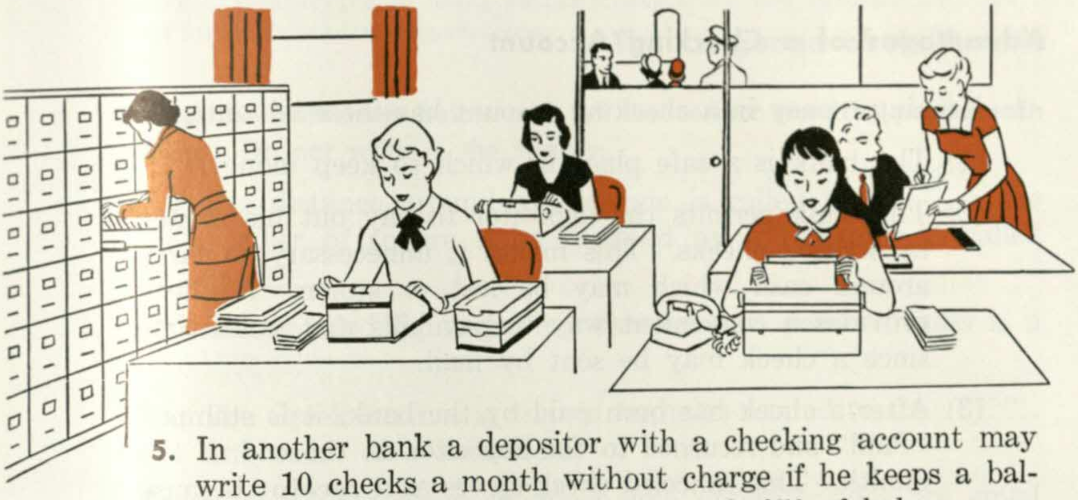
2. Banks have to earn money in order to pay for the services they render. One way that a bank earns money is to lend a large part of the money deposited in it; for these loans, the bank charges interest. If a depositor is a business man, with a balance of \$3000 or more, the bank can afford to give him considerable service without making a charge for it, since it can lend a part of the \$3000 in his account. If a depositor has a balance of less than \$200, the bank is not able to lend much of this money, and the depositor may have to pay a **service charge** to cover the expense of handling his account. In the banks in your city, what are the service charges on checking accounts? Get the details from one or more banks.

3. In one bank there is a basic charge of 50¢ a month on all checking accounts. In addition, a depositor is charged 6¢ for each deposit and 4¢ for each check he writes. The bank allows the depositor a credit toward these charges of 10¢ for each \$100 of his smallest balance during the month.

In one month Mr. Clark made 2 deposits, wrote 12 checks, and kept a balance of \$415 in his account in this bank. His service charge is found as follows: $2 \times 6¢ = 12¢$ for deposits; $12 \times 4¢ = 48¢$ for checks; these charges and the basic charge of 50¢ total \$1.10; $4 \times 10¢ = 40¢$ of credit for the balance of \$415; \$1.10 less the credit of 40¢ gives 70¢ for the service charge.

4. In the bank in ex. 3, find the service charge for 6 (\$.36) deposits, 10 checks written (\$.40), and a balance of \$325 in the account. Charges = \$1.26; credit = \$.30; service charge = \$.96

Discuss ex. 1-2 with the class, emphasizing the meaning of and the reasons for a service charge. In ex. 3 be sure the students understand how a credit of 40¢ was computed. Have them complete ex. 4 independently and then discuss the solution.



5. In another bank a depositor with a checking account may write 10 checks a month without charge if he keeps a balance of $150 in his account. For each $50 of balance over $150, 5 more checks may be written without charge. There is a charge of 5¢ for each check above those allowed, and no charge for deposits. If the balance falls below $150 during any month there is an extra charge of $1.

6. Mr. Chase has a checking account in a bank with the service charges given in ex. 5. In one month his smallest balance was $259 and he wrote 23 checks. What was his service charge? 15¢

7. Today many banks have another plan for handling small checking accounts which does not depend upon the depositor's balance. Instead of making a monthly service charge, the bank charges the depositor 15¢ for each check written, with no charge for deposits. When the depositor opens his account he buys a book of 10 blank checks for $1.50. After these checks are written, he can purchase additional checks. He must, of course, keep enough money in his account to cover any checks he writes. What bank in your city follows a plan similar to the one just described? Get the details of the plan. Have students report their findings to the class.

8. Mr. Howe has a checking account in a bank that makes the service charges given in ex. 3. In one month he made 3 deposits, wrote 9 checks, and kept a balance of $540 in his account. What was his service charge? $.54

After ex. 5-8 are discussed and completed by the students, instruct them to inquire about the practice of local banks as to service charges and report their findings to the class. Use the charges in problems similar to ex. 6 and 8.

Advantages of a Checking Account

1. Keeping money in a checking account has these advantages:

(1) The bank is a safe place in which to keep money.

(2) The bank permits the depositor to pay out his money by writing checks. This makes it unnecessary to carry around cash which may be lost or stolen. A check provides a convenient way to pay bills and saves time since a check may be sent by mail.

(3) After a check has been paid by the bank, it is stamped "Paid" and returned to the depositor at the end of the month. This canceled check serves as a **receipt** because it shows that the depositor has paid the money to the person named in the check.

(4) The bank cashes checks that a depositor receives from other persons. This is a convenience and saves time.

(5) The bank is more likely to lend money to one of its depositors.

2. It is stated above that a canceled check serves as a receipt. In what way does the person to whom the check was sent show on the canceled check that he received the money?

By his endorsement

3. Are the blank checks furnished free of charge by the bank?

4. Mr. Bell has an account in the bank. One day the manager of the bank telephoned Mr. Bell that he had "overdrawn" his account and he should make a deposit of \$50 that day. What caused the manager to telephone? Why did he ask for \$50?

5. Mr. Miller wanted to rent a house and was asked by Mr. Ward, the owner of the house, to give him a bank reference. Mr. Miller referred Mr. Ward to the Citizens Bank, where he had an account. What did Mr. Ward ask the Citizens Bank? What kind of information do you think the Citizens Bank would have concerning Mr. Miller?

6. It is said that the stubs of a checkbook furnish a record of how much you receive and how you spend it. Is this true? See Guide.

In ex. 4 be sure the students understand that "overdrawn" means Mr. Bell did not have enough money on deposit in the bank to pay all the checks he had drawn.

Give oral practice in using mathematical terms and information, and in performing certain computations.

The Language of Arithmetic

Read the statements below and tell the correct word to put in each space. Do not write in the spaces:

1. The distance around a rectangle is called its ______ (perimeter); the number of square units enclosed by a rectangle is called its ______ (area).

2. A four-sided figure having two unequal parallel bases is a ______ (trapezoid).

3. The Greek letter π is used in formulas related to the ______ (circle). One value of π is ______ ($3\frac{1}{7}$ or 3.14).

4. A four-sided figure in which two opposite sides are equal and parallel is a ______ (parallelogram).

5. All squares are ______ (rectangles) but all rectangles are not ______ (squares).

6. $A = \frac{1}{2}bh$ is the formula for the area of a ______ (triangle).

7. One formula for the circumference of a circle is ______ ($C = 2\pi r$, or $C = \pi d$).

8. In a formula, when two letters are written side by side, their values are to be ______ (multiplied).

Some students may need a redevelopment of the terms and concepts in ex. 1–8. Group students for this activity if needed.

Oral Arithmetic

9. Find the following per cents of \$40: 20% (\$8), 50% (\$20), $12\frac{1}{2}\%$ (\$5), 25% (\$10), 100% (\$40), 150% (\$60), 200% (\$80), 75% (\$30).

10. Tell what per cent of 50 each of the following numbers is: 5 (10%), 10 (20%), 25 (50%), 50 (100%), 100 (200%), 300 (600%).

11. What is the square of 2? (4) of 5? (25) of 8? (64) of 4? (16) of 9? (81)

12. Multiply each number by 10, (1) then by 100, (2) and then by 1000: (3) 216, 2.16, .216, 21.6, .0216. (1) 2160; 21.6; 2.16; 216; .216; (2) 21,600; 216; 21.6; 2160; 2.16; (3) 216,000; 2160; 216; 21,600; 21.6

13. Name the fractions that have the value $\frac{3}{4}$: $\frac{9}{12}$, $\frac{18}{21}$, $\frac{15}{20}$, $\frac{24}{27}$, $\frac{24}{32}$, $\frac{20}{24}$, $\frac{36}{48}$, $\frac{40}{60}$, $\frac{6}{8}$, $\frac{12}{16}$.

14. Find, to the nearest cent, 1% of each of the following: \$120 (\$1.20); \$13.50 (\$.14); \$4500 (\$45.00); \$99.75 (\$1.00); \$6.10 (\$.06); \$17.00 (\$.17).

Use ex. 9–14 as a quiz program activity. Let different students act as quiz master, calling on others for answers, while you note individual or class weaknesses.

Reteach how to compute interest for periods of 1 year and $\frac{1}{2}$ year

Interest (pages 118-119).

1. Mr. West wants to build an addition on his store and needs to borrow \$5000 to get the work started. Mr. Thomas will lend him that sum for 1 yr. at 4% interest. At the end of the year when Mr. West pays back the \$5000 to Mr. Thomas he will pay 4% of \$5000, or \$200, for the use of the money for 1 yr. The \$5000 borrowed is called the **principal;** the **rate of interest** is 4%; the **interest** is \$200. At the end of the year Mr. West must pay Mr. Thomas \$5000 + \$200, or \$5200; this is the sum of the principal and the interest and is called the **amount.** Emphasize the meaning of amount.

2. Mr. Jackson borrowed \$400 from Mr. Bird for 1 yr. at 6% interest. Find the interest. \$24 What was the amount Mr. Jackson paid Mr. Bird at the end of 1 yr.? \$424

Find, to the nearest cent, the interest for 1 yr. on the following:

3. \$2400 at 5% \$120 — \$1750 at $3\frac{1}{2}$% \$61.25 — \$2350 at $3\frac{1}{2}$% \$82.25

4. \$1575 at 6% \$94.50 — \$2637 at $4\frac{1}{2}$% \$118.67 — \$1125 at 3% \$33.75

5. \$1250 at 4% \$50 — \$3575 at 3% \$107.25 — \$1460 at $6\frac{1}{2}$% \$94.90

6. \$3800 at 3% \$114 — \$1900 at $4\frac{1}{4}$% \$80.75 — \$2875 at 7% \$201.25

7. Mr. Green needs to borrow \$800 for 6 mo. ($\frac{1}{2}$ yr.). Mr. North will lend it to him at 4%; this 4% is the charge for the use of the money for 1 yr. First find the interest for 1 yr. \$32 What is the interest for $\frac{1}{2}$ yr.? \$16 What is the total amount Mr. Green will pay Mr. North at the end of 6 mo.? \$816

Find the interest for $\frac{1}{2}$ yr. on the following:

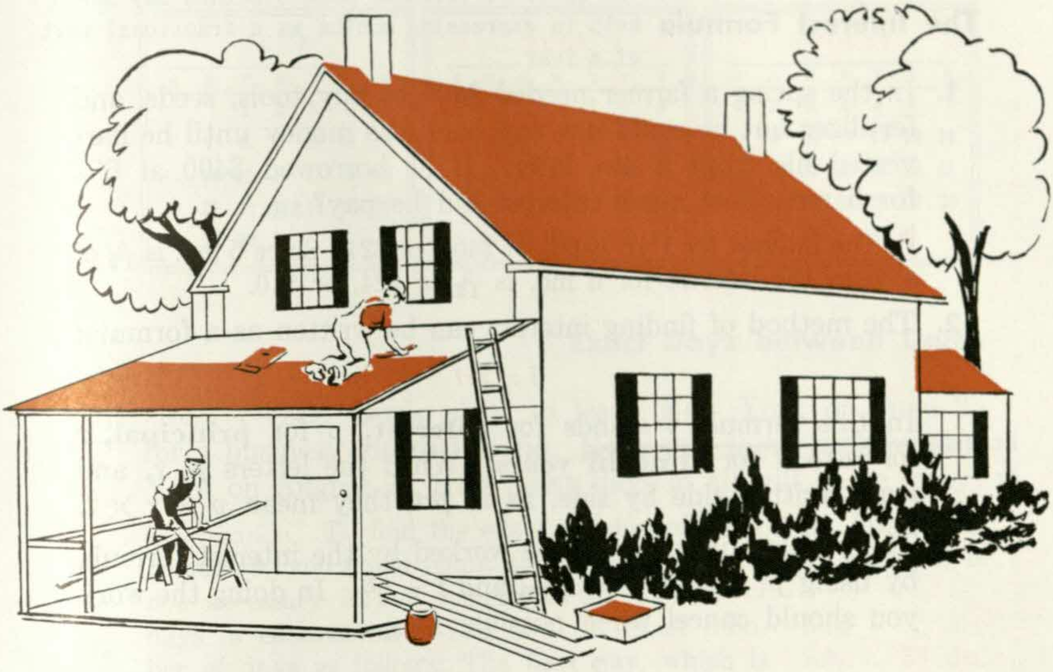
8. \$638 at 6% \$19.14 — \$3500 at $6\frac{1}{2}$% \$113.75 — \$1200 at 5% \$30

9. \$927 at 4% \$18.54 — \$2860 at 3% \$42.90 — \$1500 at $2\frac{1}{2}$% \$18.75

10. \$750 at 7% \$26.25 — \$3170 at $3\frac{1}{2}$% \$55.48 — \$1860 at 3% \$27.90

11. Suppose that Mr. Green in ex. 7 borrows \$800 for only 3 mo. What part of 1 yr. is 3 mo.? $\frac{1}{4}$ How much interest must he pay for the use of \$800 for 3 mo.? \$8

Be sure the students have a clear understanding of the terms in ex. 1. Emphasize that rate of interest always means rate for one year unless stated otherwise.



Borrowing Money for Repairs

1. Mr. Fisher owns a house that was built by his grandfather. He wants to remodel it and put in a new furnace. A builder estimated that the repairs would cost about \$3000. Mr. Fisher can borrow that sum from Mr. Crane with interest at $4\frac{1}{2}\%$. He is to pay the interest every 6 mo. How much interest will Mr. Fisher pay every 6 mo. until the money is paid back? \$67.50

2. If the first interest payment is due on April 1, when will the second interest payment be due? Oct. 1

3. If Mr. Fisher should be able to pay back \$1000 of the loan at the end of 1 yr., this will reduce the amount of interest that he has to pay. How much interest will he have to pay on the remaining \$2000 each 6 mo.? \$45

4. If Mr. Fisher is able to pay back \$1000 of the loan at the end of 1 yr., and the remaining \$2000 at the end of 2 yr., how much interest will he pay in all? \$225

Have the students do the problems independently. Then let volunteers explain the solutions at the board. Urge the students to ask questions if they have any difficulties. Have them make up original problems about interest for the class to solve.

Teach how to compute interest for a part of a year when the time is given in months, and how to use the interest formula. Some students may need help in expressing months as a fractional part of a year.

The Interest Formula

1. In the spring a farmer needed \$400 to buy tools, seeds, and fertilizer but he could not pay back the money until he harvested his crops 5 mo. later. If he borrowed \$400 at 6% for 5 mo., how much interest did he pay? \$10

   ▶ The interest for 1 yr. is 6% of \$400, or \$24. Since 5 mo. is $\frac{5}{12}$ of a year, the interest for 5 mo. is $\frac{5}{12}$ of \$24, or \$10.

2. The method of finding interest can be written as a formula:

$$i = prt$$

   In this formula i stands for **interest,** p for **principal,** r for **rate,** t for **time in years.** When the letters p, r, and t are written side by side, as in prt, they mean $p \times r \times t$.

Stress.

3. The problem in ex. 1 can be worked by the interest formula by using $p = 400$, $r = \frac{6}{100}$, and $t = \frac{5}{12}$. In doing the work you should cancel when possible: Emphasize.

$$i = \frac{\overset{\overset{2}{\cancel{4}}}{\cancel{400}}}{1} \times \frac{\overset{1}{\cancel{6}}}{\underset{1}{\cancel{100}}} \times \frac{5}{\underset{\underset{1}{\cancel{2}}}{\cancel{12}}} = 10$$

   The answer, \$10, is the same as the answer in ex. 1.

4. Mr. Porter borrowed \$225 for 8 mo. and paid interest at the rate of 5%. How much interest did he pay? \$7.50

   ▶ The time is $\frac{8}{12}$ yr. First change $\frac{8}{12}$ to $\frac{2}{3}$; it avoids extra cancellation if the fraction representing time is simplified in advance.

Using the formula, find the interest correct to the nearest cent:

5. \$300, 4%, 8 mo. \$8 — \$800, 6%, 7 mo. \$28 — \$900, 3%, 5 mo. \$11.25
6. \$450, 3%, 2 mo. \$2.25 — \$500, 4%, 9 mo. \$15 — \$800, 4%, 7 mo. \$18.67
7. \$600, 5%, 4 mo. \$10 — \$660, 7%, 1 mo. \$3.85 — \$300, 6%, 1 mo. \$1.50
8. \$500, 5%, 8 mo. \$16.67 — \$500, 3%, 5 mo. \$6.25 — \$700, 4%, 5 mo. \$11.67
9. \$850, 7%, 6 mo. \$29.75 — \$200, 6%, 7 mo. \$7 — \$500, 2%, 7 mo. \$5.83

Have the students write the formula first, then substitute values and complete the solution. Stress that when using the formula, rate is expressed as a fraction or a decimal, and time is expressed in years. Have volunteers explain the solutions at the board.

Teach how to find the exact number of days between dates. Review the number of days in each month (See the Guide).

JUNE						
S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30			

JULY						
S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

AUGUST						
S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

Exact Days between Dates

1. Problem Mr. Lewis plans to leave New York on June 25 for a business trip to Europe. He is due to be back in New York on August 3. How many days in all will he be away? 39

Explanation To find the exact number of days from June 25 to August 3, you could count the days on a calendar. But that is not necessary if you remember the number of days in each month. You find the exact number of days as follows: The **first** day, which is June 25, is not counted; since June has 30 days, there are 5 days left in June. There are 31 days in July. There are 3 days in August, since August 3, which is the last day, is counted. The exact number of days from June 25 to August 3 is 39 days, as shown above.

June	5 da.
July	31 da.
August	3 da.
	39 da.

In counting the exact number of days between two dates, remember to count the last day but not the first day. Emphasize.

Find the exact number of days within the same year from:

2. Aug. 9 to Dec. 7 120 — June 28 to July 7 9

3. Jan. 8 to Feb. 13 36 — Sept. 10 to Nov. 12 63

4. Mar. 28 to June 11 75 — Mar. 18 to June 19 93

5. May 25 to Aug. 23 90 — Apr. 27 to Nov. 29 216

6. In 1960 Mr. Lewis went to South America on business. He left New York on January 7 and returned to New York on March 1. How many days in all was he away? 54

▶ How can you tell which years are leap years?

Review the fact that leap year occurs every 4 years (when the year is exactly divisible by 4), but centenary years are not leap years unless exactly divisible by 400.

Finding Interest for Exact Days

1. When banks compute interest on loans to individuals, they usually count the exact number of days from the day that the money is borrowed to the day when it is paid back. The banks also consider 360 days as a year. While one might expect the year to be considered 365 days in cases like this, it is not the practice of banks to do so. If a bank makes a loan for 24 days, this is considered to be $\frac{24}{360}$ of a year and not $\frac{24}{365}$ of a year. Change $\frac{24}{360}$ to lowest terms. $\frac{1}{15}$

2. Problem On Dec. 19 Mr. Ray borrowed \$225 at 4% and paid it back on Feb. 5. How many (1) days did he keep the money and what was the interest (2) he paid on it? (1) 48; (2) \$1.20

Explanation Show that the exact number of days is 48 da. Using the formula, $i = prt$, you find the interest as follows:

$$i = \frac{\overset{15}{\cancel{225}}}{1} \times \frac{4}{100} \times \frac{2}{\underset{1}{\cancel{15}}} = \frac{120}{100} = 1.20$$

You see that the interest for 48 da. is \$1.20.

Remind the students to change a fractional part of a year to lowest terms.

Counting exact days between dates and letting 360 da. = 1 yr., find the interest at 6%:

3. \$600; May 3 to June 8 \$3.60 — \$2000; July 6 to Aug. 20 \$15

4. \$125; June 2 to Aug. 13 \$1.50 — \$1200; Sept. 18 to Oct. 18 \$6

5. \$280; July 2 to Oct. 3 \$4.34 — \$3500; Nov. 29 to Dec. 5 \$3.50

6. \$900; May 16 to Aug. 4 \$12 — \$2400; Apr. 6 to Aug. 19 \$54

7. In the Treasury Department of the United States Government interest is computed on the basis of 365 days to the year. Stress. This practice is also followed in certain other places. When the time is counted in this way, 90 days is considered to be $\frac{90}{365}$ of a year. Find the interest on \$3000 for 90 days at 4%, counting 365 days to the year. \$29.59 Then find the interest, counting 360 days to the year. \$30 Which way gives more interest and how much more? 360 da. gives \$.41 more.

Emphasize in ex. 1 that banks usually count 360 days to the year and the exact number of days between dates.

This method may be omitted if time is limited. Also, it should not be taught until the students have mastered the basic method.

A Short Method for Interest

There is a short method for computing interest, sometimes called the 6% method, when the interest rate is 6% and the time is 60 da. In this method 360 days are considered to be a year.

1. Problem Find the interest on $400 at 6% for 60 da.; $4.00 for 6 da. $.40

Explanation 60 da. = $\frac{1}{6}$ yr. At 6% the interest on $400 for 1 yr. is 6% of $400; so the interest for $\frac{1}{6}$ yr. is $\frac{1}{6}$ of 6%, or 1%, of $400. 1% of $400 equals $\frac{1}{100}$ of $400, or $4.00. The interest on $400 for 60 da. is $4.00. The easy way to get $4.00 is to move the decimal point in $400 two places to the left. Be sure students know why the decimal point is moved.

The interest for 6 da. is $\frac{1}{10}$ of the interest for 60 da. So the interest on $400 for 6 da. is $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$, of $400. The easy way to find $\frac{1}{1000}$ of $400 is to move the decimal point in $400 3 places to the left, which gives $.40 as the interest for 6 da.

To find the interest at 6% for 60 days, move the decimal point in the principal two places to the left; to find the interest for 6 days, move the decimal point three places to the left.

Ex. 2–3 may be done orally, but have pupils explain the work.

At 6%, find the interest for 60 da. on these amounts:

2.	$700 $7	$1200 $12	$500 $5	$695 $6.95	$80 $.80	$2240 $22.40
3.	$375 $3.75	$1145 $11.45	$100 $1	$280 $2.80	$65 $.65	$1170 $11.70

4. At 6%, find the interest for 6 da. in ex. 2 and 3. Ex. 2: $.70, $1.20, $.50, $.70, $.08, $2.24; ex. 3: $.38, $1.15, $.10, $.28, $.07, $1.17

▶ To find the interest for 6 da. on the amount $475, move the decimal point three places to the left, which gives $.475, or $.48 to the nearest cent. When you move the decimal point three places to the left in $90, first write a zero before the 90, which gives $.09. Stress. Stress.

5. Problem At 6%, find the interest on $800 for 30 da.; $4.00 90 da. $12.00

Explanation The interest for 30 da. is $\frac{1}{2}$ of the interest for 60 da. What is the interest on $800 for 60 da.? $8.00 for 30 da.? $4.00

90 da. = 60 da. + 30 da. To find the interest for 90 da., add the interest for 60 da. to the interest for 30 da., as shown at the right.

$8.00 for 60 da.
$4.00 for 30 da.
$12.00 for 90 da.

This method should be explained carefully. In ex. 1 be sure students understand why the interest for $\frac{1}{6}$ yr. is $\frac{1}{6}$ of 6% and why the interest for 6 da. is $\frac{1}{10}$ the interest for 60 da.

Using the Short Method

1. The 6% method provides a rapid way to find the interest for 60 da. and 6 da. merely by moving the decimal point in the principal. For other periods of time, it is often possible to make combinations of 60 da. and 6 da., as in ex. 2.

2. **Problem** Find the interest on $300 at 6% for 72 da.; $3.60 18 da. $.90

Explanation

$3.00 for 60 da.	$.30 for 6 da.
.30 for 6 da.	.30 for 6 da.
.30 for 6 da.	.30 for 6 da.
$3.60 for 72 da.	$.90 for 18 da.

3. Find the interest on $550 at 6% for 72 da.; $6.60 for 66 da. $6.05

4. If you know the interest for 60 da., how can you find the interest for 15 da.? for 20 da.? for 40 da.? 45 da.? $\frac{3}{4}$ of 60 da.

$\frac{1}{4}$ of 60 da. $\frac{1}{3}$ of 60 da. $\frac{2}{3}$ of 60 da.

5. At 6%, find the interest on $400 for 54 da.; for 48 da. $3.20

▶ 54 da. = 60 da. − 6 da. $3.60 48 da. = 60 da. − 12 da.

By the 6% method, find the interest at 6% on the following:

6.	$590, 66 da. $6.49	$450, 78 da. $5.85	$340, 96 da. $5.44	$710, 72 da. $8.52
7.	$860, 72 da. $10.32	$600, 42 da. $4.20	$240, 45 da. $1.80	$510, 12 da. $1.02
8.	$290, 12 da. $.58	$440, 90 da. $6.60	$950, 90 da. $14.25	$390, 66 da. $4.29
9.	$640, 36 da. $3.84	$620, 15 da. $1.55	$700, 48 da. $5.60	$582, 20 da. $1.94

10. The 6% method can also be used to find interest at rates other than 6%. To do this you first compute the interest at 6%. Then you find a part of this interest as shown in ex. 11.

11. **Problem** Find the interest on $480 for 60 da. at $2.40 3%; 2%. $1.60

Explanation The interest at 6% for 60 da. is $4.80. The interest at 3% is $\frac{1}{2}$ of $4.80, or $2.40. The interest at 2% is $\frac{1}{3}$ of $4.80, or $1.60. What is the interest at 4%? $3.20

12. Find the interest for 60 da. at 3%; 2%; 4%: See Guide.

$300	$960	$720	$540	$1440	$2040

Discuss the development given in ex. 1–5. Give the students ample opportunity to ask questions if any part of this work is not clear to them.

Present a second set of improvement tests in multiplication and in division. Have the students determine and record scores in Record Books, and compare results with previous ones.

Improving by Practice

Multiplication Test 2a. Time: 4 min. after copying.

1. 5874 × 470 = 2,760,780   4618 × 385 = 1,777,930   2903 × 576 = 1,672,128   1956 × 479 = 936,924   4795 × 806 = 3,864,770   5

Multiplication Test 2b. Time: 4 min. after copying.

2. 5073 × 493 = 2,500,989   1786 × 612 = 1,093,032   2793 × 208 = 580,944   4289 × 491 = 2,105,899   6495 × 520 = 3,377,400   5

Multiplication Test 2c. Time: 4 min. after copying.

3. 3679 × 713 = 2,623,127   2380 × 918 = 2,184,840   6848 × 672 = 4,601,856   2698 × 340 = 917,320   5716 × 908 = 5,190,128   5

Division Test 2a. Time: 5 min. after copying.

4. 13)91767 = 7059   76)62629 = $824\frac{5}{76}$ 21)78645 = 3745

5. 38)48279 = $1270\frac{1}{2}$ 94)85352 = 908   37)87727 = 2371   6

Division Test 2b. Time: 5 min. after copying.

6. 85)59927 = $705\frac{2}{85}$ 22)82896 = 3768   39)75660 = 1940

7. 17)34221 = 2013   24)71312 = $2971\frac{1}{3}$ 93)75981 = 817   6

Division Test 2c. Time: 5 min. after copying.

8. 21)96747 = 4607   83)82751 = 997   76)63847 = $840\frac{7}{76}$

9. 18)69831 = $3879\frac{1}{2}$ 42)38094 = 907   39)68835 = 1765   6

To the Teacher. In ex. 4–9, the pupils should write the quotient with a fraction when there is a remainder.

Notice that the examples should be copied before the test is begun. Group students who had errors to help them determine causes. Some may need drill on basic facts. Assign remedial work after difficulties are cleared up.

Present the procedure used in borrowing money from a bank (pages 126-127). See the Guide for a discussion of the procedure used if money is not paid when due.

Bank Loans

1. Mr. Harris has a stationery store. He finds that he can save money if he purchases supplies for his store in large quantities in March. To do this, he borrows $800 at 6% for 60 da. from the Liberty Bank in which he has a checking account. Mr. Harris gives the bank a written promise that he will pay back the $800 in 60 da. This promise is called a **promissory note** and is shown below: After the students study the note, discuss and explain the purpose of all the blank spaces on it.

$ 800 00/100 Weston, Ohio March 10, 19—

Sixty days after date I promise to pay

to the order of The Liberty Bank

Eight hundred and 00/100 Dollars

Value received

Due May 9, 19— Robert E. Harris

Mr. Harris is the **maker** of the note. The amount of money that is borrowed is called the **face** of the note and the date on which the money is to be paid is the **date of maturity.** Bank loans are usually made for short periods of time.

If the bank did not know Mr. Harris to be a reliable merchant, it might ask him to deposit stocks, bonds, or something else of value as security. Such security is called **collateral.** Another form of security that the bank might require is that Mr. Harris get another person of good financial standing to sign the note with him.

2. Find the interest on $800 at 6% for 60 da. $8 The interest is not mentioned in the note shown above because the bank Stress. collects the interest **in advance.** Mr. Harris does not get $800 on Mar. 10; instead, he receives on that date $800 less the interest that he must pay. How much money does $792 Mr. Harris receive? At the end of 60 da. he pays back the face of the note, which is $800.

Make sure students understand the meaning of the terms, and give more practice in using them. In ex. 2 point out that this is usual procedure for bank loans.

Remind the students that they may use the formula or the 6% method in ex. 3–10. Solutions for these examples should be discussed thoroughly with the class.

Bank Discount

1. The interest on a loan, which is collected in advance by the bank, is called **bank discount.** When the bank deducted $8.00 interest from the $800 Mr. Harris borrowed, it **discounted** the note; see ex. 2 on page 126. The remaining money, $792, is called the **proceeds.** The **face** of the note less the **bank discount** equals the **proceeds.** Emphasize meaning and illustrate in other examples.

2. In ex. 1 on page 118, Mr. West borrowed $5000 from Mr. Thomas for 1 yr. at 4% and the interest on this money was paid at the end of 1 yr. when the $5000 was paid back. This was a loan between two persons. In such cases the interest is often collected **at the end** of the period for which the money is borrowed, or at the end of each quarter or half year. The interest is not deducted in advance. Stress.

3. Write a promissory note, dated June 4, promising to pay the Liberty Bank $120 in 66 da. Find the bank discount at 6% ($1.32) and the date of maturity (Aug. 9). How much do you get on June 4? ($118.68) How much do you pay in 66 da.? $120

At 6%, find the bank discount (1) and the proceeds (2) on these loans. Count the exact number of days and let 360 da. = 1 yr.:

	(1)	(2)		(1)	(2)
4. $360; July 17 to Aug. 1	$.90	$359.10	$1600; Oct. 10 to Nov. 15	$9.60	$1590.40
5. $540; Jan. 12 to Feb. 11	$2.70	$537.30	$1250; May 1 to July 30	$18.75	$1231.25
6. $900; Mar. 6 to Mar. 26	$3	$897	$2640; Nov. 17 to Dec. 2	$6.60	$2633.40
7. $560; Aug. 11 to Sept. 13	$3.08	$556.92	$1880; Jan. 3 to Feb. 17	$14.10	$1865.90
8. $880; Apr. 28 to June 27	$8.80	$871.20	$2000; June 18 to Aug. 23	$22	$1978

Write the promissory notes which the persons named below make out to the banks from which they borrow. Find the bank discount at 6% (1) and the proceeds of each note: (2)

	Maker	Bank	Face	Time	(1)	(2)
9.	James Martin	First National	$275	60 da.	$2.75	$272.25
10.	Louise Small	Greene County	$600	27 da.	$2.70	$597.30

Make clear that bank discount is another term for "interest collected in advance." When interest is not collected in advance, it is called interest and not "bank discount." Give more practice in writing promissory notes if needed.

Present a review of the 4 fundamental processes with fractions.

Do You Know Your Fractions?

1. Ned rode $4\frac{1}{2}$ mi. on a bus to Yorktown and walked $\frac{3}{4}$ mi. from Yorktown to the highway where he waited for his father to drive up in a car. Then they drove $16\frac{1}{2}$ mi. to Star Lake. How many miles did Ned travel all together? $21\frac{3}{4}$

2. Dick had a board 6 ft. long from which he cut 2 pieces, each $1\frac{3}{4}$ ft. long. How many feet long was the piece of board that was left? $2\frac{1}{2}$

Remind pupils to check all work carefully.

Add. Check by going over the work:

3.	$2\frac{11}{16}$	$4\frac{1}{5}$	$5\frac{3}{4}$	$8\frac{7}{8}$	$11\frac{2}{3}$	$6\frac{5}{8}$	$2\frac{9}{10}$
	$3\frac{5}{16}$	$6\frac{1}{2}$	$7\frac{7}{12}$	$6\frac{5}{8}$	$13\frac{3}{4}$	$5\frac{1}{6}$	$7\frac{3}{5}$
	6	$10\frac{7}{10}$	$13\frac{1}{3}$	$15\frac{1}{2}$	$25\frac{5}{12}$	$11\frac{19}{24}$	$10\frac{1}{2}$
4.	$1\frac{9}{10}$	$4\frac{2}{3}$	$2\frac{7}{16}$	$5\frac{3}{4}$	$10\frac{1}{6}$	$4\frac{1}{3}$	$3\frac{7}{12}$
	$2\frac{3}{10}$	$4\frac{3}{4}$	$3\frac{3}{8}$	$2\frac{1}{2}$	$23\frac{2}{3}$	$8\frac{5}{6}$	$5\frac{3}{4}$
	$2\frac{3}{10}$	$3\frac{1}{2}$	$1\frac{5}{16}$	$8\frac{7}{8}$	$15\frac{1}{6}$	$2\frac{3}{4}$	$1\frac{2}{3}$
	$6\frac{1}{2}$	$12\frac{11}{12}$	$7\frac{1}{8}$	$17\frac{1}{8}$	49	$15\frac{11}{12}$	11

Subtract. Check by going over the work:

5.	$4\frac{11}{12}$	9	$5\frac{1}{10}$	$8\frac{4}{5}$	$12\frac{5}{6}$	$4\frac{1}{8}$	$9\frac{1}{2}$
	$3\frac{5}{12}$	$7\frac{3}{8}$	$2\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{6}$	$1\frac{5}{8}$	$3\frac{9}{10}$
	$1\frac{1}{2}$	$1\frac{5}{8}$	$2\frac{3}{5}$	$1\frac{3}{10}$	$4\frac{2}{3}$	$2\frac{1}{2}$	$5\frac{3}{5}$
6.	$6\frac{1}{5}$	$4\frac{1}{6}$	$7\frac{5}{6}$	$4\frac{3}{8}$	$20\frac{3}{4}$	$3\frac{2}{3}$	$8\frac{9}{16}$
	$2\frac{7}{10}$	$1\frac{3}{4}$	$3\frac{7}{12}$	$2\frac{3}{8}$	$15\frac{7}{8}$	$1\frac{1}{4}$	$4\frac{7}{8}$
	$3\frac{1}{2}$	$2\frac{5}{12}$	$4\frac{1}{4}$	2	$4\frac{7}{8}$	$2\frac{5}{12}$	$3\frac{11}{16}$

Find the answers:

7. $7\frac{1}{2} \times \frac{7}{12}$ $4\frac{3}{8}$ $9 \div \frac{3}{4}$ 12 $8 \div \frac{2}{3}$ 12 $\frac{15}{16} \div 10$ $\frac{3}{32}$ $6\frac{1}{4} \times 16$ 100

8. $3\frac{3}{4} \div 10$ $\frac{3}{8}$ $\frac{3}{4} \times \frac{5}{8}$ $\frac{15}{32}$ $\frac{1}{2} \times \frac{7}{8}$ $\frac{7}{16}$ $2\frac{2}{3} \div 5\frac{1}{3}$ $\frac{1}{2}$ $18 \times 1\frac{1}{4}$ $22\frac{1}{2}$

9. $5\frac{1}{4} \times 3\frac{1}{3}$ $17\frac{1}{2}$ $8 \times \frac{5}{6}$ $6\frac{2}{3}$ $\frac{7}{8} \div \frac{1}{4}$ $3\frac{1}{2}$ $1\frac{7}{8} \div 1\frac{1}{4}$ $1\frac{1}{2}$ $4\frac{1}{6} \div 6\frac{2}{3}$ $\frac{5}{8}$

10. $30 \div 3\frac{3}{4}$ 8 $\frac{2}{3} \times 5$ $3\frac{1}{3}$ $\frac{5}{6} \times 3$ $2\frac{1}{2}$ $1\frac{1}{2} \times 1\frac{1}{2}$ $2\frac{1}{4}$ $25 \div 1\frac{1}{5}$ $20\frac{5}{6}$

11. $16 \times 1\frac{7}{8}$ 30 $\frac{3}{4} \div \frac{2}{3}$ $1\frac{1}{8}$ $6 \div \frac{2}{3}$ 9 $1\frac{1}{8} \times 2\frac{2}{3}$ 3 $10 \times 1\frac{1}{3}$ $13\frac{1}{3}$

12. $2\frac{1}{2} \div 12$ $\frac{5}{24}$ $\frac{5}{8} \div \frac{3}{4}$ $\frac{5}{6}$ $\frac{3}{4} \times \frac{2}{3}$ $\frac{1}{2}$ $3\frac{3}{4} \div 1\frac{1}{2}$ $2\frac{1}{2}$ $2\frac{5}{8} \times 3\frac{1}{7}$ $8\frac{1}{4}$

First review the procedure for finding the least common denominator (page 16). Check papers and group students for further review or reteaching.

Present problems illustrating the use of a bank loan.

Borrowing Money from a Bank

1. Mr. and Mrs. Cook need \$3000 to buy a new truck for their vegetable farm. They expect to have enough money to pay for it by fall, but they want the use of the truck during the spring and summer. The bank will lend them the money at 5% interest, in advance. If they borrow the money on Mar. 5 and promise to pay it back on Oct. 1, how many days will they have the money? 210 How much interest will they pay? \$87.50 How much money do they get on Mar. 5? \$2912.50 How much money must they pay the bank on Oct. 1? \$3000 Write the promissory note that Mr. and Mrs. Cook gave the Central Bank where they borrowed the money.

2. On Oct. 1 the Cooks were able to pay back only \$2500 of the loan. They arranged to give the bank a new promissory note for \$500, dated Oct. 1 and payable Nov. 24, the interest to be paid Nov. 24 instead of in advance as usual. Write the new note and tell how much in all the Cooks paid the bank on Nov. 24. \$503.75

3. Suppose the Cooks needed to borrow \$300 more after the loan in ex. 2 had been paid. This new promissory note is dated Dec. 19, 1958, and runs for 90 da. Find the date of maturity of the note. March 19,1959

Remind the students that interest on a bank loan usually is collected in advance. Be sure they know what the face of the note is, and what the proceeds are. Check their notes carefully in ex. 1-2.

Discuss and explain personal or small loans.

Personal Loans

1. In this country many of our states have **small loan laws** that control the lending of money in small amounts. These laws make it possible for people to borrow money without furnishing the types of security required for bank loans. Personal loans are used to take care of medical bills and old debts, and to pay for automobile expenses, clothing, and household appliances. To get a personal loan, the borrower goes to a small loan company or a personal finance company, and gives references concerning his character and the nature of his job. If these references are satisfactory, the company requires the borrower to sign a promissory note and sometimes asks him to get some responsible person to sign with him. The company may also require the borrower to give security, such as a mortgage on an automobile or furniture.

2. A limit is usually set on the amount that may be loaned by small loan or personal finance companies. In a few states this limit is $300, in other states it is $500, and in some states it is $1000 or more.

3. Personal loans are paid off by making monthly payments over a certain period of time. Finance companies usually Stress. quote **equal** monthly payments that must be made to pay off, with interest, a loan of a given amount in 6 months, 12 months, and other periods of time.

4. The rate of interest charged by small loan companies is Stress. usually stated as a **rate per month** instead of as a **rate per year.** For example, in some states 3% **per month** Stress. may be charged on small loans. This rate is applied each month to the unpaid balance of the loan to find the interest due for that month. Since a loan is reduced by monthly payments, the **amount** of interest, but not the rate of interest, decreases as time goes on. Emphasize.

5. An interest rate of 3% for 1 month is the same as an interest rate of 36% for 12 mo., or 1 yr. A yearly rate of 36%

These two pages merit careful explanation and discussion. If possible, you might secure information about rates of different finance companies in your community. Concepts in ex. 4 should be explained carefully.

may seem very high, but it is not so high in view of the extra expenses and risks that go with small loans. Since small loans are paid back in monthly amounts, it requires much bookkeeping by the loan companies to keep records of these payments. Sometimes there are losses because the borrower cannot complete his payments. There is also the expense of investigating the borrower's character when he originally applies for a loan.

6. In some states a lower interest rate is used for the upper part of a small loan. For example, a rate of 3% a month may be charged on the first \$100 of the loan; 2% a month on that part of the loan between \$100 and \$300; and $\frac{3}{4}$% a month on that part between \$300 and \$600. Under this plan what would be the interest for the first month on a loan of \$450? \$8.13

7. The interest rates charged on the lower part of small loans vary from 2% to 4% a month, depending upon the state. What is the yearly rate if the monthly rate is $2\frac{1}{2}$%? 30% if the monthly rate is $3\frac{1}{2}$%? 42%

8. Find out whether your state has a law on small loans. If so, what rate of interest may be charged on such loans?

9. Personal loans may also be obtained from banks that specialize in such loans or from the personal loan departments of other banks. These loans are paid off in equal monthly payments. The interest charge is often stated as a per cent, such as 6%, of the original amount of the loan. Stress.

Emphasize the reasons for the seemingly high rates, as given in ex. 5. Do ex. 6-7 with the class. Have students report their findings for ex. 8 and also have them secure information about personal loans from banks in your community.

Teach how to find the rate of interest that is really paid on small

Interest on Unpaid Balances loans (pages 132-133).

1. Problem Mr. Rice borrowed $50 from a personal finance company which charges interest on small loans at 2% a month. Mr. Rice agreed to pay back the $50 in 5 monthly payments of $10 each, plus interest computed monthly on Stress the unpaid balances. Find the total amount of interest to be paid; also find the total monthly payment including interest. $10.60

$3.00

Explanation Mr. Rice had the use of $50 for the first month. Since he makes a payment of $10 at the end of the month, he has the use of $40 for the second month. At the end of the second month he pays another $10 so that he has the use of $30 for the third month. What amount has he to use for the fourth month? $20 for the fifth month? $10 The loan is entirely paid at the end of the fifth month. At the end of each month interest Emphasize. is charged only on the **unpaid** balance. For example, at the end of the first month interest at 2% a month is charged on $50, which is the unpaid balance; so the interest is $1.00. At the end of the second month the interest is $.80. In like manner, the interest is $.60 for the third month, $.40 for the fourth, and $.20 for the fifth. The total interest is $3.00.

	Principal Used	Monthly Interest
1st month	$50	$1.00
2nd month	40	.80
3rd month	30	.60
4th month	20	.40
5th month	10	.20
		$3.00

Stress.

Since the interest for 5 mo. is $3.00, the average interest per month is $3.00 ÷ 5, or $.60. Adding this $.60 to the regular monthly payment of $10.00, you get $10.60 as the total amount to be paid each month for 5 mo. These 5 payments of $10.60 each will pay off both the principal and the interest.

Emphasize.

2. An interest rate of 2% a month is the same as a rate of 24% a year. Show that the interest on $50 for 5 mo. at 24% a year is $5.00. Why did Mr. Rice pay only $3.00 interest? His principal kept decreasing. $i = \$50 \times \frac{24}{100} \times \frac{5}{12} = \$5$

3. Mr. Hill borrowed $60 from a loan company. He paid it back in 5 mo. at the rate of $12 a month, with interest on the unpaid balances at $2\frac{1}{2}$% a month. Find the total interest. $4.50 Also find the total monthly payment including interest. $12.90

Ex. 1 should be worked out, step by step, with the class. In ex. 2 point out the following: $5 interest is based on the whole principal being kept for 5 mo.; $3 interest is based on the principal being decreased each month by monthly payments (thus it is less).

1. Problem Mr. Burns borrowed $150 from a loan company. He repaid the loan in 6 monthly payments of $26.75 each; these payments included the monthly interest. Find the total interest charged on this loan and also the monthly rate of interest. 2% $10.50

Explanation The total amount paid back was 6 × $26.75, or $160.50. Since the amount borrowed was $150.00, the difference between $160.50 and $150.00, which is $10.50, is the total interest that was charged.

Since $10.50 is the interest for 6 mo., the **average interest** per month is $10.50 ÷ 6, or $1.75. Thus each monthly payment of $26.75 includes $1.75 in interest, which leaves $25.00 of each payment to be used to reduce the principal. The loan of $150 is reduced by $25 each month. Stress. Stress.

The table at the right shows that Mr. Burns had the use of $150 for the first month, $125 for the second month, and so on. To find the **average amount** that he had the use of each month, add the 6 amounts and divide by 6, which gives $525 ÷ 6, or $87.50. The average amount of interest paid each month was $1.75 as shown above. So, on the average, Mr. Burns paid interest of $1.75 a month for the use of $87.50. Show that $1.75 is 2% of $87.50. You see that Mr. Burns paid interest at 2% a month. A rate of 2% per month is the same as a rate of 24% per year.

$150.00	1st month
125.00	2nd month
100.00	3rd month
75.00	4th month
50.00	5th month
25.00	6th month
6) $525.00	
$ 87.50	Average

2. A personal finance company advertises that it will lend $240 and that the entire loan, including interest, may be repaid in 8 monthly payments of $34.05 each. Find the total amount of interest charged on the loan. $32.40 What is the monthly rate of interest? 3% the annual rate of interest? 36%

3. A company that makes personal loans will lend $140 with the understanding that it will be paid back in 7 payments of $22 each. These payments include the interest. Find the total interest (1) and the monthly rate of interest. (2) (1) $14; (2) $2\frac{1}{2}$%

In ex. 1 emphasize that Mr. Burns did not have use of the $150 for the entire 6 mo.

Present the formula for finding the interest paid on small loans and give practice in using it (pages 134-135). See the Guide for the derivation of the formula.

Finding the Rate by Formula

1. A quicker way to find the rate of interest in ex. 1 on page 133 is to use the following formula which applies in all cases where the loan is paid off in **monthly** payments: Stress.

$$r = \frac{24\ C}{P(n + 1)}$$

The letters in the formula stand for these values:

r = rate of interest Stress. **per year,** expressed as a decimal
C = total interest charge, expressed in dollars and cents
P = amount of original principal or loan Stress.
n = number of payments to be made to pay off the loan

In ex. 1 on page 133, show that C = \$10.50, P = \$150, and n = 6. Putting these values in the formula, you get

$$r = \frac{24 \times 10.50}{150(6 + 1)} = \frac{\overset{4}{\cancel{24}} \times 10.50}{\underset{25}{\cancel{150}} \times 7} = \frac{42.00}{175} = .24$$

Emphasize.
Thus, r = 24%. Notice that 24% is the **yearly rate** of interest. To get the **monthly rate,** divide 24% by 12, which gives 2%. So 2% is the monthly rate of interest.

2. Use the formula above to find the yearly rate of interest in ex. 2 on page 133. First find the total interest charge, which \$32.40 will be the value of C. What value will you put in the formula for P? \$240 for n? 8 After finding the yearly rate of interest, 36% also find the monthly rate. 3%

3. In ex. 3 on page 133, find the yearly rate of interest by the formula; 30% then find the monthly rate of interest. $2\frac{1}{2}\%$

4. To get a loan of \$75 from one loan company, a borrower must pay back \$13.50 each month for 6 mo. Find, to the nearest tenth of 1%, the yearly rate of interest. 27.4% About what per cent is the monthly rate? 2.3% Is this rate legal under the small loan laws? Up to 3% monthly is legal in some states.

5. Find r (1) when C = \$48.75, P = \$300, and n = 12. Then find the monthly rate of interest. (2) (1) 30%; (2) $2\frac{1}{2}\%$

Discuss ex. 1 with the class and emphasize the meaning of each letter. Do ex. 2 with the class. Have different students substitute values in the formula. Urge the students to ask questions if they are unsure of this work.

Have students bring in advertisements from loan companies. Use them to compute actual rates of interest.

Interest Rates on Personal Loans

By the formula, find the rates to the nearest tenth of 1%:

1. Some personal loan companies will lend \$300 and will permit the borrower to pay it back in any one of these ways:

   (a) By making 6 monthly payments of \$54.02 each, or 27.6%
   (b) By making 12 monthly payments of \$28.82 each, or 28.2%
   (c) By making 15 monthly payments of \$23.80 each. 28.5%

   Find the annual rate of interest in each case.

2. A large bank has a personal loan department in which loans are made to responsible persons. On a loan of \$200 the borrower must pay back \$17.34 a month for 12 mo. What annual rate of interest is charged on this loan? 7.5%

3. Mr. Weeks wants to borrow \$2000 to buy a car. The bank will lend him \$2000 for 2 yr. with the understanding that he will pay back \$91.67 each month for 24 mo. What annual rate of interest does the bank charge for this loan? 9.6%

4. In a state that has no law regulating small loans, a man borrowed \$100 and agreed to pay it back at the rate of \$12 a month for 12 mo. What annual rate of interest did he pay? 81.2%

5. A bank advertised: "We make personal loans to responsible people at 6% interest." Mr. Hall borrowed \$300 from this bank. The bank told him that the total interest charge would be 6% of \$300, or \$18. This made \$318 that he had to pay back in 12 monthly payments of \$26.50 each. What rate of interest did Mr. Hall really pay? 11.1%

6. When payments on a loan are made **weekly** instead of monthly, the formula at the right should be used. In this formula n stands for the number of **weekly** Stress. payments, P for the amount of the loan, and C for the total amount of interest that is charged. By this formula, find the annual rate of interest that a loan shark receives if he forces a workman to make 50 weekly payments of \$2.75 each to repay a loan of \$100. 76.5%

$$r = \frac{104\,C}{P(n+1)}$$

Use ex. 5 to point out that the actual interest rate may be higher than it appears to be.

Discuss installment buying including carrying charges.

Installment Buying

1. Mrs. March wants to buy an electric refrigerator. The refrigerator will cost $330 if she pays all cash when she buys it, but she does not have $330 in cash. So she buys the refrigerator on the installment plan. By this plan she pays $30 in cash when she makes the purchase and after that she pays $26.50 each month for 12 mo. How much in all will the refrigerator cost Mrs. March on the installment plan? $348 The total cost of the refrigerator on this plan is how much more than the cash price of $330? $18

2. You see that the cost on the installment plan is $18 more than the cash price. In other words, it costs $18 extra to buy the refrigerator on the installment plan. This extra charge of $18 is called the **carrying charge.** It is the extra price Mrs. March has to pay for the privilege of spreading the payments over 12 mo. and of having the use of the refrigerator while she is paying for it.

Discuss ex. 1–3 with the class. Be sure the meaning of carrying charge and additional expenses is clear. Encourage a class discussion of installment buying.

3. The extra $18 that Mrs. March pays for the refrigerator on the installment plan covers the cost to the store of this plan. The store has **additional expenses** when it sells anything on the installment plan. These expenses include the investigation of the customer's responsibility before the sale is made, the bookkeeping necessary to keep records of monthly payments, and the risk that the customer may fail to make all the payments. Is the carrying charge of $18 an extra expense to the customer who buys the machine on the installment plan? Yes

4. Mr. Williams wants to buy a new radio that sells for $40.00 cash. He can buy the radio on the installment plan by paying $4 down and $5 a month for 8 mo. What is the carrying charge if he buys the radio on the installment plan? $4

5. A watch can be bought for $60 cash, or by paying $10 down and $5 a month for 11 mo. What is the carrying charge if the watch is bought on the installment plan? $5

6. Miss Young wants to buy a television set on the installment plan. By this plan she pays $27.50 down and $15.75 a month for 18 mo. If Miss Young had saved the money before making the purchase, she could have paid $280 cash for the set. How much is the carrying charge on the set? $31

Emphasize the fact that installment prices are higher than cash prices. Have students explain when installment buying may be necessary and when it is not.

Review Problems

1. Yesterday Tom's father drove to Lee, a total distance of 275 mi. He averaged 35 mi. an hour for the first 3 hr. and then stopped $\frac{3}{4}$ hr. for lunch. At what average speed did he drive for the rest of the trip if he reached Lee 8 hr. from the time he started? 40 mi.

2. Tom says his father can average 16.5 mi. on a gallon of gasoline. How many gallons, to the nearest whole gallon, will he use on the trip of 275 mi.? 17

3. Mary Allen rode her bicycle 12 mi. in 1 hr. 40 min. Find the average time it took her to ride 1 mi. $8\frac{1}{3}$ min.

4. In our town the Community Chest set \$10,000 as the amount to be raised this year. When the drive was over, it was found that \$14,500 had been raised. What per cent more than \$10,000 was raised? 45%

5. Mr. West wants to buy some furniture for his store that is listed at \$2000 with a discount of 10%, 2%. The discount of 2% can be taken only if he can pay the bill promptly. He hasn't enough money now to pay the entire bill in time to get the 2% discount, but he will have enough if he borrows \$800 from the bank for 60 da. at 6%. Which is better, to borrow the money and take advantage of the 2% extra discount, or to pay the bill later without getting the 2% discount? How much better? \$28
Borrow the money

6. Mr. West buys a chair for \$39. He figures that his costs run about 60% of the price at which he must sell furniture. He also figures that his expenses make up about 32% of the selling price. What should be the selling price of this chair and how much profit will Mr. West make on it? \$5.20
\$65

7. A circular cement walk surrounds a circular flower garden that has a diameter of 28 ft. The width of the walk is 2 ft. How many square feet are there in the area of the walk? $188\frac{4}{7}$ Draw a diagram showing both the garden and the walk. Use $\pi = 3\frac{1}{7}$.

Check the students' work carefully and analyze errors. Discuss the problems to help them discover the causes of errors. Review or reteach a particular topic as needed. Use the developmental material in the text relating to the students' difficulties.

Present another set of improvement tests in addition.

Improving by Practice

Addition Test 3a.

Time: 4 min.

1. 356	594	898	876	513	859
435	473	682	629	829	224
116	514	533	550	748	978
282	922	801	394	686	796
249	534	668	371	660	769
326	101	674	543	851	478
927	753	260	618	482	960
109	576	152	507	989	394
319	249	375	172	446	976
3119	4716	5043	4660	6204	6434

6

Addition Test 3b.

Time: 4 min.

2. 375	392	177	599	358	831
858	655	819	782	803	946
331	809	607	575	184	225
698	488	740	367	677	554
964	357	748	325	729	900
708	587	857	706	842	424
843	672	648	657	695	695
935	822	999	312	332	773
919	678	846	949	646	756
6631	5460	6441	5272	5266	6104

6

Addition Test 3c.

Time: 4 min.

3. 584	994	597	998	689	971
396	526	969	483	968	645
533	261	774	674	847	667
804	754	648	794	854	682
887	438	575	686	627	559
945	236	789	848	996	346
806	627	627	407	257	937
119	806	941	979	709	265
269	559	380	287	276	249
5343	5201	6300	6156	6223	5321

6

To the Teacher. If individual pupils get low scores on Improvement Tests, such pupils may be assigned extra practice given on pages 358–361.

Tests should be given, scored, and recorded as others were (see pages 48–49). Have volunteers explain the work at the board so that students may find and correct errors. Plan remedial work as needed after difficulties are cleared up.

Show that the carrying charge in installment buying is the equivalent of an interest charge, and show what rate of interest this charge represents.

Installment Interest Rates

1. In ex. 1 on page 136 Mrs. March bought a refrigerator which had a **cash price** of \$330. Mrs. March bought the refrigerator on the installment plan by paying \$30 down and \$26.50 a month for 12 mo. The carrying charge on the refrigerator was \$18.

2. You can look at this problem in a new way. If the **down payment** of \$30 is subtracted from the **cash price** of \$330, you get \$300, which is the **unpaid balance.** So Mrs. March owed \$300 after making the down payment. She also owed a carrying charge of \$18 which is equivalent to an interest charge of \$18 on the loan of \$300. Thus she owed \$318 in all, which she paid off in 12 payments of \$26.50 each. By using the formula below, you can find what yearly rate of interest was paid on the loan. Stress.

\$330	Cash price
30	Down payment
\$300	Unpaid balance

$$r = \frac{24\,C}{P(n+1)}$$

r = rate of interest per year, expressed as a decimal
C = carrying charge, expressed in dollars and cents
P = unpaid balance at beginning of credit period, which equals the cash price less the down payment
n = number of payments, not counting the down payment
Stress.

3. In the above problem, show that C = \$18, P = \$300, and $n = 12$. Substitute these values in the formula and show that $r = .11$. This means that the carrying charge paid in buying the refrigerator on the installment plan was equal to an interest charge of 11% a year. Does this interest rate compare favorably with the legal rates for small loans?

4. Mrs. Smith can buy a sewing machine for \$175 cash, or by paying \$25 down and \$17 a month for 10 mo. Use the formula to find the yearly rate of interest, to the nearest whole per cent, if she buys the sewing machine on the installment plan. 29%

Follow development as given in ex. 1–2. Give the students careful instructions in substituting values in the formula. Emphasize the fact that P represents "the cash price less the down payment."

Through discussion, lead the students to see why it is important to be able to calculate rate of interest (so that you can decide whether to borrow money, pay cash, or use the installment plan).

Installment Purchases

By the formula find, to the nearest whole per cent, the yearly rate of interest that you pay if you buy these articles on the installment plan:

	Article	Cash Price	Installment Terms	
1.	Bicycle	$56.00	$6.00 down, $5.00 a month for 11 mo.	20%
2.	Phonograph	$84.00	$15.50 down, $9.50 a month for 8 mo.	29%
3.	Electric shaver	$21.50	$3.00 down, $5.00 a month for 4 mo.	39%
4.	Automatic washer	$240.00	$32.00 down, $16.00 a month for 14 mo.	12%
5.	Air conditioner	$245.00	$15.00 down, $14.50 a month for 18 mo.	17%
6.	Camera	$78.00	$8.00 down, $7.00 a month for 11 mo.	20%
7.	Outboard motor	$292.00	$31.50 down, $22.00 a month for 13 mo.	17%

8. Miss Lane can buy a necklace for $48 cash, or by making 8 monthly payments of $7.35 each, with no down payment. If she selects the monthly payment plan, find the annual rate and the monthly rate of interest. 60% annual, 5% monthly Instead of buying the necklace on this plan, would it be cheaper if she borrowed $48 from a small loan company so that she could pay all cash for the necklace? Yes She would repay the loan by making 8 monthly payments of $6 each, with interest Stress. on the unpaid balances at 2% a month. Under each plan, find the total cost of the necklace. $52.32 if borrowed, $58.80 by installments

Extend the work by using illustrations of installment buying from stores in your community.

Chapter Review

1. Why should a person who has a checking account be careful to fill out the stubs in his checkbook?

2. After receiving a check, when should you endorse it? Just before cashing it

3. When Mr. Allen bought a house from Mr. Hunt, he gave Mr. Hunt a "certified check" for $5000. Find out how a certified check differs from an ordinary check. See Guide.

4. When you are figuring interest, how many days is a year usually considered to have? 360

5. If money is borrowed from a bank, when is the interest usually paid and what is it called? In advance; bank discount

6. Suppose Mr. Gray borrows money for 6 mo. from Mr. Bell. In such cases, when is the interest usually paid? At the end of 6 mo.

7. The term "small loan" covers loans up to what amount? $300 to $1000, according to the state What are some of the legal rates of interest on such loans? 2% to 4%

8. What is meant by the term "carrying charge" as used in installment buying? The extra charge beyond the cash price

At 5%, find the interest on the following:

9. $500, 3 mo. $6.25 $650, $\frac{1}{2}$ yr. $16.25 $840, 60 da. $7.00 $1000, 90 da. $12.50

10. $450, 8 mo. $15 $900, $\frac{1}{4}$ yr. $11.25 $360, 66 da. $3.30 $2400, 18 da. $6

11. $720, 7 mo. $21 $600, 9 mo. $22.50 $300, 24 da. $1 $1800, 20 da. $5

12. A personal finance company will make a loan of $50 which can be paid back in 5 monthly payments of $10.75 each. Find the amount of interest charged for this loan. $3.75 Also find, by using the formula on page 134, the annual rate of interest charged for this loan. 30%

13. The cash price of an electric stove is $300, but it can be purchased on the installment plan by paying $50 down and $24 a month for 12 mo. Find the carrying charge on this purchase. $38 Then find the annual rate of interest paid for the installment plan, using the formula. 28%

Use the results of the review as the basis for class or individual reteaching. Try to determine the causes of errors through the students' explanations of their work. Be sure the concepts and terms in ex. 1–8 are clearly understood. Redevelop if necessary.

1. Mr. Ware needs for his farm a gasoline engine that costs \$225. He has \$75 in cash and is thinking of borrowing \$150 from Mr. Best for 9 mo. at 6%. What would this loan cost him? \$6.75

2. Instead of borrowing from Mr. Best, Mr. Ware finds that he can borrow \$150 from a loan company and pay back \$18.75 a month for 9 mo. What would this loan cost him? \$18.75

3. Mr. Ware could also purchase the gasoline engine on the installment plan, paying \$75 down and \$18 a month for 9 mo. On this plan, what would be the carrying charge? \$12 Of the three plans given in ex. 1–3, which is the cheapest? Borrowing from Mr. Best

4. Find the cost of laying a cement sidewalk 3 ft. wide and 88 ft. long at \$4.75 a square yard. \$139.33

5. When the corn production increased from 2.7 billion bushels one year to 3.1 billion the next year, what was the per cent of increase, to the nearest whole per cent? 15%

6. Mr. Lee timed himself while driving his car and found by his speedometer that he had gone 6.5 mi. in 10 min. Find Mr. Lee's speed in miles per hour. 39

7. Find the interest on \$350 for 18 da. at 6%. \$1.05

8. How many days are there between Aug. 3 and Oct. 5 of the same year? 63 between Jan. 4 and Feb. 3? 30

9. Find the net price of a bill for books amounting to \$480 if the discount is 25%, 3%. \$349.20

10. Mr. Marsh sold a house for Mr. Clay and kept \$920 of the sales price for his commission. The rest of the sales price, \$17,480, was sent to Mr. Clay. What per cent of commission did Mr. Marsh receive for making the sale? 5%

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Try to have conferences with students who had errors and help them to analyze the causes. This will also help you determine students' understanding of percentage problems. Some students may have only a mechanical mastery of a process.

Present a diagnostic test of the skills taught in Chapter 4, with practice-page references. Present a review of arithmetical terms and expressions.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row. Clear up difficulties before assigning practice pages.

Find the exact number of days from		Practice Pages
1. Nov. 17 to Dec. 2 15	July 30 to Aug. 23 24	121
2. Mar. 8 to June 6 90	May 20 to Sept. 17 120	121

Find the interest by any method:

3. \$1200, 1 yr., 3% \$36	\$480, $\frac{1}{2}$ yr., $2\frac{1}{2}$% \$6	118
4. \$3300, 5 mo., 4% \$55	\$4500, 11 mo., 5% \$206.25	120
5. \$1350, 72 da., 6% \$16.20	\$6000, 33 da., 4% \$22	122–124

At 6%, find the bank discount (1) and the proceeds: (2)

(1), (2) **6.** \$1000, 54 da. \$9, \$991	\$1750, 12 da. \$3.50, \$1746.50	\$650, 90 da. \$9.75, \$640.25	127
7. \$500, May 5 to May 20 \$1.25, \$498.75			127

The Language of Arithmetic

Lead a class discussion of students' definitions and examples of terms in ex. 8–34.

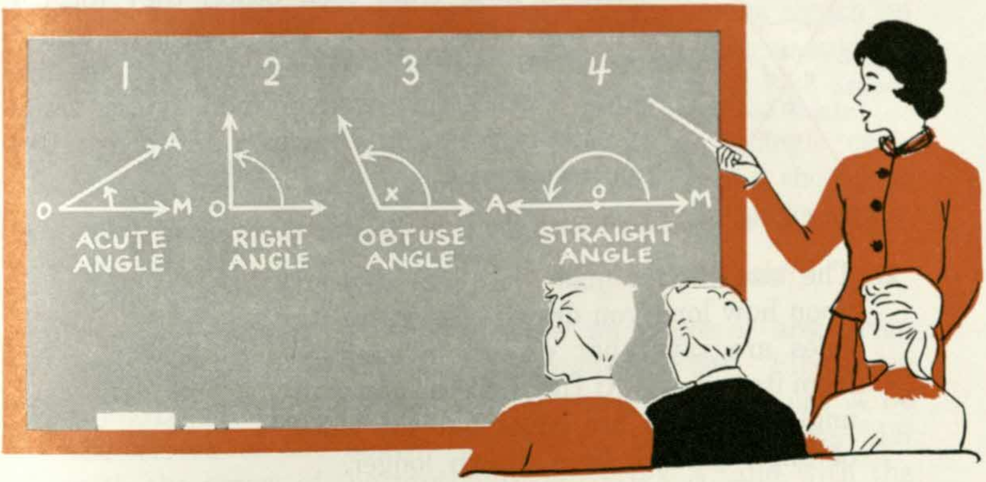
Show that you know what each word or group of words means by using it in a sentence or by giving an example of it:

8. formula
9. principal
10. proceeds
11. depositor
12. factor
13. trapezoid
14. interest
15. $33\frac{1}{3}\%$ off
16. discount
17. deposit slip
18. face of note
19. bank balance
20. endorsing check
21. endorse in full
22. certified check
23. promissory note
24. bank discount
25. carrying charge
26. small loan
27. check stub
28. list price
29. installments
30. commission
31. net price
32. ray
33. approximate
34. segment

If many students seem weak in a particular area, reteaching for whole class may be necessary. Have students make a list of terms they are unsure of.

See the Guide for the specific aims of Chapter 5 and a discussion of informal geometry. Students will need a ruler, a draftsman's triangle, a pair of compasses, and a protractor for the work in this chapter.

Chapter 5



Reading and Naming Angles

Reteach the kinds of angles and three ways of reading them.

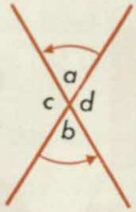
1. When two rays have the same endpoint they form an angle, as shown above. The rays are the **sides** of the angle and the endpoint O is the **vertex** of the angle. If you think of Stress. the sides as hinged at O, so that side OA can rotate from side OM, the **amount of rotation** indicates the size of the angle. When the sides are close together, the angle is small; as one side rotates from the other, the size of the angle increases. If the amount of rotation is a quarter of a circle, as in Fig. 2, the angle is a **right angle.** If the rotation is half of a circle, as in Fig. 4, the angle is a **straight angle.** An angle less than a right angle is an **acute angle** (see Fig. 1), while an angle greater than a right angle but less than a straight angle is an **obtuse angle** (see Fig. 3).

2. In Fig. 1, the angle is read angle MOA or angle AOM; when three letters are used to name an angle, the letter at the Stress. vertex is written between the other two letters. Often one letter is enough to name an angle. In Fig. 2, the angle is read angle O; in Fig. 3, the angle is read angle x where x Stress. is **inside** the angle. Give students more practice in reading angles.

Review the meaning of ray, line, and segment. The idea of angle and amount of rotation may be illustrated by spreading the legs of compasses. Have the students draw and label the different kinds of angles.

Review the measurement of angles and reteach the use of a protractor (pages 146–147). Make clear and show that a degree also refers to each of 90 small arcs into which the arc of a right angle is divided.

Measuring Angles

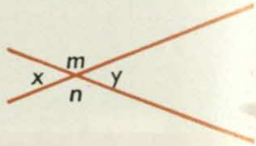


1. When 2 lines cross each other, they form 4 angles as shown at the left. Angles a and b are called **opposite** angles; these angles are equal because they were formed by the same amount of rotation. Angles c and d are also opposite angles. Be sure the meaning of opposite angles is clear.

If two lines intersect, the opposite angles are equal. Demonstrate and emphasize.

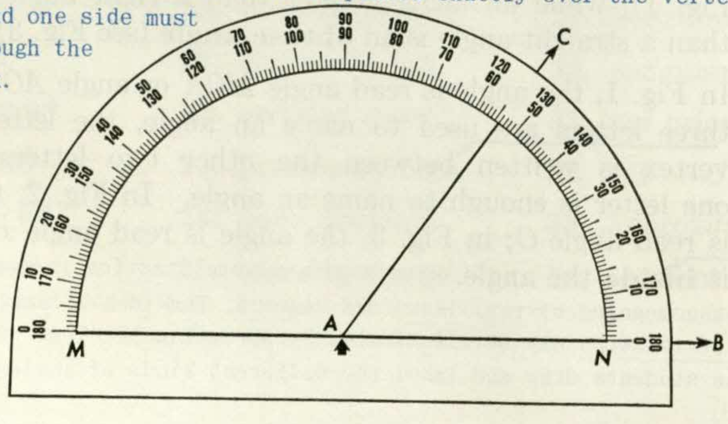
Stress.

2. The size of an angle does not depend upon how long you draw its sides, for its sides are rays that extend indefinitely from its vertex. At the right the opposite angles x and y are equal but the sides of angle y have been drawn longer.



3. The size of an angle is measured by a unit called the **degree**, which is a very small angle equal to $\frac{1}{90}$ of a right angle. The instrument used to measure the size of an angle is the **protractor**, which is shown at the bottom of the page. The curved edge of the protractor is a half-circle whose center is point A at the tip of the arrow. Point out that the 2 scales enable us to measure angles (up to 180°) in different positions.

4. To measure an angle, such as angle BAC shown below, place the protractor over the angle so that the tip of the arrow is at the vertex A and the edge MN of the protractor lies over side AB of the angle. Side AB should pass through the 180° mark on the scale of the protractor. When the protractor is in this position, side AC points to 54° on the inside scale of the protractor, so angle $BAC = 54°$. Emphasize that the center of a protractor must be placed exactly over the vertex of the angle, and one side must pass through the 180° mark.



Have students show experimentally that the ex. 2 statement is true by drawing 6 or more triangles of different sizes and shapes, and measuring the 3 angles of each triangle by means of a protractor.

1. Draw 3 large angles of different sizes, using your ruler. Then measure each angle with your protractor as shown on page 146. If either side of the angle is not long enough to meet the protractor, extend the side. Ask pupils if this affects the size of the angle.

2. Also draw a large triangle and measure each angle carefully. Find the sum of these 3 angles. The sum should be equal to or close to 180°. If not, measure the angles again.

The sum of the three angles in any triangle equals 180°.

3. A protractor can be used to draw an angle of any given size. Suppose you wish to draw an angle of 54°. First draw ray AB, which will be one side of the angle; let point A be the vertex of the angle. Place your protractor over AB with the arrow of the protractor at vertex A, and with the 180° mark on the outside scale lying on AB, as shown at the bottom of page 146. Next find the 54° mark on the inside scale of the protractor and put a dot by it on the paper. Call this dot D. Then take the protractor off the paper and draw ray AD. You now have angle BAD, which contains 54°.

4. With your protractor, construct angles of 80°, 30°, 55°, 140°, 165°, 70°, 135°, 28°, 68°, 92°, 131°.

5. An angle can be bisected by using a protractor. To **bisect** means to divide the angle into 2 equal parts. Measure the angle to be bisected and keep the protractor on the angle. Suppose it measures 60°. Take $\frac{1}{2}$ of 60°, which is 30°. On the protractor find the 30° mark and place a dot on the paper opposite this mark. Remove the protractor and draw a ray from the vertex of the angle through this dot. This ray bisects the angle, making 2 angles of 30° each.

6. With your protractor draw an angle of 50°. Then bisect the angle with your protractor. Do the same with angles of 44°, 128°, 160°, 136°, 172°, 90°, 150°.

7. Draw a large triangle and bisect each angle in it.

Post some constructions on the bulletin board for further reference.

Angles in Air Navigation

Explain the use of angles in air navigation.

1. In air navigation the direction of the straight line in which a plane flies is called the **course** of the plane and is expressed by giving the angle that the line makes with a line pointing north. These angles are always measured in the Stress **clockwise** direction from 0° to 360°. In Fig. 1, AN points north, which is called a direction of 0°. If a plane flies north, its course is 0°. In Fig. 1, AE points east. Since angle $NAE = 90°$, a plane flying east is on a course of 90°.

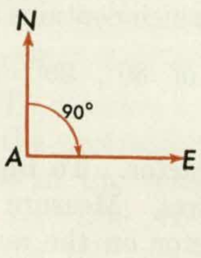


Fig. 1

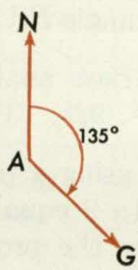


Fig. 2

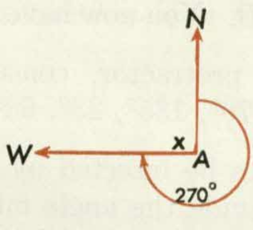


Fig. 3

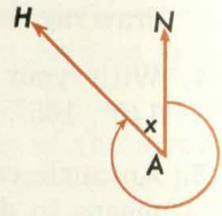


Fig. 4

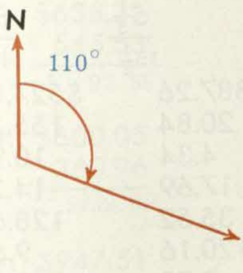
2. In Fig. 2, if a plane is flying in the direction of AG, its course is 135°; why? See Guide. In what direction does AG point? South-east In Fig. 3, AW points west. Since the large angle NAW (shown by the arrow) is 270°, a plane flying west is on a course of 270°. To get 270°, angle NAW was measured in the **clockwise** direction, as shown by the arrow; the course is **not** 90°. Stress. In Fig. 4, AH points northwest. If a plane flies northwest, what is its course? 315° In what direction is a plane flying if its course is 45°? Northeast 180°? South 125°? Southeast 360°? North

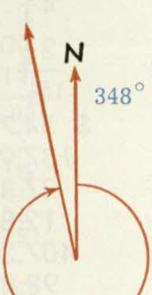
Discuss ex. 1-2 with the class, emphasizing meaning of course, how it is expressed, how these angles are measured (clockwise). Point out that we think of side AN as remaining fixed, while the other side rotates in a clockwise direction.

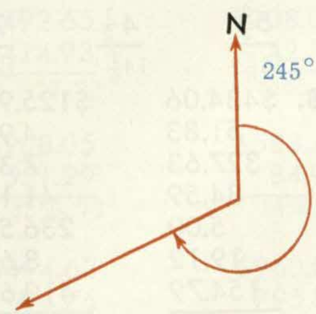
3. In Fig. 3, you cannot measure the large angle of 270° with your protractor because the scale on the protractor does not go beyond 180°, but you can measure angle x in Fig. 3. Angle x and the large angle together make 360°, so if you measure angle x and find it to be 90°, the large angle equals 360° − 90°, or 270°. In Fig. 4, if you measure angle x, how will you find the size of the large angle? 360° − 45° = 315°

4. With a protractor make drawings showing planes flying these courses: 70°, 120°, 180°, 240°, 260°, 280°, 340°, 20°.

5. With a protractor measure the angle in each figure and tell the course indicated by the arrow.







Do ex. 3 with the class, letting volunteers suggest how angles can be determined.

Present review problems and practice in addition of mixed numbers and decimals.

Problems and Practice

1. Mr. Swift's salary was \$90 a week. Then his salary was (\$108) raised 20%. He continued to receive this higher salary for 2 yr. when his salary was reduced 20% because of poor business conditions. What was his salary after the reduction? \$86.40 Jack says his salary is right back where it was 2 yr. ago, so his reduced salary is \$90 a week. Is Jack right? No

Have students explain why Jack's thinking is wrong.

2. If you use a scale of 1 in. = 50 ft., how many inches long will you draw a segment to represent 350 ft.? 7 125 ft.? $2\frac{1}{2}$

3. If you use a scale of 1 in. = 100 ft., how many feet will be represented by a line segment $1\frac{3}{4}$ in. long? 175 $2\frac{1}{4}$ in. long? 225 $3\frac{1}{2}$ in. long? 350 4 in. long? 400

4. Make a scale drawing of a trapezoid having bases of 14 ft. and 18 ft. and a height of 10 ft. Let $\frac{1}{4}$ in. = 1 ft. Then draw the trapezoid again, letting $\frac{1}{8}$ in. = 1 ft. (1) Bases: $3\frac{1}{2}$ in. and $4\frac{1}{2}$ in., height: $2\frac{1}{2}$ in.; (2) bases $1\frac{3}{4}$ in. and $2\frac{1}{4}$ in., height: $1\frac{1}{4}$ in.

Add these columns. Check the work:

5.	$9\frac{1}{4}$	$7\frac{3}{10}$	$2\frac{1}{2}$	$4\frac{3}{8}$	$9\frac{1}{4}$	$8\frac{2}{3}$	$2\frac{1}{2}$
	$4\frac{5}{16}$	$5\frac{1}{5}$	$4\frac{1}{3}$	$1\frac{1}{4}$	$6\frac{3}{8}$	$4\frac{5}{6}$	$3\frac{3}{5}$
	$13\frac{9}{16}$	$12\frac{1}{2}$	$6\frac{5}{6}$	$5\frac{5}{8}$	$15\frac{5}{8}$	$13\frac{1}{2}$	$6\frac{1}{10}$
6.	$6\frac{7}{8}$	$9\frac{5}{12}$	$6\frac{1}{2}$	$8\frac{7}{16}$	$2\frac{1}{2}$	$3\frac{3}{4}$	$7\frac{4}{5}$
	$3\frac{3}{4}$	$1\frac{3}{4}$	$2\frac{3}{10}$	$4\frac{1}{4}$	$4\frac{3}{4}$	$1\frac{5}{6}$	$3\frac{1}{2}$
	$2\frac{1}{2}$	$7\frac{5}{6}$	$2\frac{4}{5}$	$2\frac{1}{16}$	$1\frac{3}{4}$	$2\frac{1}{2}$	$6\frac{2}{5}$
	$13\frac{1}{8}$	19	$11\frac{3}{5}$	$14\frac{3}{4}$	9	$8\frac{1}{12}$	$17\frac{7}{10}$
7.	$4\frac{1}{3}$	$7\frac{1}{10}$	$5\frac{1}{2}$	$8\frac{5}{12}$	$9\frac{1}{5}$	$7\frac{2}{3}$	$1\frac{2}{3}$
	$6\frac{5}{6}$	$2\frac{3}{5}$	$6\frac{3}{4}$	$4\frac{2}{3}$	$3\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{5}{6}$
	$5\frac{2}{3}$	$4\frac{1}{2}$	$7\frac{1}{8}$	$2\frac{1}{4}$	$4\frac{3}{5}$	$5\frac{1}{3}$	$8\frac{1}{2}$
	$16\frac{5}{6}$	$14\frac{1}{5}$	$19\frac{3}{8}$	$15\frac{1}{3}$	$17\frac{3}{10}$	$15\frac{1}{2}$	14

8.	\$434.06	\$125.95	\$ 54.79	\$387.26	\$527.82
	51.83	4.92	137.92	20.84	155.15
	327.63	7.34	27.35	4.34	10.94
	84.59	41.15	12.83	17.69	14.28
	5.00	236.56	407.36	35.52	128.63
	19.12	8.63	98.95	120.16	9.58
	154.79	410.64	50.00	8.28	241.03
	\$1077.02	\$835.19	\$789.20	\$594.09	\$1087.43

Use review to spot-check for class or individual weaknesses. Plan further review or reteaching of topics as needed.

Present a set of improvement tests in subtraction. Have the students determine and record their own scores in Record Books (page 48).

Improving by Practice

Subtraction Test 3a.

Time: $3\frac{1}{2}$ min.

1.	$663.19 − 517.83	$676.18 − 27.96	$704.79 − 114.87	$400.00 − 25.99
	$145.36	$648.22	$589.92	$374.01
2.	$425.31 − 284.53	$982.75 − 525.88	$303.50 − 193.96	$100.41 − 52.77
	$140.78	$456.87	$109.54	$ 47.64
3.	$500.04 − 368.76	$626.19 − 36.62	$501.00 − 84.71	$986.35 − 682.46
	$131.28	$589.57	$416.29	$303.89 (12)

Subtraction Test 3b.

Time: $3\frac{1}{2}$ min.

4.	$953.25 − 372.49	$908.03 − 44.09	$332.72 − 64.59	$700.00 − 340.93
	$580.76	$863.94	$268.13	$359.07
5.	$200.02 − 194.13	$845.17 − 691.49	$708.02 − 429.57	$734.48 − 67.19
	$ 5.89	$153.68	$278.45	$667.29
6.	$263.77 − 137.99	$340.06 − 74.17	$900.03 − 37.09	$841.25 − 353.97
	$125.78	$265.89	$862.94	$487.28 (12)

Subtraction Test 3c.

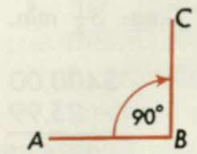
Time: $3\frac{1}{2}$ min.

7.	$638.21 − 545.63	$800.00 − 46.11	$492.65 − 414.93	$208.03 − 42.36
	$ 92.58	$753.89	$ 77.72	$165.67
8.	$600.05 − 360.96	$894.14 − 586.23	$908.05 − 551.29	$583.42 − 84.93
	$239.09	$307.91	$356.76	$498.49
9.	$947.51 − 396.76	$300.07 − 63.18	$494.62 − 94.76	$960.00 − 795.82
	$550.75	$236.89	$399.86	$164.18 (12)

Have the students compare the results of the tests with previous ones to note improvements and/or weaknesses. Help them to analyze the causes of errors and correct them. Plan remedial work as needed.

Reteach two ways of drawing perpendicular lines.

Drawing Perpendicular Lines



1. Two lines that intersect to form a right angle are perpendicular to each other. For example, *AB* is perpendicular to *BC* because angle *ABC* is a right angle. Likewise, *BC* is perpendicular to *AB*.

2. If you wish to draw a line perpendicular to line *AB* at point *B*, one way to do it is to draw an angle of 90° with your protractor, using *B* as the vertex of the angle. Another way to draw perpendicular lines is to use a **draftsman's triangle,** which has a right angle at one of its corners. A draftsman's triangle is usually made of wood or plastic but it can easily be made of cardboard. The picture on page 153 shows a draftsman's triangle in use.

3. Problem Using a ruler and a draftsman's triangle, draw a line perpendicular to line *AB* at point *C*.

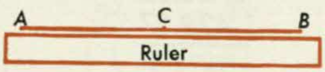


Fig. 1

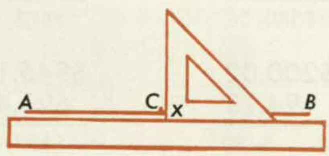


Fig. 2

Explanation Draw line *AB* with your ruler and mark a point *C* on it. Keep your ruler in place along this line, as shown above in Fig. 1. Next place your triangle on the ruler, as in Fig. 2, so that its right angle *x* has its lower side along the ruler and its vertical side touches point *C*. Then draw a line along the vertical side of the triangle. This line will be perpendicular to *AB* at *C*.

4. Draw a line *AB* and mark 4 different points on it. At each of these points draw a line perpendicular to *AB*.

Draw these rectangles. Use a draftsman's triangle to draw the right angles at the corners:

5. 6 in. by 4 in.	8 in. by 2 in.	3 in. by 5 in.
6. 7 in. by 4 in.	9 in. by 3 in.	2 in. by 4 in.

Check students' work carefully in ex. 4 before assigning ex. 5-6.

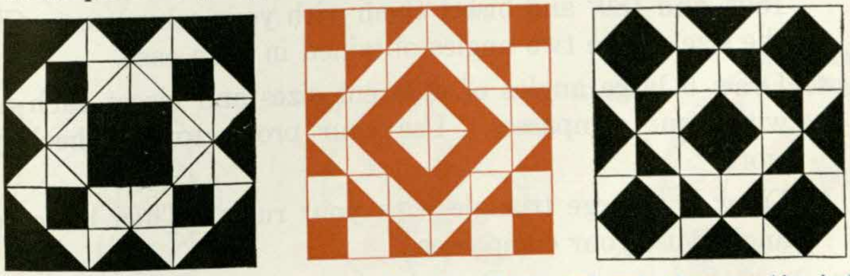
7. In the picture above, the man is holding a draftsman's triangle against the edge of a long flat ruler, called a **T-square.** The line he is drawing along the edge of the triangle will be perpendicular to the edge of the flat ruler. This man is drawing the floor plan of a house. Why are perpendicular lines used in the floor plans of houses and buildings?

Discuss fully.

8. Find some floor plans of houses in magazines or newspapers and look for lines perpendicular to each other.

9. Name some other kinds of drawings in which a draftsman would use perpendicular lines.

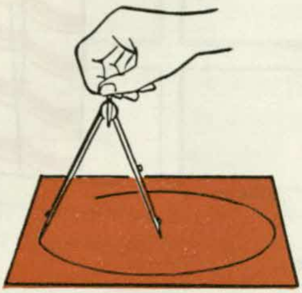
10. Draw each of the designs below, using your ruler and triangle. Make the outside square 3 in. on a side. First divide each side into 6 equal parts by points $\frac{1}{2}$ in. apart, and join these points by segments to form 36 small inside squares.



In ex. 8 have students bring examples to class and put them on the bulletin board for future reference. After they complete ex. 10, encourage them to make original designs, using their rulers and triangles.

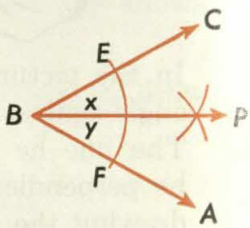
Teach how to bisect an angle by using compasses.

Bisecting an Angle



1. You know how to use compasses to draw circles. With your compasses draw a circle that has a radius of $1\frac{1}{2}$ in. To do this, spread the points of the compasses so that they are $1\frac{1}{2}$ in. apart. On the circle mark the center O and draw a radius OA. Then draw a diameter BC. How long is BC? 3 in. Mark 2 points on the circumference with the letters E and F. What is the part of the circumference EF called? An arc See page 91.

2. In ex. 5 on page 147, you learned how to bisect an angle with a protractor. You can also do this with compasses without measuring the size of the angle to be bisected. To bisect angle ABC, place the sharp point of the compasses on B as center and, with any convenient radius, draw an arc cutting the sides of the angle at E and F. Then with EF as radius, draw an arc with E as center and another arc with F as center. These arcs cross at P. Draw ray BP and it will be the **bisector** of angle ABC. Stress. This bisector divides angle ABC into 2 equal angles x and y, so angles x and y are each half of angle ABC.



3. With your protractor draw an angle of 70° and bisect the angle with your compasses. Then, with your protractor, measure the two angles obtained by bisecting 70°. Is each angle the right size? Also draw angles of 90°, 130°, 50°, 150°, and 180° and bisect them with your compasses. Check the size of the two angles obtained in each case.

4. Draw 5 large angles of different sizes and bisect each angle with your compasses. Use your protractor to check your work.

5. Draw any large triangle with your ruler. Then bisect each angle with your compasses.

In ex. 2 emphasize the fact that the new method does not require measurement of the angle and is independent of errors in measurement.

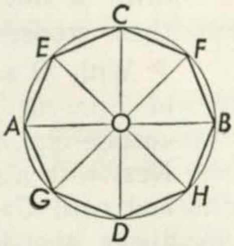
Show how to draw a regular octagon using the method of bisecting an angle. Emphasize the meaning of regular octagon as given in ex. 1.

Using the Octagon in Designs

Stress.

1. A **regular octagon** is a figure having eight equal sides and Stress. eight equal angles. You draw a regular octagon as follows:

▶ First draw a circle having a radius of 1 in.; next draw a diameter *AB*. Then with your draftsman's triangle draw diameter *CD* perpendicular to *AB*. You now have four right angles at the center *O*. With your compasses bisect angle *BOC*, which will give *OF* as the bisector; then draw *OG* in the opposite direction. Also bisect angle *COA*, which will locate points *E* and *H* on the circle. You now have 8 points on the circle which divide the circle into 8 equal parts. Connect these 8 points with segments and you will have a regular octagon.

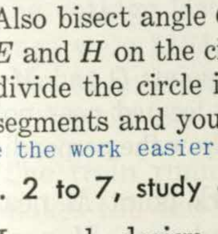


Make the work easier by having the students use a radius of $1\frac{1}{2}$ in. or 2 in.

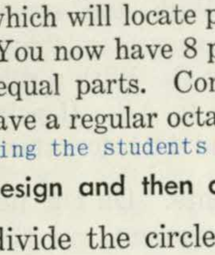
In ex. 2 to 7, study each design and then copy it:

In each design, first divide the circle into 8 equal parts as shown in ex. 1. Design 2 shows how to start designs 3 and 4. Design 5 shows how to draw design 6. In design 5 first draw the two overlapping squares. Then draw the arcs that form the petals by using the mid-points of the sides of the squares as centers and a radius equal to one half the side of each square.

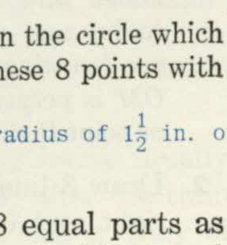
2.



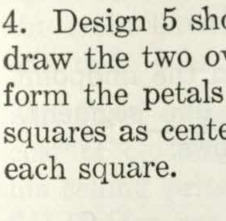
3.



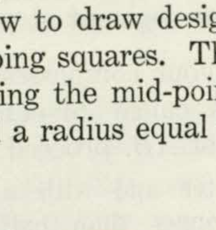
4.



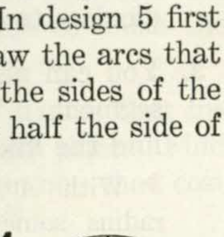
5.



6.



7.



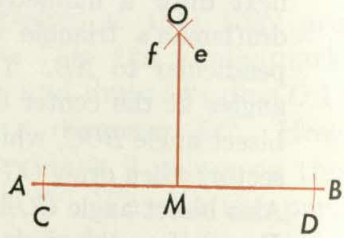
See the Guide for instructions for drawing the design in ex. 7. Have the students make their own designs for display after they complete this work.

Teach how to use a ruler and compasses to draw perpendicular segments.

A New Way to Draw Perpendiculars

1. Instead of using a draftsman's triangle, you can use your compasses to draw one line perpendicular to another. To draw a line perpendicular to line AB at point M, follow these steps:

▶ With M as a center and a radius of 1 in. or $1\frac{1}{2}$ in., draw two arcs cutting line AB at points C and D. Next with a somewhat longer radius and with C as center, draw arc e above the line. With the same radius and with D as center, draw arc f cutting arc e at O. With a ruler join points O and M; then OM is perpendicular to AB. If point M is located near point A or point B, first extend AB before starting to draw the perpendicular.

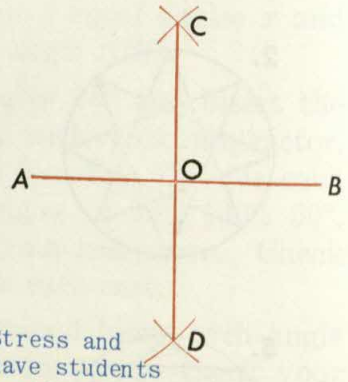


2. Draw 3 line segments. On the first, mark point M near the right end of the segment; on the second, put M near the middle; on the third, put M at the left end. Draw perpendiculars to each line segment at point M.

3. You can also use your compasses to find the midpoint of a segment. This is called **bisecting** a line segment. To find the midpoint of AB, proceed as follows: Stress.

▶ With A as center and with a radius somewhat longer than half of AB, draw a short arc above and another one below AB. With B as center and with the same radius, draw arcs above and below AB; these arcs should intersect the first arcs at points C and D. Using your ruler, draw CD, which will cut AB at O; then O is the midpoint of AB. CD bisects AB and is also perpendicular to AB. Test your work by measuring AO and OB.

Stress and have students verify this.



4. Draw 3 segments of different lengths and bisect them. Check the work by measuring with a ruler.

Present a review of problem solving and computational skills (pages 157–158).

Review Problems

1. Mr. King built a cottage at Clear Lake to rent. He spent \$6500 to build it and rents it at \$85 a week during the summer. Last summer the cottage was rented 12 weeks and the rest of the year it was closed. Expenses connected with the cottage last year were \$165.75 and the taxes and insurance were \$194.25. Find, to the nearest whole per cent, the rate of interest Mr. King received last year on the \$6500 he has invested in the cottage. 10%

2. The train running between Denton and Westbrook usually makes the 480-mile trip in 10 hr. Yesterday, after the train had traveled 3 hr. at its usual speed, there was a delay of $1\frac{1}{2}$ hr. due to an accident. How many miles per hour did the train have to run on the rest of the trip to finish the trip only $\frac{1}{2}$ hr. late? 56

3. Mr. Field owns a men's clothing store. He figures that his cost of clothing should be 65% of his selling price. Find his selling price for a coat that cost \$39; \$60 for one that cost \$41.60; \$64 for one that cost \$55.90. \$86

4. A set of books that have a cash price of \$75 can be purchased on the installment plan by paying \$15 down and \$6.75 a month for 10 mo. Find the carrying charge. \$7.50 This charge is equivalent to paying what annual rate of interest? 27.3% See page 140 for the formula for finding the installment rate.

5. Mary bought a box of writing paper for \$.71 that had been marked \$1.45. Find, to the nearest whole per cent, the per cent of reduction in the price of this paper. 51%

6. Mrs. Gates paid \$.74 for a bag containing 18 oranges. Was this price cheaper than \$.53 a dozen and if so, how much cheaper was it per dozen? $\$.03\frac{2}{3}$ cheaper

First review the meaning of the letters in the formula for finding the rate of interest in installment buying (page 140).

Emphasize the importance of checking each example carefully.

Mixed Practice

Find these sums:

1.	\$2765.49	.625	$16\frac{2}{3}$	43.5	1 lb. 3 oz.
	618.12	.450	$4\frac{1}{4}$	21.3	1 lb. 8 oz.
	500.00	.875	$12\frac{1}{2}$	2.7	2 lb. 1 oz.
	1225.75	.125	$14\frac{5}{12}$	14.4	2 lb. 8 oz.
	96.60	.500	$11\frac{5}{6}$	10.2	1 lb. 14 oz.
	937.36	.385		7.9	10 oz.
	\$6143.32	2.960	$59\frac{2}{3}$	100.0	9 lb. 12 oz.

Divide. Round off quotients to the nearest hundredth: Stress.

2. $18\overline{)32.7}$ 1.82 $6.5\overline{)133.9}$ 20.6 $.25\overline{)1.375}$ 5.5 $.4\overline{)236}$ 590 $.007\overline{)32.2}$ 4600

3. $.09\overline{)216}$ 2400 $8.6\overline{)100}$ 11.63 $.17\overline{)1.274}$ 7.49 $.5\overline{).12}$ .24 $.125\overline{)6.13}$ 49.04

Multiply or divide as indicated:

4. $2\frac{5}{8} \times 4\frac{2}{3}$ $12\frac{1}{4}$ $18 \div 4\frac{1}{2}$ 4 $\frac{9}{16} \times \frac{2}{3}$ $\frac{3}{8}$ $\frac{15}{16} \times 2\frac{2}{3}$ $2\frac{1}{2}$

5. $16 \times 6\frac{3}{4}$ 108 $5\frac{1}{4} \times 10$ $52\frac{1}{2}$ $\frac{9}{10} \div 6$ $\frac{3}{20}$ $\frac{5}{12} \div 3\frac{1}{3}$ $\frac{1}{8}$

6. $5\frac{1}{2} \div \frac{11}{12}$ 6 $4\frac{1}{2} \div 1\frac{1}{4}$ $3\frac{3}{5}$ $6 \times \frac{7}{8}$ $5\frac{1}{4}$ $\frac{11}{12} \div \frac{11}{16}$ $1\frac{1}{3}$

Find the number to put in each space:

7. 3% of \$75 is ∴ \$2.25 28 is ∴ % of 80 35 24 is 15% of ∴ 160

8. 7% of \$86 is ∴ \$6.02 72 is ∴ % of 96 75 77 is $3\frac{1}{2}$% of ∴ 2200

9. 4% of \$.25 is ∴ \$.01 26 is ∴ % of 65 40 38 is 25% of ∴ 152

Find the interest:

10. \$400, 3%, 1 yr. \$12 \$850, 6%, $1\frac{1}{2}$ yr. \$76.50 \$390, 5%, 2 yr. \$39

11. \$600, 4%, 7 mo. \$14 \$720, 5%, 11 mo. \$33 \$630, 2%, 14 mo. \$14.70

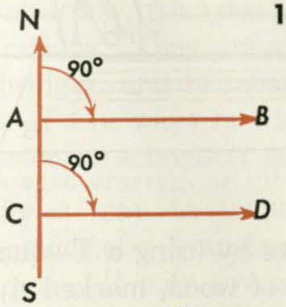
12. \$750, 6%, 6 da. \$.75 \$500, 6%, 57 da. \$4.75 \$800, 3%, 36 da. \$2.40

13. Write these numbers in our number system: CXLII, 142 MCMLI, 1951 MDCCCLXXV, 1875 MCMXXIX, 1929 XXXIV, 34 DCCCCXIII, 913 CDVIII, 408 CMLXXXV. 985

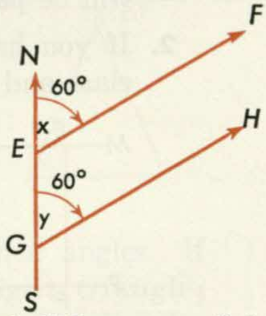
Check the answers with the class, letting volunteers explain their work at the board. If many students evidence difficulties with a particular example, reteaching for the whole class may be necessary. Plan individual remedial work as needed.

Reteach and extend the work on parallel lines first presented in Grade 7 (pages 159-160). In discussion, emphasize that parallel lines are lines having the same direction.

Parallel Lines



1. On page 148 you learned that the direction of a line can be indicated by giving the angle that the line makes with a line pointing north. In the figure at the left, line *NS* points north and lines *AB* and *CD* each make an angle of 90° with line *NS*. In what direction does line *AB* point? line *CD*? East Do the two lines point in the same direction? Yes The lines *AB* and *CD*, which have the **same direction, are parallel lines.** Will *AB* and *CD* ever meet? No



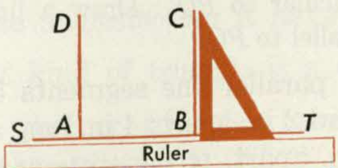
2. The figure at the right shows lines *EF* and *GH* each drawn at an angle of 60° from line *NS*. Do *EF* and *GH* have the same direction? Yes Are lines *EF* and *GH* parallel? Yes

The angles x and y in the figure at the right are called **corresponding angles.** In this figure, angle x and angle y are equal, and lines *EF* and *GH* are parallel. Would the lines be parallel if the angles were unequal? No

If two lines make equal corresponding angles with a third line, the two lines are parallel. Emphasize after understanding is assured.

3. One way of drawing parallel lines is as follows:

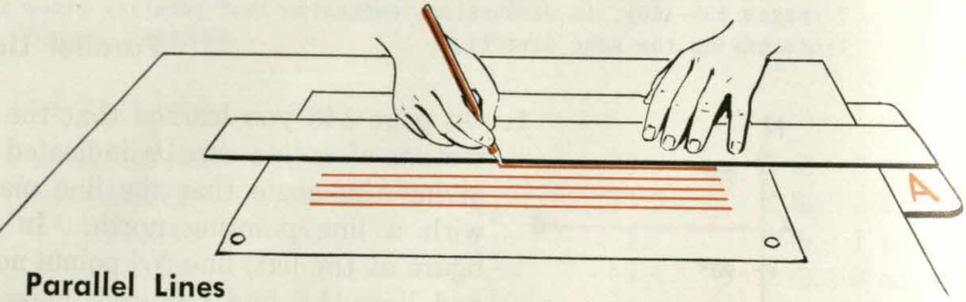
▶ Draw line *ST* and mark points *A* and *B* on it. Lay your ruler along *ST* and with your draftsman's triangle draw *AD* perpendicular to *ST*. Then slide the triangle along the ruler to point *B* and draw *BC* perpendicular to *ST*. You have made the corresponding angles *A* and *B* equal, so lines *BC* and *AD* are parallel. Stress.



Be sure the students can explain why the following is true.

If two lines are each drawn perpendicular to a third line, the two lines are parallel.

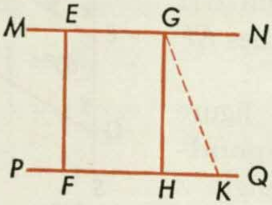
Discuss ex. 1-2 carefully with the class. Be sure the meaning of corresponding angles is understood.



Parallel Lines

1. You may also draw parallel lines by using a T-square, which is a long flat ruler with a piece of wood, marked A, attached Stress. perpendicularly to it. As part A slides along the edge of the drawing board, lines drawn along the edge of the ruler will be parallel. Explain why. Lead a class discussion of this point.

2. If you have a T-square and drawing board, bring them to class and show how to use them.



3. In the figure at the left, lines MN and PQ are parallel. Segments FE and HG are each perpendicular to PQ. Each is $\frac{3}{4}$ in. long. If you draw other perpendiculars to PQ, they will also be $\frac{3}{4}$ in. long, because parallel lines are the same distance apart at all points. Stress.

▶ EF and GH are perpendicular to both PQ and MN. The distance between parallel lines is always measured on a line perpendicular to the parallels because this gives the shortest distance. Stress. It is not measured by segment GK, which is not perpendicular to the parallels, because GK is longer than GH.

4. Draw two lines parallel to each other and 1 in. apart.

▶ As in the figure in ex. 3, draw PQ. Then draw FE and HG each 1 in. long and each perpendicular to PQ. Draw a line through points E and G; it will be parallel to PQ.

5. With your T-square, draw parallel line segments 5 in. long and $\frac{1}{2}$ in. apart; 3 in. long and 2 in. apart; 4 in. long and $1\frac{1}{2}$ in. apart; $5\frac{1}{2}$ in. long and $2\frac{1}{2}$ in. apart. If the students do not have T-squares, have them use the method on page 159 (ex. 3).

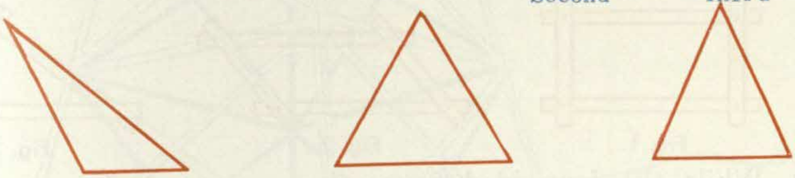
6. Draw six pairs of parallel lines, using the methods explained in ex. 2 and 3 on page 159.

The concepts in ex. 1 and 3 should be clearly explained and demonstrated further as needed.

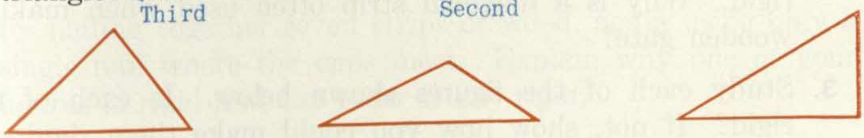
Names of Triangles

1. As you know, triangles are often used in making designs and decorations. They are also used in navigation, in constructing bridges, and in measuring heights and distances. Make a list of five ways in which you have seen triangles used at home or in school. If possible, have students illustrate their lists with drawings or pictures.

2. Triangles may be named according to their sides. If all three sides of a triangle are equal, it is called an **equilateral triangle.** If only two sides are equal, the triangle is called an **isosceles triangle.** If the three sides are of different lengths, it is called a **scalene triangle.** Study the figures below. Which triangle is equilateral? Second isosceles? Third scalene? First



3. Triangles may also be named according to their angles. If a triangle has a right angle, it is called a **right triangle;** if it has an obtuse angle, it is called an **obtuse triangle;** if all of the angles are acute angles, it is called an **acute triangle.** Study the figures below. Which one is a right triangle? Third an obtuse triangle? Second an acute triangle? First



4. Can a triangle have two right angles? No Can it have two obtuse angles? No Can it have two acute angles? Yes

5. What kind of triangle is a draftsman's triangle? Right

6. Tell whether triangles having these sides are equilateral, isosceles, or scalene: 3″, 2″, 4″; Scalene 7″, 5″, 5″; Isosceles 4″, 4″, 4″. Equilateral

7. Tell whether triangles having these angles are obtuse, acute, or right: 40°, 23°, 117°; Obtuse 81°, 11°, 88°; Acute 27°, 90°, 63°. Right

Be sure the meanings of the names of triangles are clear to the students. The students should explain the answers for ex. 4 (the sum of the angles in a triangle equals 180°).

Teach an important property of triangles. Strips of stiff cardboard and paper fasteners could be used for the experiment in ex. 1.

The Triangle as a Rigid Figure

1. Make a four-sided figure by nailing four strips of wood together as shown in Fig. 1. In nailing the strips, use only a single nail at each corner and do not clinch the nail. Show that you can change the shape of this four-sided figure by pushing the sides together, as in Fig. 2. Now fasten three strips together to form a triangle, as shown in Fig. 3. Can you change the shape of the triangle by pushing the sides together? Is the triangle a stiff, or rigid, figure?

The above experiment shows that if a triangle is constructed from three sides of given length, the shape of the triangle cannot be changed.

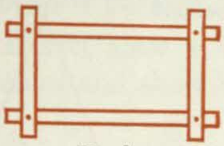


Fig. 1

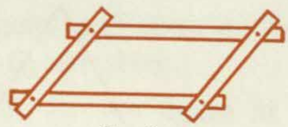


Fig. 2

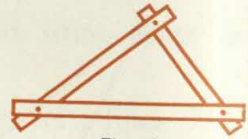


Fig. 3

2. While the four-sided figure that you made is not a rigid figure, you can make it so by placing a strip diagonally across the figure and fastening it at the corners, as shown at the right. Can the sides be pushed together now? Since the figure is made up of two rigid triangles, the entire figure will be rigid. Why is a diagonal strip often used when making a wooden gate?

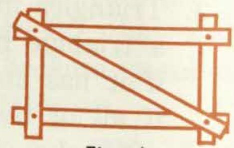
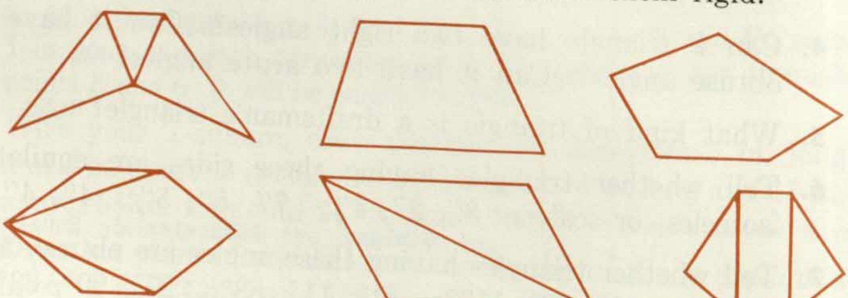
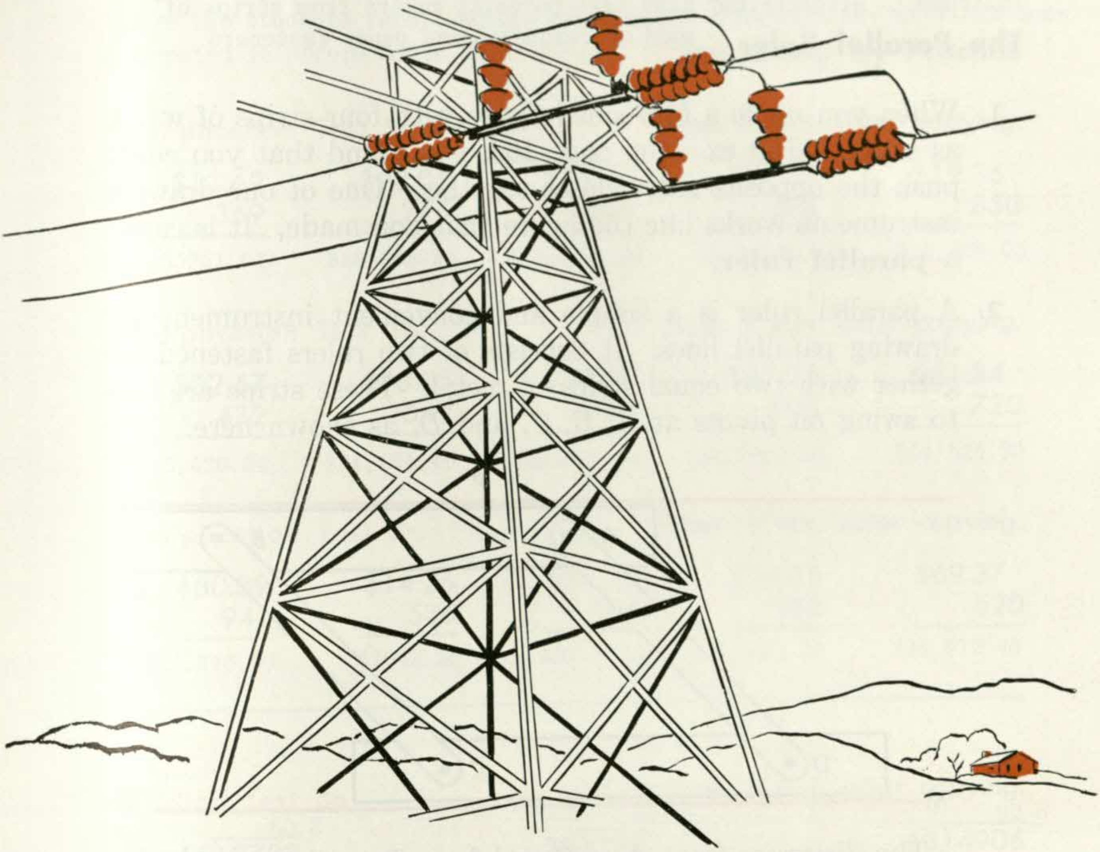


Fig. 4

3. Study each of the figures shown below. Is each of them rigid? If not, show how you could make them rigid.



Have students perform the experiment in ex. 1 and lead them to discover that the triangle is rigid. Then discuss ex. 2 with the class, and emphasize that for a figure to be rigid it must be made up entirely of triangles.



4. Make a rigid figure by nailing together at their ends seven strips of wood, using only a single nail at each point where the ends meet. Then make another figure that is not rigid by nailing together seven strips of wood, again using only a single nail where the ends meet. Explain why one of your figures is rigid and the other is not rigid.
5. The picture above shows the use of triangles in the construction of a tall steel tower. In constructing this steel tower, why were so many triangles used? Find at least ten triangles that form part of the tower.
6. Describe three other structures you have seen where triangles are used to make the structures rigid.
7. Find some pictures of this kind of structure in magazines or newspapers and bring them to class.

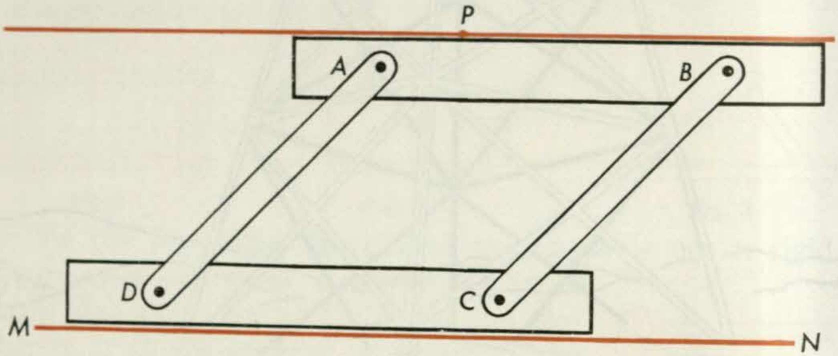
Ex. 4 may be done as a homework assignment and discussed in class the next day. Discussion of ex. 5-6 should emphasize the use of triangles to make structures rigid.

Introduce and describe the parallel ruler. If possible, display this instrument. Students may also make parallel rulers from strips of wood or cardboard and paper fasteners.

The Parallel Ruler

1. When you made a four-sided figure with four strips of wood, as suggested in ex. 1 on page 162, you found that you could push the opposite sides closer together. One of our drawing instruments works like the figure that you made. It is called a **parallel ruler.**

2. A parallel ruler is a simple and convenient instrument for drawing parallel lines. It consists of two rulers fastened together with two equal strips of metal. These strips are free to swing on pivots at $A, B, C,$ and $D,$ as shown here.



The distances from A to B and from D to C are the same, and the lengths AD and BC are equal. Therefore the figure $ABCD$ is a parallelogram when the rulers are in any position. Why? The edges of the two rulers are always parallel and can be used to draw parallel lines. Parallel rulers are used in drafting and in navigation.

3. To draw a line through the point P parallel to MN, you place the edge of one ruler along MN and hold that ruler firmly in place. Then you move the other ruler so that its edge just reaches the point P. Now draw a line along this edge through P. This will be the required line parallel to MN and passing through P.

4. How will you draw a line through P parallel to MN with a parallel ruler if the distance from P to MN is more than the greatest distance between the edges of its two rulers?

Review the fact that the opposite sides of a parallelogram are parallel and equal. Lead the students to see that any 4-sided figure having equal opposite sides must be a parallelogram. Have the students use their own homemade parallel rulers to draw parallel lines.

Present a set of improvement tests in multiplication and in division. Have the students record scores and compare them with the previous ones as noted in Record Books (page 48).

Multiplication Test 3a.

Time: 4 min. after copying.

1.	$13.69 × 369	$98.05 × 275	$28.74 × 569	$27.48 × 803	$19.36 × 850
	$5051.61	$26,963.75	$16,353.06	$22,066.44	$16,456.00

5

Multiplication Test 3b.

Time: 4 min. after copying.

2.	$32.67 × 472	$48.05 × 649	$39.57 × 736	$67.25 × 908	$61.84 × 720
	$15,420.24	$31,184.45	$29,123.52	$61,063.00	$44,524.80

Multiplication Test 3c.

Time: 4 min. after copying.

3.	$80.39 × 941	$14.26 × 536	$54.96 × 104	$94.85 × 382	$69.37 × 520
	$75,646.99	$7643.36	$5715.84	$36,232.70	$36,072.40

5

Division Test 3a.

Time: 5 min. after copying.

4. 45)2738 60.8   38)1987 52.3   53)4906 92.6

5. 27)2293 84.9   84)7385 87.9   72)6426 89.3   6

Division Test 3b.

Time: 5 min. after copying.

6. 51)5087 99.7   36)2985 82.9   93)9289 99.9

7. 64)6283 98.2   49)3926 80.1   87)7374 84.8   6

Division Test 3c.

Time: 5 min. after copying.

8. 32)3075 96.1   78)6753 86.6   44)4397 99.9

9. 93)9268 99.7   65)5091 78.3   29)1952 67.3   6

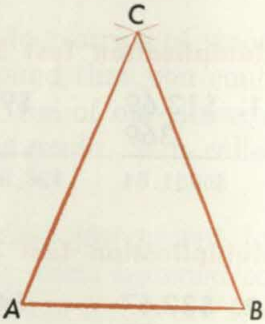
To the Teacher. In ex. 4–9, pupils should find quotients to the nearest tenth.

Papers should be carefully checked before returning them to students. Urge the students to find and correct their own mistakes. Give students help in analyzing causes of errors.

Show how to draw a triangle when the lengths of the three sides are given, and teach experimentally some properties of triangles

Drawing and Studying Triangles

(pages 166–168).



1. You can easily draw a triangle if you know the lengths of its three sides. Suppose you wish to draw an isosceles triangle whose base is 3 in. long and whose equal sides are each 4 in. long. Stress meaning. The **base** of a triangle is the side on which it stands. The triangle is drawn as follows:

▶ First draw base AB 3 in. long. Then, with a radius of 4 in. and with A as center, draw an arc above AB as shown in the figure. With the same radius, and with B as center, draw another arc cutting the first one at C. Draw AC and BC. You now have triangle ABC whose base is 3 in. long and whose other two sides are each 4 in. long. This is an isosceles triangle because $AC = BC$. Check your work by measuring the sides with a ruler.

2. In ex. 1, angles A and B are called the **base** angles because they are the angles at each end of the base of the triangle. Measure angles A and B with your protractor. How do these angles compare? Draw 3 different isosceles triangles. Measure the base angles of each of these triangles. Are the base angles of an isosceles triangle always equal?

Emphasize after demonstration.

The base angles of an isosceles triangle are equal.

3. With your ruler and compasses draw an equilateral triangle whose sides are each 3 in. long. With your protractor measure each of the three angles of this triangle. What do you discover about these angles? All are 60°. Draw another equilateral triangle whose sides are each 4 in. long, and measure the angles. Make a statement that you think is true about the three angles of an equilateral triangle. All are equal.

4. Draw a triangle whose sides are 3 in., 4 in., and 5 in. What kind of triangle is this if you name it according to its sides? Scalene With your protractor measure the largest angle in this triangle; how many degrees does it contain? 90° What kind of triangle is this if you name it from this largest angle? Right Draw another triangle whose sides are $1\frac{1}{2}$ in., 2 in., and $2\frac{1}{2}$ in.

Have students actually draw triangles and measure angles as suggested. Emphasize the meaning of <u>base</u> and <u>base angles</u>. Stress the new property of equilateral triangle shown in ex. 3.

1. On page 147 you measured the angles of a triangle with a protractor and found that the sum of the three angles is always 180°.

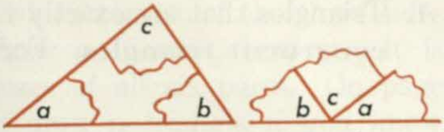


Fig. 1 Fig. 2

Another way to show this is to draw a large triangle on paper and cut it out. Then tear off the three angles as shown in Fig. 1 and place the angles together as shown in Fig. 2. What kind of angle is the large angle formed from putting together the three angles? Straight Does this show that the sum of the three angles of a triangle is 180°? Yes, since straight angle equals 180° (amount of rotation is half of a circle).

2. In ex. 1 on page 166, the base angles of the isosceles triangle are each 68°. How can you tell the number of degrees in the other angle without measuring it? Subtract the sum of the base angles from 180° to get 44°.

3. One of the base angles of an isosceles triangle is 45°. Tell the size of the other two angles of that triangle. 45° and 90°

4. In ex. 1 on page 166, angle *C* is called the vertex angle. If the vertex angle of an isosceles triangle equals 40°, how many degrees are there in each of the base angles? Each is 70°.

5. If a triangle has one right angle, what kinds of angles are the other two angles? Acute Why? What is the sum of these other two angles? 90 Why? If an acute angle of this triangle equals 30°, what is the size of the other acute angle? 60°

Draw triangles having sides of these lengths:

6. 4 in., 3 in., $2\frac{1}{2}$ in.	$2\frac{1}{4}''$, $3''$, $4\frac{1}{2}''$	$3''$, $1''$, $2\frac{1}{2}''$
7. 3 in., 4 in., 3 in.	$3\frac{1}{2}''$, $3\frac{1}{2}''$, $3\frac{1}{2}''$	$6''$, $3''$, $4''$
8. 2 in., 4 in., 3 in.	$4\frac{1}{2}''$, $3\frac{1}{2}''$, $5''$	$2''$, $4\frac{1}{2}''$, $3''$ 45°, 45°, 90°
9. 4 in., 5 in., 2 in. 22°, 50°, 108°	$3''$, $4''$, $4''$ 68°, 44°, 68°	$3''$, $4\frac{1}{4}''$, $3''$ 60°, 60°, 60°
10. 6 in., 5 in., 4 in. 41°, 56°, 83°	$6''$, $6''$, $3\frac{1}{2}''$ 34°, 73°, 73°	$2''$, $2''$, $2''$

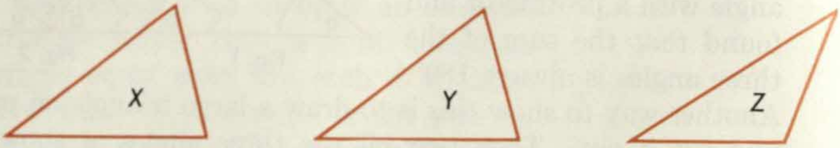
11. In ex. 9 and 10, measure the angles in each triangle. Check by seeing if the sum of the angles equals 180°. See answers above.

Have students perform the experiment in ex. 1 and discuss the results with them. Be sure students understand the basis of the answers (the sum of the angles of a triangle is 180°) to ex. 2–5.

Have the students check the pronunciation and meaning of congruent in their dictionaries.

Congruent Triangles

1. Triangles that are exactly the same size and shape are called **congruent triangles.** For example, triangles X and Y are



congruent, but triangles Y and Z are not congruent. If you have drawn two triangles, like X and Y, and wish to see if they are congruent, you can cut them out and place one over the other. If triangle X exactly fits over triangle Y, then they are congruent.

2. With your ruler and compasses draw two triangles each having sides of 5 in., $4\frac{1}{2}$ in., and 3 in. If possible, draw these triangles on cardboard. When the drawings are finished, cut out the two triangles and place one on top of the other. Do the two triangles have exactly the same size and shape? Are they congruent triangles?

3. Using sides of the lengths given in ex. 2, see if you can draw a triangle that will have a different size and shape from the ones you drew in ex. 2. Try this in different ways, first using one of the given sides as base and then another one of these sides as base. Do you always get a triangle of the same size and shape as those in ex. 2? This shows that a triangle of only one size and shape can be drawn from three given sides. Try this experiment again with two triangles each having sides of $4\frac{1}{2}$ in., 6 in., and $7\frac{1}{2}$ in.

4. The experiments in ex. 2 and 3 also show that a triangle is a **rigid** figure. If you could change the shape of a triangle by pushing its sides together, then it would be possible to have triangles of two different shapes from the same set of sides.

Emphasize after demonstration.

If the three sides of one triangle are respectively equal to the three sides of another triangle, the triangles are congruent.

Have the students perform the experiment in ex. 2–3 under your direction and lead a class discussion of the results. Try to lead the students to make the statement given in ex. 4 before presenting it.

Present a summary of certain facts about triangles. Test the students' understanding of important geometric terms.

The Parts of a Triangle

1. Every triangle has six parts, which consist of the three sides and the three angles. To draw a triangle, however, it is not necessary to know the sizes of all six parts. On page 166 you found that you can draw a triangle if you know only the lengths of the three sides. Then, after drawing the triangle, you can find the sizes of the three angles by measuring each with a protractor.

2. On page 171 you will learn how to draw a triangle when the three known parts are two sides and one angle. Also, on page 176, you will learn how to do this when the three known parts are one side and two angles. In each case, after the triangle is drawn, you can find the sizes of the other three parts by measuring them.

The Language of Arithmetic

Show that you know what each word or group of words means by making a drawing of it:

3. trapezoid
4. acute angle
5. right triangle
6. regular octagon
7. scalene triangle
8. straight angle
9. opposite angles
10. congruent triangles
11. angle of 90 degrees
12. bisector of a line segment
13. equilateral triangle
14. bisector of an angle
15. angle of 250 degrees
16. parallel lines 1 inch apart
17. isosceles triangle with 50-degree base angles
18. perpendicular lines drawn with compasses
19. right triangle that is also isosceles
20. hexagon inscribed in a circle
21. circle having a radius of 2 in.

The work in ex. 3–21 should be carefully checked to see if redevelopment of terms is necessary.

Mixed Practice

1. Find the interest on \$2000 for 36 da. at 6%; \$12 at 4%. \$8

2. Subtract \$275.53 from the sum of \$150 and \$235.25. \$109.72

3. Find the average of $11\frac{1}{2}$, $9\frac{2}{3}$, $12\frac{1}{2}$, and $10\frac{5}{6}$. $11\frac{1}{8}$

4. If 35% of a number is 315, find the number. 900

5. Find the sum of $9\frac{1}{3}$, $12\frac{1}{2}$, $6\frac{5}{12}$, $3\frac{3}{4}$, $4\frac{1}{6}$, $9\frac{1}{2}$, $2\frac{3}{4}$. $48\frac{5}{12}$

6. Find the quotient when 35.78 is divided by .004. 8945

7. Find these products: $9\frac{3}{4} \times 6\frac{2}{3}$; 65 $12 \times 8\frac{1}{5}$; $98\frac{2}{5}$ $\frac{7}{10} \times 15$. $10\frac{1}{2}$

8. Find these quotients: $12\frac{1}{2} \div 3\frac{3}{4}$; $3\frac{1}{3}$ $17 \div 1\frac{1}{3}$; $12\frac{3}{4}$ $\frac{5}{8} \div \frac{3}{4}$. $\frac{5}{6}$

9. Find, to the nearest tenth of a mile, what part of a mile is 1500 ft.; .3 3750 ft.; .7 4300 ft.; .8 4725 ft. .9

10. Which is greater, $2\frac{1}{2}$% of \$2500 or $1\frac{3}{4}$% of \$3800? $1\frac{3}{4}$% How much greater is it? \$4

11. Team A won 47 games out of 50; .94 team B won 72 games out of 75. .96 Which team had the better standing?

12. In an isosceles triangle, one of the base angles is 40°. Find the size of the other two angles. 40° and 100°

13. Draw the isosceles triangle in ex. 12 if the base is 2 in. Give another name to the triangle. Obtuse

14. Are $\frac{1}{2}$ of 1% of \$250 and 50% of \$250 equal? No If not, what is the difference between them? \$123.75

15. Tell which have the same answer: $\frac{3}{4}$ of \$80, $7\frac{1}{2}$% of \$800, 750% of \$8000, $\frac{3}{4}$ of 1% of \$8000, 75% of \$80. All but 750% of \$8000

16. By the formula, find the annual rate of interest that is charged if a loan of \$75 is paid back in 6 monthly installments of \$13.50 each. Give the result to the nearest whole per cent. 27%

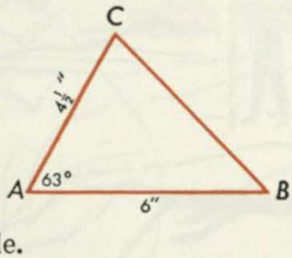
17. Round off \$381,496,000 \$381.5 million and \$94,602,000 \$94.6 million to the nearest tenth of one million dollars and then find what per cent, to the nearest whole per cent, the second number is of the first. 25%

Show how to draw a triangle when given two sides and the included angle.

Another Way to Draw a Triangle

1. You can also draw a triangle if the three known parts are two sides and the angle included between those sides. For example, you can draw a triangle if you know that side $AB = 6$ in., side $AC = 4\frac{1}{2}$ in., and angle $A = 63°$. The steps are as follows:

▶ First draw side AB 6 in. long. Next, with a protractor, draw angle A equal to 63°, using AB as one side of the angle. Then on the other side of the angle make AC $4\frac{1}{2}$ in. long. Draw a segment connecting points C and B. You now have triangle ABC, which is the required triangle.



2. On paper, each member of the class should follow the directions in ex. 1 and draw the triangle carefully. Then the class members should cut out their triangles. Are they exactly alike in size and shape? Are they congruent? Can only one triangle be drawn, using these 3 known parts?

Emphasize after understanding is assured.

If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of another triangle, the triangles are congruent.

Draw triangles having two sides and the included angle as follows. Then measure the sizes of the other three parts:

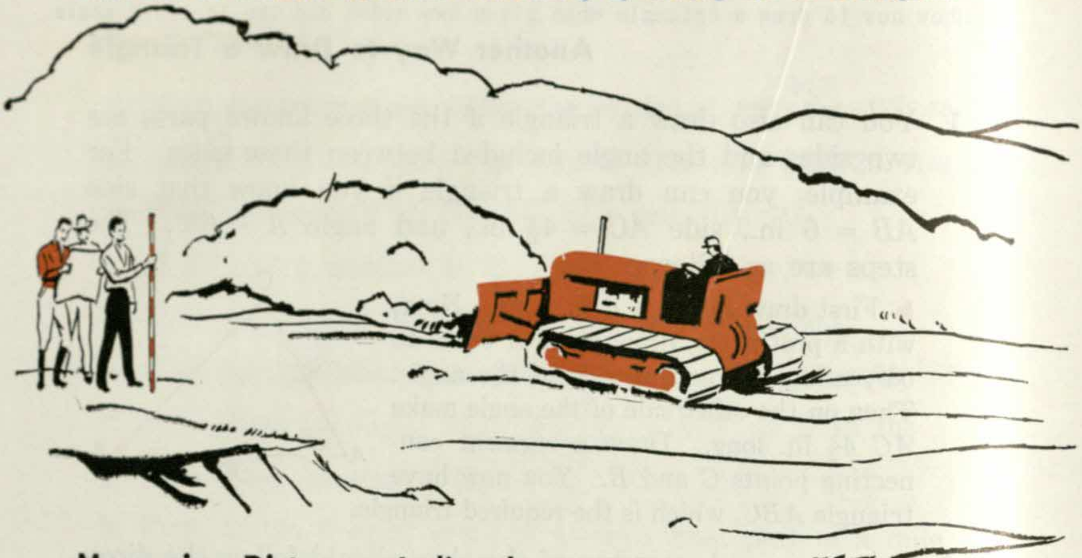
3. 4 in., $2\frac{1}{4}$ in., 60° 86°, $3\frac{1}{2}$ in., 34° — $2\frac{1}{4}$ in., 2 in., 90° 42°, 3 in., 48°
4. 3 in., $4\frac{1}{4}$ in., 59° 77°, $3\frac{3}{4}$ in., 44° — $4\frac{1}{2}$ in., 3 in., 115° 40°, $6\frac{3}{8}$ in., 25°
5. 6 in., $3\frac{1}{2}$ in., 40° 105°, 4 in., 35° — 7 in., 8 in., 30° 89°, 4 in., 61°
6. 5 in., $3\frac{1}{2}$ in., 18° 129°, 2 in., 33° — 2 in., 3 in., 115° 40°, $4\frac{1}{4}$ in., 25°

Discuss ex. 7 first before the students make the drawing.

7. If it is not convenient to draw a triangle full size in order to find the length of the third side, make a drawing to scale, letting 1 in. equal 100 ft., or 500 ft., or some other convenient amount. Using a scale drawing, find the length of the third side of a triangle if two of the sides are 800 ft. and 450 ft. and the included angle is 60°. Let 1 in. = 200 ft. $3\frac{1}{2}$ in., or 700 ft.

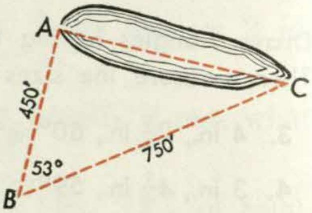
Demonstrate ex. 1 on the board and discuss the steps with the students. Be sure they understand the meaning of included angle. Then have the students draw their triangles and compare them (ex. 2). Work in ex. 7 is very important and more practice should be given.

Show how to measure distances indirectly by making use of triangles.



Measuring Distances Indirectly

1. Suppose you wish to measure the distance from one end of a pond to the other, such as the distance AC shown below. You cannot measure it **directly** with a steel tape because of the water, but you can find this distance **indirectly** if you form a triangle ABC by driving stakes into the ground at A, B, and C.

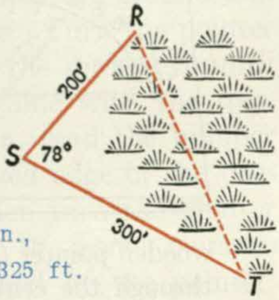


▶ In triangle ABC you can find the distance AC if you know the angle at B and the lengths of AB and BC. By pacing, suppose you find that $AB = 450$ ft. and $BC = 750$ ft. Then measure the angle at B with a field protractor like that described on page 174. Suppose angle B equals 53°. Using these measurements, make a scale drawing of triangle ABC, letting 1 in. = 100 ft.; then $4\frac{1}{2}$ in. = 450 ft. and $7\frac{1}{2}$ in. = 750 ft. First draw BC $7\frac{1}{2}$ in. long. At B draw an angle of 53°; then make BA $4\frac{1}{2}$ in. long. Connect A and C. If you measure AC, you find it is 6 in. long; therefore $AC = 600$ ft. How many feet long is the pond? 600

2. In ex. 1, suppose that $AB = 350$ ft., $BC = 450$ ft., and angle $B = 76°$. Make a scale drawing, letting 1 in. = 100 ft. Then find the distance AC. 500 ft.

Have the students discuss other examples where measuring indirectly would be necessary. Then discuss thoroughly the explanation in ex. 1 and have students make the scale drawing. Be sure they understand the use of the scale.

3. Jim wants to find the distance RT across a swamp. He makes a triangle RST by driving stakes into the ground at R, S, and T. By pacing, he finds that $RS = 200$ ft. and $ST = 300$ ft. With his field protractor he finds that angle S equals 78°. Find the distance RT across the swamp. Make a scale drawing, letting 1 in. represent 50 ft. or 100 ft. $3\frac{1}{4}$ in., or 325 ft.



4. In ex. 3, suppose that Jim had found that $RS = 150$ ft., $ST = 250$ ft., and angle $S = 83°$. Make a scale drawing, letting 1 in. = 50 ft., and find the distance RT. 275 ft.

5. In ex. 3, suppose that Jim had found that $RS = 550$ ft., $ST = 400$ ft., and angle $S = 67°$. Make a scale drawing, letting 1 in. = 100 ft., and find the distance RT. 550 ft.

6. The man in the picture above is using a transit to measure the angle formed by the transit and two distant points. The vertex of the angle is the point where the transit stands.

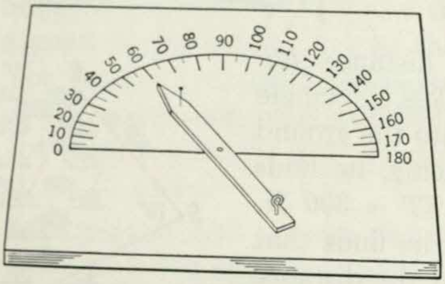
If a transit is available, display it in class and explain how it is used.

Show how to make a field protractor and how to use it in measuring distances indirectly (pages 174–175). If a large paper protractor is not available one can be made (see the Guide).

Making a Field Protractor

1. The picture on pages 172 and 173 shows a transit in use. This instrument is used by surveyors and engineers to measure angles out-of-doors. The basic parts of a transit are two protractors and a small telescope. One protractor is used to measure horizontal angles, and the other protractor is used to measure vertical angles. The telescope is used to sight some distant object. When surveyors lay out highways or locate the boundaries of farms they measure angles. They also measure angles in many other situations. Can you name some of them? Encourage class discussion of these.

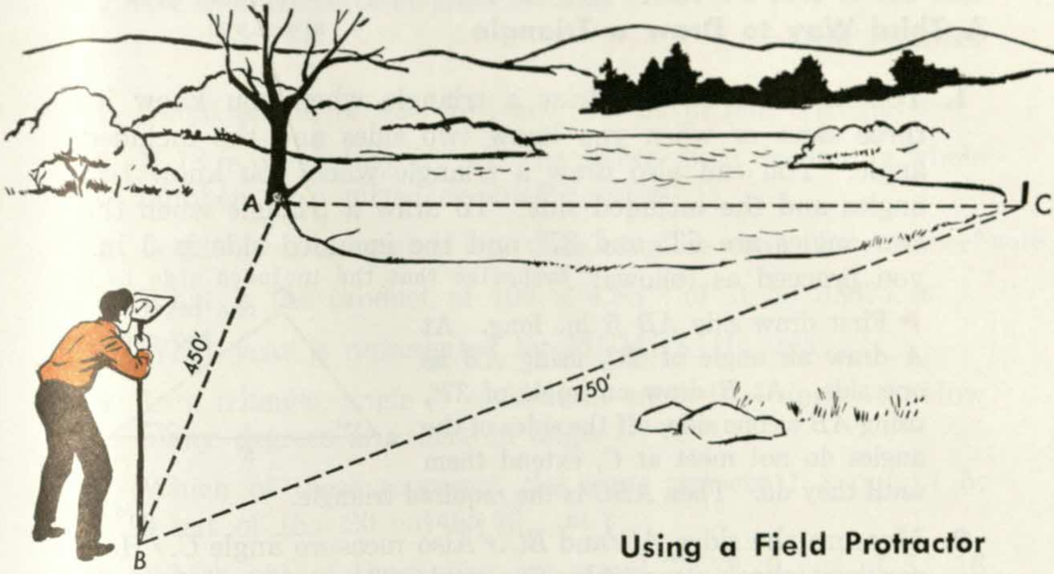
2. For your purposes in indirect measurement, angles may be measured with a homemade instrument, called a **field protractor.** Such an instrument is shown below. You can easily make one by following these directions:



▶ Paste a semicircular protractor about 8 in. in diameter on a piece of board about 10 in. square and $\frac{1}{2}$ in. thick, as shown in the picture at the left. The degrees on the protractor should be marked from 0° to 180°. Fasten a narrow wooden pointer (about $7\frac{1}{2}$ in. long) to the board by a screw passing through the center of the protractor so that the pointer may be revolved about the screw as a center. For sights, set a pin vertically at the front end of the pointer and a piece of tin having a vertical slit, or a very small screw eye, at the other end of the pointer. Fasten the board very firmly to a camera tripod or to a single stick that may be pushed into the ground, as shown in the picture on page 175. In using the field protractor, place it over the point on the ground that is the vertex of the angle you are measuring. Stress.

Circular protractors 8 in. or 14 in. in diameter, printed on paper or Bristol board, are sold by dealers in drawing materials. These protractors consist of a complete circle of 360°. By cutting one of these protractors through the center, you obtain 2 semicircular protractors from which two field protractors may be constructed.

Discuss the difference between horizontal and vertical angles. Then discuss the directions in ex. 2 thoroughly and, if possible, make a field protractor with the class.



Using a Field Protractor

1. The picture above shows how angle B in ex. 1 on page 172 is measured with a field protractor. Set up the field protractor at B, pushing the pointed stick into the ground and keeping the protractor as level as possible. Turn the pointer toward the tree at A and sight through the screw eye until the pin at the end of the pointer is in line with the tree. When the pointer is in the right position, read the number of degrees to which it points on the curved edge of the protractor. Suppose the reading is 45°. Then turn the pointer toward C and sight the stake at C in the same manner. Note the number of degrees now indicated by the pointer. Suppose the reading is 98°. Then angle B equals $98° - 45°$, or 53°. Be sure students understand why you subtract 45° from 98°.

After angle B is found, the distances from B to A and from B to C are paced off. These three measurements are then used to find AC as shown in ex. 1 on page 172.

▶ When you are using the field protractor, it is important to set it up so that the board of the protractor is as level as possible. You may consider the board to be level if a marble laid on it does not roll off.

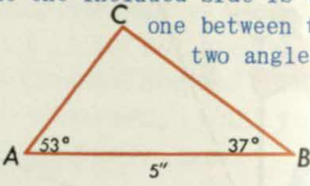
2. How large is angle B if the readings on the protractor are 30° and 90°? 60° if they are 35° and 160°? 125° 15° and 45°? 30°

If possible, give students an opportunity to use a field protractor in measuring some distances on the school grounds.

Show how to draw a triangle when two angles and the included side are given.

A Third Way to Draw a Triangle

1. You have learned to draw a triangle when you know its three sides or when you know two sides and the included angle. You can also draw a triangle when you know two angles and the included side. To draw a triangle when the two angles are 53° and 37° and the included side is 5 in., you proceed as follows: Emphasize that the included side is the one between the two angles.



▶ First draw side AB 5 in. long. At A draw an angle of 53°, using AB as one side. At B draw an angle of 37°, using AB as one side. If the sides of the angles do not meet at C, extend them until they do. Then ABC is the required triangle.

2. Measure the sides AC (3 in.) and BC. (4 in.) Also measure angle C. (90°) How can you check your value for angle C? 53° + 37° = 90°, 180° − 90° = 90°

3. Draw two triangles each having angle $A = 90°$, angle $B = 45°$, and side $AB = 3$ in. Then cut out the two triangles and place one on top of the other. Do the triangles have the same size and shape? Are they congruent? If you drew more triangles with these same dimensions, would they all be the same size and shape?

Emphasize after understanding is assured.

If two angles and the included side of one triangle are respectively equal to two angles and the included side of another triangle, the triangles are congruent.

Emphasize the importance of accuracy in drawing triangles.

Draw triangles with two angles and the side included between them as given below. Then measure the size of the other three parts:

4.	$2\frac{1}{2}$ in., 60°, 60° ($2\frac{1}{2}$ in., $2\frac{1}{2}$ in., 60°)	6 in., 40°, 34° ($3\frac{1}{2}$ in., 4 in., 106°)	$4\frac{1}{2}$ in., 53°, 90° ($7\frac{1}{2}$ in., 6 in., 37°)
5.	$3\frac{3}{4}$ in., 60°, 90° ($7\frac{1}{2}$ in., $6\frac{1}{2}$ in., 30°)	5 in., 41°, 83° (6 in., 4 in., 56°)	$5\frac{1}{2}$ in., 77°, 44° ($4\frac{1}{2}$ in., $6\frac{1}{2}$ in., 59°)
6.	$4\frac{1}{4}$ in., 45°, 45° (3 in., 3 in., 90°)	4 in., 98°, 30° ($2\frac{1}{2}$ in., 5 in., 52°)	$3\frac{1}{2}$ in., 60°, 60° ($3\frac{1}{2}$ in., $3\frac{1}{2}$ in., 60°)
7.	5 in., 90°, 41° ($4\frac{3}{8}$ in., $6\frac{5}{8}$ in., 49°)	2 in., 34°, 106° (3 in., $1\frac{3}{4}$ in., 40°)	$2\frac{1}{2}$ in., 83°, 33° ($2\frac{3}{4}$ in., $1\frac{1}{2}$ in., 64°)
8.	4 in., 26°, 65° ($3\frac{5}{8}$ in., $1\frac{3}{4}$ in., 89°)	6 in., 39°, 24° ($2\frac{3}{4}$ in., $4\frac{1}{4}$ in., 117°)	3 in., 56°, 74° ($3\frac{3}{4}$ in., $3\frac{1}{4}$ in., 50°)
9.	3 in., 96°, 36° ($2\frac{3}{8}$ in., 4 in., 48°)	3 in., 65°, 32° ($1\frac{5}{8}$ in., $2\frac{3}{4}$ in., 83°)	$2\frac{1}{2}$ in., 50°, 71° ($2\frac{3}{4}$ in., $2\frac{1}{4}$ in., 59°)

Lead the students to making the statement given in ex. 3 before presenting it.

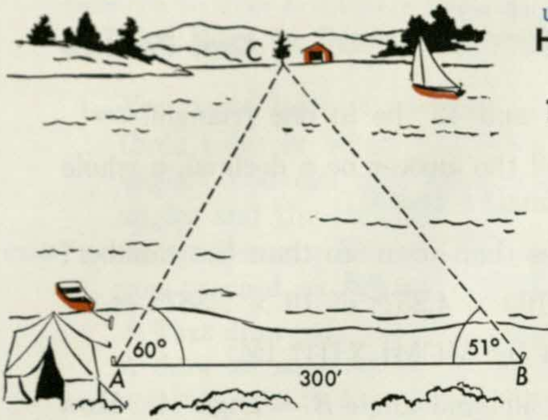
Provide an oral review of previous work.

Oral Practice

1. Could angles of 62°, 75°, and 43° be in one triangle? Yes
2. If 18 is divided by 23, will the answer be a decimal, a whole number, or a mixed decimal? A decimal
3. Is 125% of a number less than or more than the number? More
4. What is the product of 100 × 4.85? 485 of 10 × .386? 3.86
5. What year is represented by MCMLXIII? 1963
6. In a triangle, angle $A = 50°$ and angle B = angle A. How many degrees are there in angle C? 80°
7. Which of these represent the same per cent? 3 out of 5; 9 out of 15; 20 out of 30.
8. Which one of these does not equal 75%? 6 out of 8; 16 out of 20; 9 out of 12; 24 out of 32. 16 out of 20
9. Which of these represents the largest per cent? 26 out of 35; 26 out of 32; 26 out of 29. Do not do the work. See Guide.
10. Which per cent will be the smallest? 37 out of 50; 37 out of 45; 37 out of 53. Do not do the work.
11. Is 75% of 200 the same as $\frac{3}{4}$ of 1% of 200? No Give a reason for your answer but do not do the work. $\frac{3}{4}$ of 1% is less than 1%, while 75% is 75 times 1%.
12. Is the interest on $100 at 6% for 30 da. more than, equal to, or less than the interest on $100 at 3% for 60 da.? Do not find the answer. See Guide. They are equal.
13. If 60% of the selling price of an article is the cost, and 33% of the selling price covers expenses, what per cent of the selling price is the profit? 7%
14. What does 6 × 2 × 5 × 0 × 8 × 3 equal? 0
15. What does 6 + 2 + 5 + 0 + 8 + 3 equal? 24
16. If a ball team wins 60 games and loses 40 games during the season, is its percentage .667, .600, or .150?
17. Does 19.6% of 275 equal 5.39, 53.9, or 539? Find your answer by estimating.

Instruct the students to explain their answers, so that you can judge the degree of understanding.

Teach how to use the method of drawing triangle (page 176) in measuring distances indirectly.



How Far to the Boathouse?

1. Tom wanted to know the distance from his camp to the boathouse across the river. Because of the water, he cannot measure the distance by pacing, so he finds it by making a scale drawing of a triangle as shown below.

▶ At point A, Tom drives a stake into the ground. Then, starting at A he paces a distance of 300 ft. along the sandy shore and drives another stake into the ground at B; this makes $AB = 300$ ft. AB is called the **base line.** Tom now places his field protractor at A and moves the pointer so that it points to the tree C near the boathouse; he measures angle CAB and finds it to be 60°. Then, in the same way, he sets the field protractor at B and measures angle CBA, which he finds to be 51°. Tom can now draw triangle ABC to scale since he knows angles A and B and the included side AB. From the scale drawing, the distance AC across the river can be found; see ex. 2 below.

2. Make a scale drawing of the triangle in ex. 1, letting 1 in. = 100 ft. How many inches will represent 300 ft.? 3 When the drawing is finished, measure AC; how many inches long is it? $2\frac{1}{2}$ What is the distance across the river? 250 ft.

3. Using a scale of 1 in. = 100 ft., make scale drawings to find the distance across the river if $AB = 250$ ft., angle $A = 50°$, angle $B = 71°$; if $AB = 275$ ft., angle $A = 90°$, angle $B = 50°$. 325 ft. 275 ft.

4. Find the distance across the river if $AB = 200$ ft., angle $A = 70°$, angle $B = 60°$. In the scale drawing, let 1 in. = 50 ft. 225 ft.

5. On page 174 you were given directions for making a field protractor. By the method explained above, use your field protractor and find the distance across your schoolyard **without** crossing it. Do this as a class project.

Have the students do ex. 2 under your supervision. Emphasize the fact that the actual distance is 250 ft., not $2\frac{1}{2}$ in.

Present an oral true-false test on the concepts and facts of Chapter 5.

True or False?

Read each statement carefully and then tell whether you think it is true or false. Draw diagrams if necessary: Stress.

1. A triangle can be drawn in which the three angles have these sizes: 90°, 110°, 30°. False
2. An angle that is labeled *ABC* may also be called angle *B*. True
3. An angle that is larger than 180° cannot be directly measured with a protractor of the kind shown on page 146. True
4. The size of an angle depends upon how long you draw its sides. False
5. An angle may be bisected by using compasses. True
6. Obtuse angles are formed when one line is drawn perpendicular to another line. False
7. An equilateral triangle has three equal angles. True
8. A right triangle may also be an isosceles triangle. True
9. A triangle cannot contain more than one right angle. True
10. The triangle is a rigid figure. True
11. A triangle of only one size and shape can be drawn if the lengths of the three sides are given. True
12. Parallel lines will meet if they are extended far enough. False
13. Congruent triangles are triangles of exactly the same size and shape. True
14. Only one triangle can be drawn if two sides and the included angle are given. True
15. If two angles and the included side of one triangle are respectively equal to two angles and the included side of another triangle, the triangles are not congruent. False
16. A triangle may contain angles of 48°, 111°, and 21°. True
17. If you know the size of one base angle in an isosceles triangle, you can find the size of each of the other angles. True

To prevent guessing and to discover the causes of difficulties, have the students give the reasons for their answers. Use the developmental material in the text relating to students' difficulties if reteaching is necessary.

Provide a general review of problem solving.

Review Problems

1. Frances bought a watch that was marked \$48.75. In addition to that price, she had to pay a federal tax of 10% and a sales tax of 2%. What was the total cost of the watch? \$54.61 Each tax is computed on the marked price.
2. In a recent year there were about 133.6 million telephones in the world and about 70.6 million of them were in the United States. What per cent, to the nearest whole per cent, of the world's telephones were in the United States that year? 53%
3. The installment price of a television set is \$195 and the cash price is \$177. Mr. Williams finds that he can pay cash if he borrows \$150 which he knows he can pay back in 6 mo. How much will the television set actually cost if he borrows the \$150 for 6 mo. at 5%? \$180.75 How much less will this be than the installment price? \$14.25
4. In Rockland electricity is sold at the rate of \$.95 for the first 15 kw-hr or less, 4¢ per kw-hr for the next 35 kw-hr, 3¢ per kw-hr for the next 50 kw-hr, and $2\frac{1}{2}$¢ per kw-hr for all over 100 kw-hr. At this same rate, find the charge for 88 kw-hr; \$3.49 for 143 kw-hr; \$4.93 for 218 kw-hr. \$6.80
5. There are 5 rectangular flower gardens each with a perimeter of 32 ft. The dimensions of the gardens are as follows: (1) $4' \times 12'$; (2) $5\frac{1}{2}' \times 10\frac{1}{2}'$; (3) $6' \times 10'$; (4) $6\frac{1}{2}' \times 9\frac{1}{2}'$; (5) $8' \times 8'$. Are all the areas the same size? If not, which area is the smallest and which one is the largest? (1) 48 sq. ft.; (2) $57\frac{3}{4}$ sq. ft.; (3) 60 sq. ft.; (4) $61\frac{3}{4}$ sq. ft.; (5) 64 sq. ft.; (6) smallest, $4' \times 12'$; largest, $8' \times 8'$
6. The Star Store sells 25 sheets of carbon paper for \$.75 or 100 sheets of the same kind for \$2.50. If Miss Baker can use 100 sheets, how much will she save by buying 100 sheets at one time? \$.50
7. Mr. Carver received \$725 as his commission for selling a house. This was 5% of the selling price of the house. At what price did Mr. Carver sell the house? \$14,500

Discuss the solutions to the problems with the entire class, letting volunteers explain their answers. Students who had errors should be encouraged to explain their solutions so that you can determine the causes.

Chapter Review

1. With your ruler and protractor, draw a right angle, (90°) an acute angle, a straight angle, (180°) and an obtuse angle. Tell how many degrees each angle contains.
2. Draw an angle of 80° and bisect it with compasses.
3. Make a drawing to show the direction in which an airplane is flying if its course is 260°. See Guide.
4. Draw a segment 3 in. long and bisect it with compasses.
5. Draw a segment 3 in. long and mark a point above it. From that point draw a perpendicular to this line segment, using your draftsman's triangle.
6. Draw two parallel lines cut by a third line and put the numbers 1 to 8 in the angles that are formed. Then, by using numbers, write two pairs of opposite angles and two pairs of corresponding angles.
7. Using your ruler and compasses, draw an equilateral triangle. Tell the size of each angle in this triangle.
8. Draw an isosceles triangle having a base of 3 in. and base angles that are each 50°. What is the size of the other angle of this triangle? 80°
9. Tell whether or not a triangle could contain three angles of the following sizes: 90°, 45°, 45° Yes; 90°, 60°, 30° Yes; 80°, 50°, 60° No; 100°, 25°, 55° Yes; 34°, 62°, 89° No; 90°, 15°, 90°. No
10. With your ruler and compasses, draw a triangle in which the sides are $2\frac{1}{2}$ in., 3 in., and $4\frac{1}{2}$ in.
11. With your ruler and protractor, draw a triangle having an angle of 40° included by sides of 3 in. and $3\frac{1}{2}$ in. Then measure the other two angles and the length of the third side of the triangle. $2\frac{1}{4}$ in., 82°, 58°
12. Draw a triangle in which two of the angles are 52° and 30° and the side between these angles is $2\frac{1}{2}$ in. Measure the third angle and the other two sides. $1\frac{1}{4}$ in., 2 in., 98°

Check the students' work carefully and note the kinds of errors. Students who do well on this review might like to work on original designs and constructions. Group others for reteaching or review as needed.

A Problem Test

1. In the business section of a city, land is valued at \$3 per square foot. Find the value of a rectangular lot 65′ by \$19,500 100′, a triangular lot with a base of 96′ and a height of \$12,096 84′, and a lot shaped like a trapezoid with bases of 80′ and 120′ and a height of 60′. \$18,000

2. Find the average weight of boxes weighing $2\frac{1}{2}$ lb., $1\frac{15}{16}$ lb., $2\frac{1}{8}$ lb., $1\frac{7}{8}$ lb., $2\frac{1}{4}$ lb., $2\frac{3}{16}$ lb., $1\frac{1}{2}$ lb., and $1\frac{5}{8}$ lb. 2 lb.

3. Mr. Jackson borrowed \$500. He paid the money back in 12 monthly installments of \$43.33 each. Find the charge for this loan. \$19.96

4. Mr. Hill bought a table with a list price of \$50 at a discount of 20%. At what price should he sell the table if expenses and profit are 40% of the selling price? \$66.67

5. After deducting his commission of \$820 from the sale of a house, Mr. Turner paid \$15,580 to the former owner of the house. What was the per cent of the commission? 5%

6. On May 5 John Ward borrowed \$880 from the bank at 6% interest, to be repaid July 7. Find the bank discount. \$9.24

7. In ex. 6, what amount did Mr. Ward receive from the bank on May 5 \$870.76 and what amount did he pay the bank on July 7? \$880

8. Jean bought a box of 500 sheets of typewriter paper for \$2.50. She used 30 sheets of it to write a class report. How much did the paper for the report cost? 15¢

9. When the price of a quart of milk increases from 21¢ to 25¢, find, to the nearest whole per cent, the per cent of increase in the price. 19%

10. At 2 grapefruit for 25¢, find the selling price of a box of 72 grapefruit if there is no spoilage. \$9

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

After correcting the papers and noting the errors, return the papers so that the students can find and correct their mistakes. Through individual conferences, try to determine the causes of errors. Plan remedial work as needed.

Present diagnostic tests (pages 183-184) with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row. Emphasize the importance of checking all work.

Divide and check the work: (Practice Pages)

1. $187{,}448 \div 72$ $2603\frac{4}{9}$ — $174{,}960 \div 216$ 810 — 12
2. $466{,}204 \div 58$ 8038 — $407{,}295 \div 189$ 2155 — 13

Find the answers. Add in ex. 3; subtract in ex. 4:

3. $6\frac{3}{8} + 2\frac{1}{4} + 5\frac{1}{2} = 14\frac{1}{8}$ — $3\frac{1}{6} + 4\frac{1}{3} + 8\frac{1}{2} = 16$ — $2\frac{3}{4} + 1\frac{7}{16} + 6\frac{5}{8} = 10\frac{13}{16}$ — $1\frac{7}{12} + 2\frac{3}{4} + 3\frac{1}{6} = 7\frac{1}{2}$ — $5\frac{3}{4} + 2\frac{5}{6} + 2\frac{1}{3} = 10\frac{11}{12}$ — 15, 16
4. $7 - 2\frac{3}{5} = 4\frac{2}{5}$ — $9\frac{7}{8} - 3\frac{1}{8} = 6\frac{3}{4}$ — $5\frac{3}{10} - 2\frac{1}{2} = 2\frac{4}{5}$ — $1\frac{1}{16} - \frac{3}{4} = \frac{5}{16}$ — $6\frac{1}{2} - 4\frac{1}{3} = 2\frac{1}{6}$ — 15
5. $6\frac{2}{3} \times \frac{9}{10}$ 6 — $\frac{5}{6} \times 1\frac{1}{5}$ 1 — $\frac{2}{3} \times \frac{1}{4}$ $\frac{1}{6}$ — 17
6. $25 \div \frac{15}{16}$ $26\frac{2}{3}$ — $\frac{3}{4} \div \frac{9}{10}$ $\frac{5}{6}$ — $\frac{7}{8} \div 1\frac{1}{3}$ $\frac{21}{32}$ — 18, 19

Divide. If you continue to have a remainder, round off the quotient to the nearest hundredth:

7. $27\overline{)13.5}$ .5 — $32\overline{)6.848}$ .214 — $43\overline{)240.1}$ 5.58 — $64\overline{)8.025}$ .13 — 27
8. $.09\overline{)8.13}$ 90.33 — $5.1\overline{).1363}$ .03 — $.76\overline{)9.126}$ 12.01 — $.87\overline{)5916}$ 6800 — 29, 30

Change these decimals to per cents:

9. .24 24% — .875 $87\frac{1}{2}\%$ — 1.25 125% — .1625 $16\frac{1}{4}\%$ — 10.45 1045% — 43, 50
10. .07 7% — .085 $8\frac{1}{2}\%$ — 3.64 364% — .0775 $7\frac{3}{4}\%$ — 4.125 $412\frac{1}{2}\%$ — 43, 50

Find the answers to the nearest cent:

11. $37\frac{1}{2}\%$ of \$216 \$81.00 — $10\frac{3}{4}\%$ of \$325 \$34.94 — 42
12. 130% of \$135 \$175.50 — 244% of \$9.85 \$24.03 — 50

The topics covered include: four fundamental operations with fractions, division of whole numbers and decimals, percentage.

Notice that mastery of this work should be assured before beginning new topics.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Pages for that row.

Find the answers to the nearest cent:

			Practice Pages
1.	4.2% of \$55 \$2.31	3.5% of \$95 \$3.33	43
2.	16.7% of \$465 \$77.66	23.6% of \$827 \$195.17	43
3.	$\frac{1}{2}$% of \$375 \$1.88	$\frac{3}{4}$ of 1% of \$550 \$4.13	45
4.	.3% of \$860 \$2.58	$\frac{7}{10}$ of 1% of \$795 \$5.57	45
5.	127$\frac{1}{2}$% of \$640 \$816.000	215$\frac{3}{4}$% of \$1600 \$3452.00	50
6.	136.4% of \$750 \$1023.00	170.4% of \$2500 \$4260.00	50

Find the number to put in each space. In ex. 9, find the nearest whole per cent; in ex. 10, find the nearest tenth of 1%:

7.	60 is 15% of .400	6 is 12% of .50.	64, 66
8.	28 is 40% of .70.	9 is 75% of .12.	64, 66
9.	13 is .24. % of 55	8 is .42. % of 19	52
10.	32 is 38.9% of 83	8 is 11.9% of 67	53, 54

Find the selling price. The expenses and profit are given as per cents of the selling price:

	Cost	Exp.	Profit		Cost	Exp.	Profit		
11.	\$87	36%	6%	\$150	\$99	32%	8%	\$165	73
12.	\$222	36%	8$\frac{1}{2}$%	\$400	\$92	35%	7$\frac{1}{2}$%	\$160	73
13.	\$342	38%	5%	\$600	\$88	38%	7%	\$160	73

Find the bank discount on these loans:

14.	\$250, 6%, 6 mo. \$7.50	\$925, 5%, 1 yr. \$46.25	127
15.	\$860, 6%, 66 da. \$9.46	\$580, 6%, 15 da. \$1.45	127
16.	\$300, 5%, 90 da. \$3.75	\$750, 6%, 72 da. \$9	127

Tests should be checked carefully and kinds of errors noted. Help the students to analyze errors and determine causes (carelessness, lack of information, and so on). Reteach as needed before assigning practice pages.

See the Guide for the specific aims of Chapter 6.

Chapter 6

Savings and Investments

Present a discussion of ways of investing money.

1. Every person should try to save part of his earnings. When a sum of money has been saved, there is the problem of investing it in a safe place where it will earn more money. There are a number of ways of investing money so that it will produce a profit. One of these ways is to buy shares of stock in an important industry, such as a large motor company. If you invest money in a sound business, it often earns good returns for you.

2. Another way to invest money is to buy bonds that pay a fixed amount of interest each year. Bonds are sold by states, counties, and cities when it is necessary to raise money to build schools, roads, or bridges. The United States Government issues Savings Bonds which have been widely sold during recent years. Point out that corporations also sell bonds, called "industrial bonds." Explain the meaning of this term.

3. Another investment that many families try to make as soon as possible is the purchase of a home. This is often a family's first investment.

Discuss ex. 1–3 with the students and ask them to tell of other ways of investing savings. Point out that when you buy shares you become a part owner of a company; when you buy bonds you become a creditor.

Show how a corporation is formed and how it operates. This material should be discussed thoroughly with the students. Point out that "par value" is the

Forming a Corporation

arbitrary price put on shares when they are first sold in order to raise money. It may not be the actual value of the shares.

1. Fred and Tom Johnson learned to make wooden toys and decided to form the Johnson Toy Company to raise money so that they could make toys to sell. They needed $300 to buy an electric saw, tools, a workbench, and other necessary equipment. In order to raise $300, they sold **shares** in this company at $1 a share. The **par value** of the shares was $1. When a company like the Johnson Toy Company is legally organized, it is called a **corporation** and the shares are called **stock.** To raise $300, how many shares did the company have to sell? 300

2. Fred bought 25 shares, $25 Tom bought 20 shares, $20 Jim bought 5, $5 and Henry bought 3. $3 How much did each of the 4 boys pay for his shares? Other boys and girls bought all the rest of the shares. They all became **stockholders,** or owners, of the company. Emphasize this concept of ownership.

3. At the end of the first year of business there was a net profit of $50. It was decided to use $20 of this amount for improvements in the business and to divide the remaining $30 among the stockholders in the form of **dividends.** The dividend on each share was $\frac{1}{300}$ of $30*, or $.10. Since each share had a par value of $1, what per cent of $1 was the dividend of $.10 a share? 10% *Be sure the students understand why the dividend was $\frac{1}{300}$ of $30.

4. Find the total dividend received by each of the boys in ex. 2. $2.50, $2.00, $.50, $.30 Find the dividend on 18 shares; $1.80 on 9 shares; $.90 on 35 shares; $3.50 on 40 shares. $4.00

5. Mary had not bought any stock when the company was formed, but after the first dividend was paid she tried to buy 5 shares from Fred. He said he would charge her $1.25 for each share, which was above the par value, or **above par.**

6. The second year the price of materials increased so much that there was no profit and several of the boys wanted to sell their shares of stock. No one was anxious to buy it and George offered to sell his 10 shares at $.80 each, which was **below par.**

7. No one person would have money enough to build and own a large railroad or a steel company. But when many people invest their savings in such companies, there is money enough to carry on the business, to expand it, and to make more money. In a recent year 17,010,000 persons in America owned shares of stock in our corporations. These investors were executives, clerks, farmers, shopkeepers, professional men, workmen, housewives, and just about all other kinds of workers you could mention.

Emphasize the meaning of "above par" and "below par," and why people buy or sell stocks above or below par. Point that "no par value" stock is also issued.

Describe the kinds of stock issued by corporations. If possible, display a stock certificate of a corporation so that the students can see information on it.

Kinds of Stock

1. The Johnson Toy Company is an example of how a corporation is formed. A large business is owned and run along the same lines but becomes much more complex. If a group of men know how to make a good automobile or airplane, they may wish to organize a corporation to get the money necessary to produce it in quantity. The company is chartered by the state government and then sells shares of stock to many people. One of our large corporations has about 281,000,000 shares of stock which are owned by a great many people. Another large corporation has about 197,000,000 shares of stock. A person who buys stock receives a **stock certificate** stating the number of shares he has bought.

2. A corporation may issue 6000 shares of stock with a par value of \$100, thus getting \$600,000 to use in the business. The par value is not always \$100; it may be \$50, \$25, \$1, or there may be no fixed par value at all. Emphasize.

3. When the company makes a profit, a part of this profit is used for expansion of the business and the rest of it is divided among the stockholders as dividends. These dividends are often paid quarterly instead of annually. If a dividend of \$.75 a share is declared each quarter by a corporation, \$3.00 how much is this per year? If a man owns 50 shares of this stock, how much will he receive per quarter? \$37.50

4. Some corporations have two kinds of stock, **preferred stock** and **common stock.** Preferred stock has a fixed **yearly** rate of dividend, like 4% or 5%. A 4% preferred stock with a par value of \$100 pays \$4 a year; if this dividend is paid quarterly, \$1 is paid each quarter. When profits are distributed, the dividends on the preferred stock must be paid first. Any remaining profits are then divided among the stockholders of the common stock. If business is good, the dividends on the common stock might be even higher than on the preferred stock, but if business is poor there may be no dividend at all on the common stock.

Emphasize the difference between preferred and common stock. Point out that, in the case of liquidation payments, the holders of preferred stock are paid first, but that preferred stockholders usually do not have voting rights.

Present information about the New York Stock Exchange and the way in which stocks are bought and sold. See the Guide for a further discussion of the New York Stock Exchange.

1. Stocks of hundreds of different corporations are bought and sold on the New York Stock Exchange. This is a market place for investors from all over the world. **Brokers** who are members of the Stock Exchange do the buying and selling for their customers. If a man in Denver wishes to buy some stock, he gives the order to a Denver broker who telegraphs the order to a New York broker who is a member of the Stock Exchange.

2. The buying or selling price of a stock is called its **market price.** The market price may increase when a corporation pays a good dividend and is reliable; the market price may decrease when a company is insecure and the dividend is low or not paid at all. The market prices of important stocks are printed daily in newspapers. Find a page of stock reports in your newspaper and bring it to class.

3. Below are some **stock quotations** from an evening newspaper. These facts are given for each stock: the number of shares sold during the day, the first price, the highest price, the lowest price, and the price at the end of the day. In the column marked **net change,** $+\frac{1}{2}$ means that today's last price was $\frac{1}{2}$ point higher than yesterday's last price; thus a stock that closed at 103 today closed at $102\frac{1}{2}$ yesterday. Similarly, $-\frac{3}{8}$ means that today's last price was $\frac{3}{8}$ point lower than that of yesterday. The number in parentheses after each stock is the annual dividend expressed in dollars and based on the last dividend declared. Pf. means **preferred** stock. The price of each stock is given in dollars and fractions of a dollar. Thus, $58\frac{3}{4}$ means $\$58\frac{3}{4}$, or \$58.75, and $145\frac{7}{8}$ means $\$145.87\frac{1}{2}$.

Sales	Stock and Dividend Rate	First	High	Low	Last	Net Change
8200	American Radio, pf. ($4\frac{1}{2}$)	$103\frac{1}{4}$	$103\frac{1}{4}$	$102\frac{5}{8}$	103	$+\frac{1}{2}$
6300	Martin Motors (3)	$58\frac{3}{4}$	$59\frac{1}{2}$	$58\frac{1}{4}$	$58\frac{1}{2}$	$-\frac{3}{8}$
9000	United Foods (6.80)	146	148	$145\frac{7}{8}$	$146\frac{3}{4}$	$+1$

Have students bring to class the financial pages of newspapers and show them how to read stock quotations. Emphasize that the price is given for one share. Give students practice in explaining the prices quoted.

Teach students how stocks are bought and sold through brokers.

Buying and Selling Stocks

1. Stocks are usually sold in 100-share lots. When a broker buys or sells stock for a customer, he charges a commission, called **brokerage**, for his services. Brokerage commissions vary from time to time and depend upon the market price of the stock. The broker's commission for buying or selling 100 shares of stock is figured in this way:

Price per Share	Brokerage per 100 Shares
From \$1 to \$4	\$ 3 plus 2 times price per share
From \$4 to \$24	\$ 7 plus price per share
From \$24 to \$50	\$19 plus $\frac{1}{2}$ price per share
\$50 or more	\$39 plus $\frac{1}{10}$ price per share

(But not more than \$75 per 100 shares, or less than \$6)

For example, if a stock sold at \$35 a share, the brokerage was \$19 plus $\frac{1}{2}$ of \$35, or \$19 + \$17.50, which is \$36.50. The broker's commission is given for 100 shares, so if 300 shares are purchased, the commission must be multiplied by 3. If the brokerage for 100 shares was \$36.50, then the brokerage for 300 shares of the same stock was 3 × \$36.50, or \$109.50.

Stress.

Certain taxes are also charged on stock transactions; in this book, these taxes are omitted. See the Guide for a discussion of these taxes.

2. Problem Mr. Wood asks his broker to buy for him 100 shares of stock at $93\frac{1}{4}$. Find the total cost of this stock including the brokerage. \$9373.33

Explanation The price per share is \$93.25. Since it is greater than \$50, the brokerage for 100 shares is \$39 plus $\frac{1}{10}$ of \$93.25, which is \$39 + \$9.33, or \$48.33.

The cost of 100 shares is 100 × \$93.25, or \$9325. The total cost is found by adding \$48.33 and \$9325, which gives \$9373.33.

3. If Mr. Wood had bought 300 shares of this stock, what would have been the brokerage? \$144.99 What would have been the total cost? \$28,119.99

4. Mr. Carter had his broker buy for him 200 shares of oil stock at $57\frac{3}{8}$. What was the broker's commission for buying this stock? \$89.48 Find the total amount Mr. Carter paid for this stock, including the brokerage. \$11,564.48

In ex. 1 discuss the reasons why brokers charge a commission.

Supplement the work on these pages with problems based on actual stock quotations from newspapers.

5. Problem Mr. Allen asks his broker to sell 100 shares of stock at 40. How much does he receive for his stock after paying the brokerage? $3961

Explanation The broker's commission for 100 shares is $19 plus $\frac{1}{2}$ of $40, or $39. The 100 shares sell for $4000. So Mr. Allen receives $4000 – $39, or $3961.

6. Fewer than 100 shares of stock are usually considered an **odd lot** of stock. Odd lots of stock may be purchased through special brokers for an additional commission. This commission is $12\frac{1}{2}$¢ a share for stocks selling under $40 and 25¢ a share for stocks selling at $40 or more. For what reasons do people buy odd lots of stock?

Find the total cost of buying these shares. (2) **First find the brokerage for each transaction:** (1)

(1), (2)

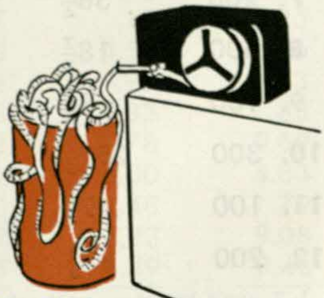
7. 100 shares, Adams Fruit, at $56\frac{1}{4}$ $44.63 $5669.63

8. 200 shares, Erie Coal, at $43\frac{7}{8}$ $81.88 $8856.88

9. 100 shares, Iowa Steel, at 113 $50.30 $11,350.30

10. 300 shares, Eastern Drug, at $17\frac{3}{8}$ $73.14 $5285.64

11. 200 shares, Southern Oil, at $84\frac{3}{4}$ $94.96 $17,044.96



In ex. 5 emphasize that the brokerage is subtracted from the amount received when selling stock, but added to the amount paid when buying stock. In ex. 6 point out that a 100-share lot is called a "round lot."

Show how profits and losses are determined when stocks are bought and sold.

Profits and Losses

1. Mr. Hunt bought 100 shares of stock of a well-managed company at $49\frac{3}{4}$. Several years later he sold this stock at $53\frac{1}{2}$. At the rates given on page 190, what was the broker's commission when the stock was purchased? $43.88 when it was sold? $44.35 What was Mr. Hunt's total gain? $286.77

2. In ex. 1, suppose Mr. Hunt had received dividends totaling $6.50 per share while he held the stock. What was his gain if these dividends are taken into consideration? $936.77

3. A stranger interested Mr. Bell in buying some stock of an oil company in Alaska. The stranger claimed that the company owned valuable oil wells and that it would make enormous profits in a few years. Mr. Bell bought 100 shares of this stock at 14, the brokerage being $21. A year later Mr. Bell learned that many of the stranger's statements were not true so he sold the stock at $1 a share, the brokerage being $6. What was Mr. Bell's loss? $1327 Was Mr. Bell investing his money or speculating? Speculating

4. In ex. 3, do you think Mr. Bell received any dividends? No

Find the profit or loss on the following transactions:

	Total Number of Shares	Price at Which Bought	Brokerage per 100 Shares When Bought	Dividends Received per Share	Price at Which Sold	Brokerage per 100 Shares When Sold	
5.	200	$49\frac{1}{2}$	$43.75	$2.50	$60\frac{3}{4}$	$45.08	(P) $2572.34
6.	100	77	46.70	4.50	$92\frac{1}{2}$	48.25	(P) $1905.05
7.	200	$38\frac{1}{4}$	38.13	1.50	$23\frac{1}{4}$	30.25	(L) $2836.76
8.	500	$18\frac{7}{8}$	25.88	None	18	25.00	(L) $691.90
9.	100	$3\frac{1}{8}$	9.25	.60	$7\frac{3}{4}$	14.75	(P) $498.50
10.	300	$252\frac{1}{2}$	64.25	10.00	$260\frac{1}{8}$	65.01	(P) $4899.72
11.	100	$49\frac{1}{4}$	43.63	3.00	64	45.40	(P) $1685.97
12.	200	$83\frac{5}{8}$	47.36	None	$78\frac{7}{8}$	46.89	(L) $1138.50

Do ex. 1-3 step by step with the class. Emphasize that to find the actual cost of stock you must add brokerage; to find returns from the sale of stock you must subtract brokerage. Remind the students that the brokerage given in ex. 5-12 is for each 100 shares.

Present a set of improvement tests in addition. Tests are to be given, scored, and recorded as others were (see pages 48-49, 99, 139).

Improving by Practice

Addition Test 4a.

Time: 4 min.

1.	$1.35	$9.97	$3.62	$1.14	$9.68	$ .28
	.42	7.01	.79	3.23	.51	4.54
	7.53	.85	4.57	.70	1.34	5.78
	4.02	.78	.45	9.68	2.40	2.59
	.87	8.44	.50	5.05	.98	.77
	6.51	.53	5.68	.26	4.22	5.89
	.46	2.89	4.81	4.08	.94	9.78
	3.52	3.66	.32	9.34	7.19	3.46
	1.64	.63	.97	.81	2.07	.09
	4.14	1.25	3.58	2.32	2.13	7.69
	$30.46	$36.01	$25.29	$36.61	$31.46	$40.87

6

Addition Test 4b.

Time: 4 min.

2.	$6.31	$1.32	$4.86	$5.90	$8.79	$9.33
	.92	3.06	8.00	.58	.53	.18
	2.53	5.98	.73	.48	6.12	6.97
	7.08	.91	4.72	7.73	.94	8.25
	.75	.87	7.06	.96	5.20	6.77
	4.66	8.35	.99	2.28	.69	8.06
	.67	1.84	.39	2.17	8.57	6.68
	3.75	.47	4.85	.85	1.31	.46
	1.92	5.00	.76	.91	.81	8.77
	.42	2.35	3.52	3.14	.46	6.59
	$29.01	$30.15	$35.88	$25.00	$33.42	$62.06

6

Addition Test 4c.

Time: 4 min.

3.	$3.18	$6.64	$6.47	$5.45	$8.95	$2.55
	.64	.87	.99	.27	.14	8.25
	.93	5.42	.84	.85	4.59	5.36
	9.68	.36	4.31	1.32	.92	4.63
	.73	7.05	.66	2.96	8.33	.83
	9.10	.91	1.52	.53	.78	9.69
	.56	4.15	7.77	9.21	6.00	4.83
	1.42	2.23	.42	4.40	.48	.79
	.76	.60	.33	.81	1.23	9.08
	3.07	.34	4.13	.55	4.49	7.46
	$30.07	$28.57	$27.44	$26.35	$35.91	$53.47

6

Have the students compare their scores with previous ones. Give individual help in analyzing errors and plan remedial work as needed.

Teach how to find the rate of income received from stocks that have been purchased.

Rate of Income on Stocks

1. People with money to invest often buy stocks of reliable corporations because the income they receive from dividends is the same as earning interest on their money. The rate of income depends upon the amount of the annual dividend and the price paid for the stock.

2. Mr. White owns one share of stock in the Willson Motor Co., which he bought at $69\frac{1}{2}$. This year his dividend was \$5. What was his rate of income on this investment, disregarding brokerage? 7.2%

▶ Mr. White has received \$5 on an investment of \$69.50. \$5 is 7.2% of \$69.50, to the nearest tenth of 1%. So Mr. White earned 7.2% on his investment this year.

The first number below is the price paid per share of stock and the second number is the annual dividend. Omitting brokerage, find, to the nearest tenth of 1%, the rate of income on each stock: Stress.

3. 50, \$3.00 6.0%
4. 80, \$4.00 5.0%
5. 60, \$4.00 6.7%
6. 96, \$6.00 6.3%
7. 60, \$5.00 8.3%
8. 40, \$1.50 3.8%
9. 80, \$4.40 5.5%
10. 80, \$6.00 7.5%
11. 55, \$4.00 7.3%
12. 99, \$3.85 3.9%
13. 92, \$8.00 8.7%
14. 75, \$4.50 6.0%
15. 60, \$3.90 6.5%
16. 37, \$1.60 4.3%
17. 125, \$6.00 4.8%
18. $71\frac{3}{4}$, \$7.50 10.5%
19. $33\frac{3}{4}$, \$1.45 4.3%
20. $23\frac{1}{2}$, \$1.30 5.5%
21. 183, \$7.00 3.8%
22. $95\frac{1}{2}$, \$3.40 3.6%
23. $46\frac{1}{2}$, \$2.00 4.3%

24. The \$2 preferred stock of the Handy Machine Company is now selling at $46\frac{3}{4}$, while the common stock of that same company is selling at 60. The annual dividend on the preferred stock is \$2 and that on the common stock is \$3.25. Find, to the nearest tenth of 1%, the rate of income on each stock. 4.3%; 5.4% Give possible reasons why the common stock is selling higher than the preferred and also why the common stock is paying a higher dividend. See Guide.

Notice that this work is an application of percentage. Some students may need a quick review of this type of percentage problem.

Present a general review of the various skills taught thus far.

Problems and Practice

1. Mr. Stone received a bill of \$186 for 10 doz. pairs of gloves to sell in his store. He was allowed a discount of 2% on this bill for prompt payment. The parcel post charge for sending the gloves was \$.90, which Mr. Stone had to pay. Find the cost of each pair of gloves. \$1.53 How much should Mr. Stone charge for a pair of gloves if he must allow 40% of the selling price for expenses and profit? \$2.55

2. The installment price of a radio is \$25 down and 10 monthly payments of \$5.50 each. If you pay all cash for this radio, you can get it for 20% less than the installment price. How much can you save by paying cash? \$16

3. Which represents the better rate of discount, \$75 reduced to \$63, (16%) or \$80 reduced to \$66? ($17\frac{1}{2}$%)

4. Last year the Athletic Association had 160 members. This year they have 328 members. What per cent greater is the membership this year than last? 105%

Find each quotient correct to the nearest hundredth:

5.	$28\overline{)127.3}$ 4.55	$.19\overline{)2.075}$ 10.92	$.06\overline{)73.3}$ 1221.67	$8.4\overline{)111}$ 13.21
6.	$71\overline{)8.004}$ .11	$2.6\overline{).1173}$ .05	$.56\overline{)4.88}$ 8.71	$.07\overline{)24.8}$ 354.29

Find these sums. Check by going over the work:

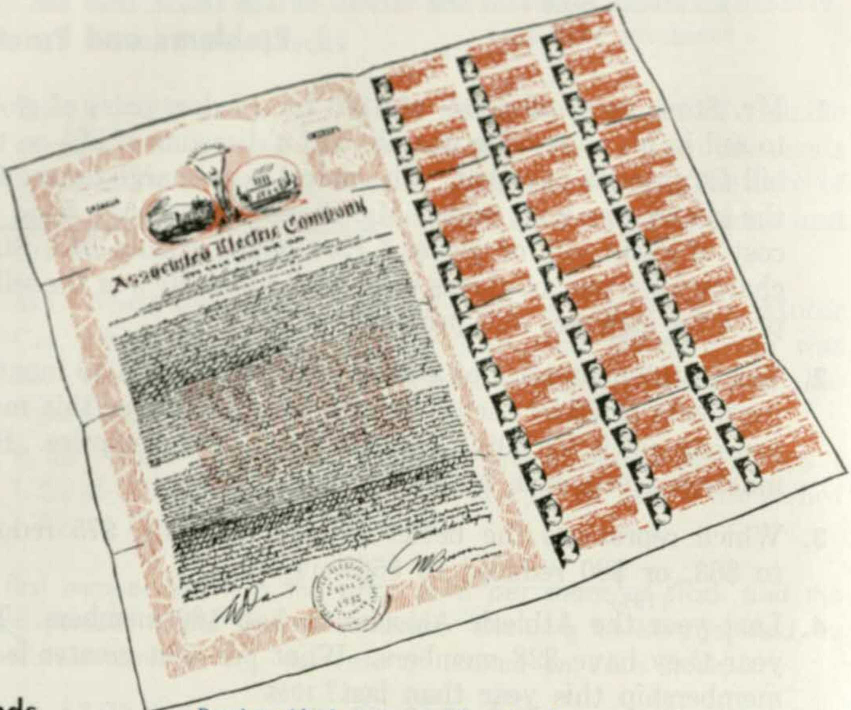
7.	$8\frac{3}{4}$	$2\frac{3}{5}$	$1\frac{1}{2}$	$6\frac{5}{8}$	$7\frac{9}{10}$	$3\frac{1}{6}$	$4\frac{9}{16}$
	$4\frac{1}{2}$	$9\frac{1}{2}$	$8\frac{5}{6}$	$7\frac{1}{4}$	$4\frac{3}{5}$	$7\frac{3}{4}$	$6\frac{3}{8}$
	$1\frac{5}{8}$	$1\frac{7}{10}$	$3\frac{3}{4}$	$2\frac{2}{3}$	$5\frac{1}{2}$	$6\frac{1}{2}$	$2\frac{3}{4}$
	$14\frac{7}{8}$	$13\frac{4}{5}$	$14\frac{1}{12}$	$16\frac{13}{24}$	18	$17\frac{5}{12}$	$13\frac{11}{16}$

Find the missing numbers. Give the per cents to the nearest whole per cent:

8.	9 is 60 % of 15	4 is 2% of 200	8% of 150 is 12
9.	5 is 22 % of 23	9 is 5% of 180	6% of 235 is 14.1
10.	8 is 11 % of 70	7 is 8% of 87.5	4% of 350 is 14

Check the papers carefully and note the kinds of errors. Watch for group errors as an indication that reteaching of a topic to the entire class may be necessary. If errors are computational, give more practice in this area.

Introduce the topic of bonds and the terms used in connection with them.



Bonds

Review that corporation bonds are called "industrial bonds" to distinguish them from other kinds.

1. Another way to invest savings is in the **bonds** of a corporation, a city, a state, or the United States Government. If a corporation wishes to raise money to expand its business, it can borrow money by issuing bonds. A bond is a kind of promissory note in which a corporation or a government promises to pay a certain sum at a stated time. The bonds of a corporation are often secured by a mortgage on its property. Bond owners have no share in the business of the corporation; they are merely lending money to the corporation and getting interest on the loan.

Stress meaning.

2. Each bond has a date when it is due to be paid. This date is called the **date of maturity.** The face value of a bond is usually $1000, but bonds are also issued in denominations of $100 and $500; United States Savings Bonds come in denominations as low as $25. In this work on bonds, each bond is understood to be a $1000 bond unless otherwise stated in the problem. Emphasize.

Discuss the fact that bonds are usually more dependable than stocks.
Stress the fact that bonds pay a fixed rate of interest (ex. 3).
Point out that bond owners are creditors and have no voting power.

3. Every bond pays a fixed rate of interest, such as 4% per year. The interest on a 4% bond equals 4% of $1000, or $40, a year. This interest is paid semiannually; thus $20 is paid each half year. The interest on a bond is usually paid by means of small **coupons** attached to the bond and dated 6 mo. apart. On a 4% bond, each coupon is like a check for $20. On page 196 you see a coupon bond with the coupons attached. When a coupon is due, the owner cuts it off the bond and usually deposits it in his bank. Some bonds are **registered bonds** and have no coupons; in such cases the corporation has the owner's name and mails a check to him for the interest. Be sure the difference between a coupon bond and a registered bond is clear to the students.

4. Bonds are described by giving the rate of interest, the year of maturity, and sometimes other facts. For example, "Central Gas $3\frac{1}{2}$s, 1978" pay $3\frac{1}{2}$% interest and are due in 1978. "Ohio Electric 1st 3s, 1974" are 3% bonds, secured by a first mortgage, and due in 1974. Point out that the s in $3\frac{1}{2}$s refers to the plural bonds.

5. The bond at the top of page 196 has a face value of $1000 and pays interest at 5%. The interest coupons are on the right side of the bond. These coupons are dated Jan. 1 and July 1 of each year from the year the bond is issued to the year it becomes due. Each coupon represents the interest on $1000 at 5% for a half year. What is the value of each coupon? $25 How much interest does this bond pay per year? $50

6. There are 45 coupons left on this bond. If one coupon is cut off each half year, how many years will it be before the bond matures? Bond matures 22 yr. after the first of the 45 coupons is due.

7. A bond like the one shown on page 196 should be kept in the bank in a safe deposit box since this bond is not registered in the name of the owner. Ask pupils why this bond should be kept in bank.

8. Bonds are often issued by a city or state in order to obtain the money needed to build schools, highways, tunnels, and bridges. Try to find out whether your city or state has recently issued bonds and for what purpose the bonds were issued. Also find the rate of interest and the year that the bonds are due.

Use the financial pages of a newspaper to give students more practice in interpreting bond descriptions (ex. 4). Let volunteers explain the answers to ex. 5–6. Have the students follow the suggestion in ex. 8 and report their findings to the class.

Show how much interest one gets from a bond.

Interest on Bonds

1. The Hall Company issued $425,000 worth of 40-year, 5% bonds in denominations of $1000 each. How many bonds were issued? 425 How much interest does the company pay on these bonds each 6 mo.? $10,625

2. Westfield County needed $800,000 to build a school. It raised the money by issuing $3\frac{1}{2}\%$ bonds with a value of $1000 each, interest payable every 6 mo. How much interest did the county pay on these bonds the first year? $28,000

3. In ex. 2, all the bonds did not mature the same year. Beginning 5 yr. after the date of issue, 20 of these bonds matured each year. Why was this arrangement made? See Guide.

4. A state needed $130,000,000 to build a tunnel under a river. It raised the money by issuing $3\frac{3}{4}\%$ bonds with a face value of $1000 each. How many bonds were issued? 130,000 How much interest was paid at the end of the first 6 mo.? $2,437,500

State the rate of interest, (1) **the year due,** (2) **and the amount of interest** (3) **paid each 6 mo. on these $1000 bonds:**

(1), (2), (3)

5. American Oil 5s, 1974 — 5%, 1974, $25	Coast R.R. $3\frac{1}{2}$s, 1985 — $3\frac{1}{2}\%$, 1985, $17.50
6. Ohio Power $4\frac{1}{2}$s, 1971 — $4\frac{1}{2}\%$, 1971, $22.50	Bell Airlines 6s, 1995 — 6%, 1995, $30.00
7. Union Coal 4s, 2003 — 4%, 2003, $20.00	Star Steel $2\frac{3}{4}$s, 1976 — $2\frac{3}{4}\%$, 1976, $13.75
8. Central Gas 3s, 1990 — 3%, 1990, $15.00	Hill Motors $3\frac{1}{2}$s, 1981 — $3\frac{1}{2}\%$, 1981, $17.50

9. If you buy a 4% bond, will you get the same amount of interest each year that you have the bond? Yes* Is this rate of interest guaranteed, or can the company issuing the bond change the rate of interest? It is guaranteed.

*Point out that this is true as long as the company is sound.

10. Does common stock always pay the same rate of dividend? No Does a 4% preferred stock always pay a 4% dividend if the company makes enough profit to pay it? Yes

11. In a good company, which income is most dependable, that of its preferred stock, of its common stock, or of its first mortgage bonds? Bonds Give reasons for your answer.

Use ex. 9–11 to initiate a discussion of relative merits of bonds, common and preferred stocks. See the Guide for a discussion of ex. 11.

Show how bonds are bought and sold on the stock exchange through a broker. Emphasize the fact that a bond pays a fixed amount of interest, based on the face value of the bond, regardless of the current price of the bond.

Buying Bonds

1. Many bonds, like stocks, are bought and sold on a stock exchange. The prices of bonds are given daily in the newspapers in the form shown below:

Sales	Name	High	Low	Last
36	Texas Rubber 3s, 1978	$88\frac{1}{2}$	$87\frac{3}{4}$	88
60	Newton Oil $4\frac{1}{2}$s, 1996	$100\frac{1}{8}$	100	$100\frac{1}{8}$
14	Dayton R.R. $3\frac{3}{8}$s, 1980	$103\frac{5}{8}$	103	103

You see that the Texas Rubber bond is quoted at $88\frac{1}{2}$. If you buy a bond at $88\frac{1}{2}$, you pay \$$88\frac{1}{2}$, or \$88.50, for each \$100 of face value of the bond. Thus you pay 10 × \$88.50, or \$885.00, for a \$1000 bond. Likewise, if you buy a \$1000 bond at $103\frac{5}{8}$, it costs 10 × \$103.625, or \$1036.25.

Emphasize.

2. At what price does a \$1000 bond sell if it is quoted at $82\frac{1}{2}$? \$825.00 at $71\frac{3}{4}$? \$717.50 at $92\frac{5}{8}$? \$926.25 at $103\frac{1}{8}$? \$1031.25 at $110\frac{7}{8}$? \$1108.75 at $87\frac{1}{4}$? \$872.50 at $100\frac{3}{8}$? \$1003.75

3. If you wish to buy or sell a bond, you give the order to a broker. The broker's fee is \$5 each for 1 or 2 bonds, \$4 each for 3 bonds, \$3 each for 4 bonds, and \$2.50 each for 5 or more bonds. Find the total cost, including brokerage, of 2 bonds at $88\frac{3}{8}$; \$1777.50 of 4 bonds at $87\frac{3}{4}$; \$3522 of 7 bonds at $102\frac{5}{8}$. \$7201.25

4. Using the table in ex. 1, find the cost of a Newton Oil bond quoted at $100\frac{1}{8}$, including \$5 brokerage. \$1006.25 How much interest does this bond pay each 6 mo.? \$22.50

5. Find the cost of 5 Dayton Railroad bonds at $103\frac{5}{8}$, including brokerage at \$2.50 each. \$5193.75

6. The price of a \$1000 bond gets nearer and nearer to \$1000 as the date of maturity approaches. For example, if you buy a good bond at 95, which is due in 1985, the price of the bond will gradually rise from 95 to 100 as 1985 approaches. Likewise, if you buy a bond at 104, due in 1985, its price will gradually drop to 100 by 1985. Explain why this is so. When a \$1000 bond matures, the owner will receive only \$1000.

The concept in ex. 6 should be made clear to the students. See the Guide for an important discussion of ex. 6.

Teach how to find the rate of income on a bond. Introduce and explain the term current yield.

Rate of Income on Bonds

1. Problem Mrs. Grant bought a 4% \$1000 bond at 86. What annual rate of income does she get on her investment?

Explanation The annual interest on this bond is \$40 and the investment Mrs. Grant made was \$860. This means that her money is earning $\frac{40}{860}$, or 4.7% a year. The 4.7% is called the **current yield** of the bond. You see that the current yield is higher than the rate of interest stated in the bond because the purchase of the bond was made at a price below par. Emphasize.

Stress meaning.

$$\begin{array}{r} .0465 \text{ or } 4.7\% \\ \$860\overline{)\$40.0000} \\ 34\,40 \\ \hline 5\,600 \\ 5\,160 \\ \hline 4400 \\ 4300 \\ \hline 100 \end{array}$$

2. Suppose that Mrs. Grant paid 115 for the bond in ex. 1. Find the current yield. 3.5%

▶ Since the price of the bond is now \$1150, the current yield on the price is $\frac{40}{1150}$, or 3.5%. This is less than the rate stated in the bond because the bond was bought above par. Emphasize.

3. Mr. Mills bought a $2\frac{1}{2}$% \$1000 bond at 101. How much did he pay for it, disregarding brokerage? \$1010 How much interest does he get annually? \$25 Find, to the nearest tenth of 1%, the current yield of the bond. 2.5%

4. Miss King purchased a $3\frac{1}{2}$% \$1000 bond at 82. How much is the annual interest on this bond? \$35 Find, to the nearest tenth of 1%, the current yield of the bond. 4.3%

If there are no apparent difficulties, have the students complete ex. 5-14.

Find the current yield, to the nearest tenth of 1%, if \$1000 bonds are purchased as given below. Do not include brokerage:

5. A 3% bond at $89\frac{1}{4}$ 3.4%

6. A 5% bond at $83\frac{1}{2}$ 6.0%

7. A 5% bond at 114 4.4%

8. A 4% bond at 85 4.7%

9. A 3% bond at $105\frac{5}{8}$ 2.8%

10. A $4\frac{1}{2}$% bond at 123 3.7%

11. A $3\frac{1}{2}$% bond at 72 4.9%

12. A $2\frac{3}{8}$% bond at $99\frac{1}{8}$ 2.4%

13. A $3\frac{1}{4}$% bond at $75\frac{1}{8}$ 4.3%

14. A $4\frac{1}{2}$% bond at $108\frac{7}{8}$ 4.1%

Explain that "current yield" is the rate of interest bonds pay based on the actual price paid for the bond. Emphasize that it may be larger or smaller than the given rate, as noted in ex. 1-2.

Present information about United States Savings Bonds of Series E (pages 201-202).

United States Savings Bonds

1. The investments discussed earlier in this chapter require that a person have several hundred dollars to invest. Many people would have difficulty in saving such amounts of money. The United States Government has Savings Bonds that come in denominations of \$25, \$50, \$100, \$500, and \$1000. This makes it possible for anyone with an average income to buy bonds. The Savings Bonds most people buy are the bonds of Series E, which are on sale at most banks.

Emphasize.

Stress.

2. The Savings Bonds of Series E originally matured 10 yr. after they were purchased. They now mature 7 yr. and 9 mo. after they are purchased. These bonds do not pay interest each year; instead, the interest accumulates from year to year and is paid in a lump sum at the end of 7 yr. and 9 mo. When you buy a \$25 bond, you do not pay \$25 for it; a \$25 bond is sold at \$18.75. At the end of 7 yr. and 9 mo., when the bond matures, the United States Government pays you \$25 for it. So your total interest on this bond is \$25.00 – \$18.75, or \$6.25. This means that your \$18.75 earns \$6.25 interest in 7 yr. and 9 mo. Emphasize.

3. A \$100 Savings Bond of Series E costs you only \$75. You get \$100 for this bond 7 yr. and 9 mo. later; thus your investment of \$75 earns \$25 interest in 7 yr. and 9 mo. If you get \$25 interest in 7 yr. and 9 mo., how much interest is that per year? \$3.23 (First change 7 yr. 9 mo. to $7\frac{3}{4}$ yr.) What rate of simple interest per year are you getting on your investment of \$75? Find the rate to the nearest tenth of 1%. 4.3%

4. The prices of Series E Savings Bonds of each denomination are shown at the right. If you buy 2 bonds at \$18.75 each, how much in all will you get back at the end of 7 yr. 9 mo.? \$50 What is the total amount of interest that your investment will have earned? \$12.50 If you buy 3 bonds at \$37.50 each, what is the total amount of interest you will get in 7 yr. 9 mo.? \$37.50

\$25 bonds cost	\$18.75
\$50 bonds cost	\$37.50
\$100 bonds cost	\$75.00
\$500 bonds cost	\$375.00
\$1000 bonds cost	\$750.00

Emphasize the fact that interest accumulates and is paid in one lump sum.

Discuss ex. 1 with the class to be sure all understand the table. Have students report on the latest redemption values of Savings Bonds if they have been changed.

United States Savings Bonds

1. In order to collect the full amount of interest on a Series E Savings Bond, you must hold the bond until the end of 7 yr. 9 mo.; but it is possible to redeem the bond before that time and get part of the interest. The redemption values of a $100 bond at the end of each year are given in the table. Remember that this bond cost you only $75.00. If you redeem this bond after holding it 4 yr., the table shows that you will get $86.00 for it. How much interest in all will your $75 have earned in these 4 yr.? $11.00 How much is this per year? $2.75

Redemption Values of $100 Bond That Costs $75.00	
If Redeemed After	Its Value Is
1 yr.	$76.76
2 yr.	79.60
3 yr.	82.64
4 yr.	86.00
5 yr.	89.60
6 yr.	93.28
7 yr.	97.08
7 yr. 9 mo.	100.00

2. If you redeem the bond after holding it 5 yr., how much will you get for it $89.60 and how much interest in all will your $75 have earned? $14.60 How much interest is this per year? $2.92

3. How much interest per year will your $75 have earned if you redeem the bond after holding it 6 yr.? 3.04\frac{2}{3}$ after holding it 7 yr.? 3.15\frac{3}{7}$

4. Compare the yearly amount of interest in ex. 1 with that in ex. 2. Then compare these amounts with those in ex. 3. Jack says that the longer you hold the bond, the greater the yearly interest becomes. Is Jack right? Yes Is it to your advantage to hold these Savings Bonds as long as you can? Yes

5. From the table above you can find the redemption value of a $1000 bond by multiplying each value by 10. For example, at the end of 4 yr. the redemption value of a $1000 bond is $860. Find the redemption value at the end of 5 yr.; $896.00 at the end of 7 yr. $970.80

6. Make a table like the one above to show the redemption values of a $500 bond. See Guide.

Emphasize that the longer you hold bonds, the higher the rate of interest you receive. Point out also that bonds held for 10 or 20 years after the date of maturity receive additional interest.

Present a set of improvement tests in subtraction. Tests are to be given, scored, recorded as others were (see pages 65, 113, 151).

Improving by Practice

Subtraction Test 4a.

Time: 4 min.

1.	$2428.38	$6001.06	$8276.15	$8000.46	
	631.69	1253.08	3749.46	136.49	
	$1796.69	$4747.98	$4526.69	$7863.97	
2.	$7002.06	$5495.14	$4950.21	$3000.07	
	3015.47	646.29	1376.86	952.48	
	$3986.59	$4848.85	$3573.35	$2047.59	
3.	$5434.76	$7000.00	$7301.05	$5123.82	
	137.91	1738.47	726.79	1749.48	12
	$5296.85	$5261.53	$6574.26	$3374.34	

Subtraction Test 4b.

Time: 4 min.

4.	$5129.16	$3205.07	$9329.71	$6000.50	
	164.29	1345.13	943.59	3399.28	
	$4964.87	$1859.94	$8386.12	$2601.22	
5.	$4000.02	$7547.54	$8900.95	$5263.48	
	684.37	148.46	5796.59	3525.96	
	$3315.65	$7399.08	$3104.36	$1737.52	
6.	$9689.21	$8401.02	$2974.31	$5000.00	
	6831.28	7218.58	357.85	145.29	12
	$2857.93	$1182.44	$2616.46	$4854.71	

Subtraction Test 4c.

Time: 4 min.

7.	$5000.06	$7472.53	$5070.30	$2288.15	
	4379.27	819.68	2981.42	978.58	
	$ 620.79	$6652.85	$2088.88	$1309.57	
8.	$6395.14	$1000.01	$6874.38	$6603.70	
	469.37	570.37	2374.76	4705.50	
	$5925.77	$ 429.64	$4499.62	$1898.20	
9.	$9000.03	$1184.37	$8600.68	$7683.31	
	1036.93	416.83	258.73	6964.22	12
	$7963.10	$ 767.54	$8341.95	$ 719.09	

To the Pupil. This is the fourth set of Improvement Tests in subtraction. Try to get a score of 10 on each test.

Group students who had errors on these tests. Have them do computations orally so that you can determine the causes of errors. Plan remedial work (including a drill on facts, if needed) after difficulties are cleared up.

Present a review of the four fundamental operations with fractions.

Fraction Review

1. Mr. Penn has made a scale drawing of a building lot of irregular shape. In the drawing, the 4 sides of this lot have these lengths: $1\frac{1}{2}$ in., 1 in., $2\frac{1}{4}$ in., and $2\frac{1}{2}$ in. How many feet long is each side of the lot if the scale of the drawing is 1 in. = 100 ft.? 150; 100; 225; 250

2. The average length of Tom's step is $2\frac{1}{2}$ ft. If he paces off a distance of 350 ft., how many steps will he take? 140 Bill's step is only 2 ft. 4 in. How many more steps will he take than Tom to pace off 350 ft.? 10

3. Miss Lake sent a telegram of 23 words. The charge was \$1.45 for the first 15 words and $6\frac{1}{2}$¢ for each extra word. There was also a 10% Government tax on the cost of the telegram. Find the entire cost of sending this telegram. \$2.17

Find the answers. Check by going over the work:

4.	$6\frac{1}{2} + 3\frac{3}{4}$ $10\frac{1}{4}$	$2\frac{1}{2} \div \frac{2}{5}$ $6\frac{1}{4}$	$\frac{7}{8} - \frac{1}{2}$ $\frac{3}{8}$	$8 \times 5\frac{1}{4}$ 42
5.	$1\frac{1}{2} \times 1\frac{1}{2}$ $2\frac{1}{4}$	$7\frac{3}{4} + \frac{5}{8}$ $8\frac{3}{8}$	$\frac{5}{6} + \frac{3}{4}$ $1\frac{7}{12}$	$\frac{1}{3} \div 5\frac{1}{3}$ $\frac{1}{16}$
6.	$9\frac{3}{4} \div 3\frac{1}{4}$ 3	$6\frac{2}{3} - 4$ $2\frac{2}{3}$	$\frac{3}{8} \times \frac{4}{5}$ $\frac{3}{10}$	$\frac{5}{8} \times 11$ $6\frac{7}{8}$
7.	$12 - 4\frac{3}{4}$ $7\frac{1}{4}$	$4\frac{1}{2} + \frac{2}{3}$ $5\frac{1}{6}$	$9 \div \frac{1}{3}$ 27	$\frac{3}{4} - \frac{5}{12}$ $\frac{1}{3}$
8.	$1\frac{1}{2} + \frac{3}{10}$ $1\frac{4}{5}$	$\frac{9}{10} \div 3$ $\frac{3}{10}$	$\frac{2}{3} - \frac{1}{2}$ $\frac{1}{6}$	$\frac{2}{3} \times 7\frac{1}{2}$ 5
9.	$\frac{15}{16} + 2\frac{1}{4}$ $3\frac{3}{16}$	$1\frac{1}{2} \times \frac{2}{3}$ 1	$\frac{5}{8} - \frac{1}{8}$ $\frac{1}{2}$	$\frac{3}{8} \div \frac{9}{16}$ $\frac{2}{3}$
10.	$4\frac{1}{2} - 2\frac{3}{8}$ $2\frac{1}{8}$	$1\frac{1}{6} \div \frac{2}{3}$ $1\frac{3}{4}$	$\frac{7}{8} + \frac{5}{6}$ $1\frac{17}{24}$	$\frac{2}{3} \times 18$ 12
11.	$5\frac{2}{5} \div 18$ $\frac{3}{10}$	$2\frac{3}{4} + \frac{4}{5}$ $3\frac{11}{20}$	$\frac{3}{4} \times \frac{1}{2}$ $\frac{3}{8}$	$\frac{1}{4} - \frac{3}{16}$ $\frac{1}{16}$
12.	$10 \div 4\frac{1}{6}$ $2\frac{2}{5}$	$3\frac{5}{6} - \frac{2}{3}$ $3\frac{1}{6}$	$\frac{5}{6} \times \frac{4}{5}$ $\frac{2}{3}$	$\frac{3}{8} + 2\frac{7}{8}$ $3\frac{1}{4}$
13.	$2\frac{1}{4} \times 10$ $22\frac{1}{2}$	$\frac{7}{10} + \frac{4}{5}$ $1\frac{1}{2}$	$6 \div \frac{2}{3}$ 9	$8 - 7\frac{3}{4}$ $\frac{1}{4}$

Write in columns and add:

14. $9\frac{1}{8} + 1\frac{1}{2} + 8\frac{1}{4} + 3\frac{3}{4} + 4\frac{7}{16} + 4\frac{1}{2}$ $31\frac{9}{16}$ $\qquad 5\frac{1}{2} + 6\frac{9}{10} + 4\frac{4}{5}$ $17\frac{1}{5}$

15. $1\frac{1}{2} + 2\frac{5}{6} + 1\frac{11}{12} + 7\frac{5}{12} + 5\frac{5}{6} + 2\frac{1}{3}$ $21\frac{5}{6}$ $\qquad 2\frac{3}{4} + 7 + 8\frac{7}{16}$ $18\frac{3}{16}$

Use the results of the review as an indication of the need for class or individual reteaching. Have volunteers explain the solutions at the board so that others may find and correct their mistakes. If problem situations cause difficulty, provide more practice.

Acquaint the students with the services of savings and loan associations (see the Guide for important background information).

Savings and Loan Associations

1. A savings and loan association makes loans to persons who wish to buy a house or remodel an old house. Such loans are usually paid back in monthly installments including interest. By this arrangement, the amount the borrower owes the association becomes smaller and smaller each month until the entire loan is paid off. This plan is also to the advantage of the savings and loan association because the risk involved in lending the money becomes less and less each month.

2. Mr. Smith borrows \$10,000 from a savings and loan association to build a house. When the loan is made, the association has a risk of \$10,000. How much risk does the association have 2 yr. later when Mr. Smith has paid back \$2000 on this loan? \$8000

3. Each savings and loan association also has a savings and investment department where people may deposit their savings. Interest on these deposits is usually paid at the rate of 3% to $4\frac{3}{4}$% a year. The money that the association lends to build homes comes from these savings accounts. Emphasize.

4. Mr. Gray deposits \$20 a month regularly with a savings and loan association which pays interest at 4%. At the end of 5 yr. the principal and the interest together amount to \$1325.54. How much of this money did Mr. Gray deposit (1) and how much is interest (2)? (1) \$1200; (2) \$125.54

5. In ex. 4, if Mr. Gray had continued these regular deposits for another 5 yr. he would have \$2942.96 in all, including interest. How much of this amount is interest? \$542.96

6. There are 6358 savings and loan associations in the United States. They have 32,431,000 savings accounts with deposits totaling \$70,838,000,000. About how many savings accounts does each association have, on the average? 5101 What is the average size of these saving accounts? \$2184.27

7. If there is a savings and loan association in your city, ask for booklets describing its services.

Emphasize the important functions of these associations. Have different students explain the solutions. Ask them to follow the suggestion in ex. 7 and discuss their findings with the class.

Show how money is borrowed from savings and loan associations to build a house and how it is paid back.

Borrowing Money to Build a House

1. Mr. Hill is building a house that will cost $18,000. He has $6000 in cash and has borrowed the balance of $12,000 from a savings and loan association at 6%. He will pay this debt of $12,000 by making monthly payments of $86 each, which will be as convenient as paying rent. Each month a part of the $86 will be used to pay interest on the amount he still owes and the rest will be used to reduce the debt.

Emphasize.

2. The details of the above plan are shown below:

	A Unpaid Balance	B Total Monthly Payment	C Interest on Unpaid Balance	D Amount Used to Reduce Debt
1st month	$12,000.00	$86	$60.00	$26.00
2nd month	11,974.00	86	59.87	26.13
3rd month	11,947.87	86	59.74	26.26
4th month	11,921.61	86	59.61	26.39
5th month	11,895.22	86	59.48	26.52

The first month Mr. Hill owes $12,000, as shown in the first line of the table. The interest on $12,000 for 1 mo. at 6% is $60. At the end of the first month he makes a payment of $86, from which the interest of $60 is deducted. This leaves $26 to be used to reduce the debt of $12,000. Subtracting $26 from $12,000, you get $11,974, which is the new unpaid balance.

The second month Mr. Hill owes $11,974, as shown in the second line of the table. The interest on $11,974 for 1 mo. at 6% is $59.87. Deducting $59.87 from $86, you get $26.13, which is used to reduce the unpaid balance of $11,974. This leaves $11,947.87 as the new unpaid balance. Show how each item in the last three lines of the table is obtained. Do this step by step with the students.

3. Study Column A in the table; why is the unpaid balance getting smaller each month? Study Column C; why is the interest getting smaller each month? Study Column D; why are these amounts becoming larger each month? In the first line of the table, add the amount in Column C to the amount in Column D; what is the sum? Add the same

Ex. 1–4 should be carefully explained to the students and studied by them. Show how each item is computed in the table in ex. 2. Emphasize that the $86 covers interest as well as a payment to reduce the debt.

items in each of the other lines of the table; why is this sum always the same?

4. In ex. 2, the interest on \$11,974 for 1 mo. at 6% is \$59.87. This interest is computed as follows: If the interest is 6% for 1 yr., it is $\frac{1}{12}$ of 6%, or $\frac{1}{2}$%, for 1 mo. $\frac{1}{2}$% is the same as $\frac{1}{2}$ of 1%. So the interest on \$11,974 for 1 mo. is $\frac{1}{2}$ of 1% of \$11,974. 1% of \$11,974 is \$119.74; $\frac{1}{2}$ of \$119.74 is \$59.87. Compute the interest in the other lines of Column C.

Ask why.

5. Continue the table in ex. 2 by computing the items for the sixth, seventh, and eighth months. See Guide.

6. If Mr. Hill continues his payments of \$86 a month for 20 yr., his entire debt of \$12,000 will be paid off and he will own the house. When the debt on a house is paid off in monthly payments, it is customary to add enough to each monthly payment to cover fire insurance and taxes. This extra amount depends upon the city or town in which the house is located.

Be sure the students understand all the relationships in ex. 3 and emphasize that interest becomes smaller each month while the payment on the debt becomes larger. Have them note that Column A shows that risk to the association becomes smaller each month.

Show how the monthly payment on a loan depends on the size of the loan, the rate of interest, and the time taken to repay the loan.

Making Loans on Houses

1. On page 206, Mr. Hill repaid his loan of \$12,000 in monthly payments of \$86 each. If he had made larger payments, he would have repaid the loan sooner. If he had made smaller payments, it would have taken longer to repay the loan.

2. Savings and loan associations and banks have tables like the one below which show how large each monthly payment must be in order to repay a \$1000 loan within a certain time.

Rate of Interest	Monthly Payment on Each \$1000 of Loan if Loan Is Repaid in			
	10 yr.	15 yr.	20 yr.	25 yr.
5%	\$10.61	\$7.91	\$6.60	\$5.85
$5\frac{1}{2}$%	\$10.86	\$8.18	\$6.88	\$6.15
6%	\$11.11	\$8.44	\$7.17	\$6.45
$6\frac{1}{2}$%	\$11.36	\$8.72	\$7.46	\$6.76

This table shows that monthly payments of \$10.61 each will repay a \$1000 loan in 10 yr. at 5%, while monthly payments of \$7.91 each will repay it in 15 yr. At 5%, what is the monthly payment on a \$1000 loan if the time is 20 yr.? \$6.60 25 yr.? \$5.85 At $5\frac{1}{2}$%, what is the monthly payment on a \$1000 loan if the time is 10 yr.? \$10.86 15 yr.? \$8.18 20 yr.? \$6.15

3. The table can also be used to find the monthly payments on loans larger than \$1000. For example, on a loan of \$5000 the monthly payment is 5 times as much as that shown in the table. What will be the monthly payment if you borrow \$5000 at 5% to be paid off in 15 yr.? \$39.55 in 20 yr.? \$33.00 What will be the monthly payment on a loan of \$3000 at $6\frac{1}{2}$% to be paid off in 10 yr.? \$34.08 in 15 yr.? \$26.16 in 25 yr.? \$20.28

4. Mr. Case borrows \$9000 from a bank at 5% to build a house. If he repays the loan in 20 yr., how much is the monthly payment? \$59.40 How much in all will he pay the bank in 20 yr.? \$14,256 What is the total amount of interest in 20 yr.? \$5256

5. In ex. 4, what is the monthly payment (1) and the total amount of interest (2) if Mr. Case repays the loan in 15 yr. instead of 20 yr.? (1) \$71.19; (2) \$3814.20

Have the students notice through a discussion of ex. 5 that the longer you take to pay off a loan, the larger the amount of total interest you pay.

Supplement this work with additional problems that can be solved by using the table in ex. 2.

6. Mr. Wells wants to buy a house valued at \$16,500. A bank will make him a loan equal to $66\frac{2}{3}\%$ of the value of the house. If he repays this loan in 15 yr., what will be his monthly payment with interest at 5%? \$87.01

7. Mr. Lewis is planning to buy a house valued at \$15,000. A savings and loan association will make him a loan equal to 80% of the value of the house. If he repays this loan in 20 yr., what will be his monthly payment with interest at $5\frac{1}{2}\%$? \$82.56 How much cash will Mr. Lewis need in order to buy this house? \$3000

Discuss the concepts in ex. 8 thoroughly.

8. The Federal Housing Administration insures a loan made on a house if the house meets certain standards. This insurance protects the bank, or other lending agency, that makes the loan if anything prevents the borrower from making his monthly payments. It is possible to get a larger loan on a house when the loan is insured. The interest rate on FHA loans cannot be greater than $5\frac{1}{4}\%$. The borrower must also pay the cost of the loan insurance, which is $\frac{1}{2}$ of 1% each year on the unpaid balance of the loan. Have the students check the current regulations for FHA loans.

9. Mr. and Mrs. Black are planning to build a new house which will cost \$20,000. They can get an FHA loan at their local savings bank for 85% of the cost of their house. Find the monthly payment they must make to repay the loan in 20 yr. with interest at 5%. Omit the cost of insurance. \$112.20

Explain the meaning of compound interest and show how it is computed.

Compound Interest

1. If you put your money in a savings bank and leave it there for some time, it will earn interest for you. Savings banks often pay interest semiannually (twice a year) or quarterly (four times a year). The rates of interest vary between $2\frac{1}{2}\%$ and $4\frac{1}{4}\%$. If this interest is left in the bank, it also earns interest. When interest is paid on the principal and also on the interest that is left in the bank, it is called **compound interest.**

2. Jean's birthday comes on Jan. 1. When she was 10 yr. old, her father put $300 in the savings bank for her as a birthday present. The bank pays 4% interest, compounded semiannually. Jean is now 13 yr. old. How much has she in the bank now if she left all the interest in the bank? Jean did not deposit or withdraw any money during this time.

Amt., age 10	$300.00
Int., $\frac{1}{2}$ yr.	6.00
Amt., age $10\frac{1}{2}$	306.00
Int., $\frac{1}{2}$ yr.	6.12
Amt., age 11	312.12
Int., $\frac{1}{2}$ yr.	6.24
Amt., age $11\frac{1}{2}$	318.36
Int., $\frac{1}{2}$ yr.	6.36
Amt., age 12	324.72
Int., $\frac{1}{2}$ yr.	6.48
Amt., age $12\frac{1}{2}$	331.20
Int., $\frac{1}{2}$ yr.	6.62
Amt., age 13	337.82

The interest is 4% a year but it is paid each half year, so the interest for $\frac{1}{2}$ yr. is 2% of the principal. Jean's account starts with $300.00. The interest for $\frac{1}{2}$ yr. is 2% of $300.00, or $6.00. Jean leaves this interest in the bank, so she has $306.00 in the bank at age $10\frac{1}{2}$. This $306.00 then earns interest for $\frac{1}{2}$ yr., which is $6.12; Jean leaves this $6.12 in the bank, so she has $312.12 in the bank at age 11. The interest on $312.12 is computed on $312.00 and not on $312.12 since savings banks do not allow interest on fractional parts of a dollar. In the table, show how each of these amounts of interest is computed: $6.36, $6.48, $6.62.

Stress.

Jean's money increases each half year, as shown in the table. At age 13, she has $337.82 in the bank, which is $37.82 **more** than she had at age 10. This $37.82 is the total **compound interest** that Jean's savings account of $300 has earned in 3 yr.

Emphasize the fact that the interest is added to the principal (making it larger) before the interest for the next period is computed. Do all computations for ex. 2 at the board so that students can see how each step is obtained.

3. In ex. 2, suppose Jean had withdrawn the interest from the bank at the end of each half year, leaving only the $300 in the bank; how much interest would she have received each half year? $6.00 Show that the total interest she would have received in the 3 yr. would have been $36.00. This $36.00 is called **simple interest.** In ex. 2, the total compound interest is $37.82. The difference between the simple and the compound interest in this case is $1.82. This difference would be much larger if the money remained in the bank for a much longer period, such as 10 yr. or 20 yr.

4. Continue the computations in the table in ex. 2 to age 16. See Guide. What is the total amount Jean has in the bank then? $380.36 How much compound interest has her $300 earned during the entire 6 yr.? $80.36 Find the simple interest on her $300 for the 6 yr. $72 How much greater is the total compound interest than the total simple interest? $8.36

5. Mr. Reed put $400 in a savings bank that paid 3% interest, compounded semiannually. If he left all the interest in the bank, how much in all did he have in the bank at the end of 4 yr.? $450.56 You see that it requires considerable work to compute compound interest. See Guide. This work is much reduced by using compound interest tables as explained on page 212.

Ex. 3 should also be done and discussed with the class. Be sure the students understand why simple interest (the principal remains the same) is less than compound interest (the principal keeps getting larger).

Teach the use of a table in computing interest (pages 212-213).

Using a Compound Interest Table

Table Showing Amount of $1 at Compound Interest						
Years or Periods	$\frac{3}{4}$%	1%	$1\frac{1}{2}$%	2%	3%	4%
1	1.00750	1.01000	1.01500	1.02000	1.03000	1.04000
2	1.01506	1.02010	1.03023	1.04040	1.06090	1.08160
3	1.02267	1.03030	1.04568	1.06121	1.09273	1.12486
4	1.03034	1.04060	1.06136	1.08243	1.12551	1.16986
5	1.03807	1.05101	1.07728	1.10408	1.15927	1.21665
6	1.04585	1.06152	1.09344	1.12616	1.19405	1.26532
7	1.05370	1.07214	1.10984	1.14869	1.22987	1.31593
8	1.06160	1.08286	1.12649	1.17166	1.26677	1.36857
9	1.06956	1.09369	1.14339	1.19509	1.30477	1.42331
10	1.07758	1.10462	1.16054	1.21899	1.34392	1.48024
16	1.12699	1.17258	1.26899	1.37279	1.60471	1.87298
20	1.16118	1.22019	1.34686	1.48595	1.80611	2.19112
24	1.19641	1.26973	1.42950	1.60844	2.03279	2.56330
30	1.25127	1.34785	1.56308	1.81136	2.42726	3.24340
40	1.34835	1.48886	1.81402	2.20804	3.26204	4.80102

1. The table above gives to 5 decimal places the amount to which $1 will grow at compound interest. Emphasize that it is compounded annually.

Stress. ▶ Suppose you wish to find the amount to which $1 will grow in 6 yr. at 3%, compounded annually. In the table, hold a ruler horizontally under the 6 in the left-hand column; the number just above the ruler in the 3% column is 1.19405. This shows that $1 will grow to $1.19 in 6 yr. at 3%.

2. From the table, find the amount to which $1 grows in 5 yr. at $1\frac{1}{2}$%; $1.08 in 4 yr. at 2%; $1.08 in 30 yr. at 3%; $2.43 in 40 yr. at 3%. $3.26 In each case the interest is compounded annually.

3. Find the amount to which $300 will grow in 16 yr. at 2%, compounded annually. $411.84

▶ The amount of $1 in 16 yr. at 2% is $1.37279. Then the amount of $300 is 300 × $1.37279, or $411.84.

4. How much is the compound interest in ex. 3? $111.84 How much is the simple interest on $300 at 2% for 16 yr.? $96 What is the difference between the two kinds of interest in this case? $15.84

5. Find the amount to which $400 will grow in 20 yr. at 2%; $594.38 $650 in 10 yr. at 3%; $873.55 $800 in 30 yr. at $1\frac{1}{2}$%; $1250.46 $1500 in 24 yr. at 3%; $3049.19 $750 in 40 yr. at 4%. $3600.77

Students must be given careful instructions on the use of the table. Have them follow the directions in ex. 1. Check to see that all locate the interest. Do ex. 2-3 with the class in the same way.

1. A savings bank pays interest at 3%, compounded semiannually, on sums of money deposited there. Find the amount to which $500 will grow in 10 yr. at 3% compounded semiannually. $673.43

▶ Since 3% is the yearly rate of interest, the rate for each half year, or for each interest period, is $1\frac{1}{2}\%$. Since there are 2 interest periods in each year, there are 20 interest periods in 10 yr. In the table on page 212, find 20 in the column marked "Years or Periods" and place a ruler under 20. Then find the number just above the ruler in the $1\frac{1}{2}\%$ column. You find the number 1.34686. So $1 grows to $1.34686 in 10 yr., or in 20 interest periods. In the same time $500 will grow to $500 \times \$1.34686$, or to $673.43. Be sure you can explain why you use 20 periods and the $1\frac{1}{2}\%$ column instead of using 10 periods and the 3% column.

2. If interest is compounded semiannually, how many interest periods are there in 5 yr.? 10 in 8 yr.? 16 in $4\frac{1}{2}$ yr.? 9 in 20 yr.? 40 If interest is compounded semiannually, what is the interest rate for each period if the annual rate is 4%? 2% 3%? $1\frac{1}{2}\%$ 2%? 1% $1\frac{1}{2}\%$? $\frac{3}{4}\%$ 6%? 3%

Find the amounts by the table at 4% compounded semiannually:

	Principal	Time			Principal	Time			Principal	Time	
3.	$300	5 yr.	$365.70	**6.**	$900	10 yr.	$1337.36	**9.**	$1800	20 yr.	$3974.47
4.	$700	8 yr.	$960.9	**7.**	$650	20 yr.	$1435.2	**10.**	$1250	15 yr.	$2264.20
5.	$575	4 yr.	$673.7	**8.**	$325	12 yr.	$522.74	**11.**	$2800	10 yr.	$4160.66

12. Do ex. 3–5 again, at 3% compounded semiannually. $348.16; $888.29; $647.73

13. Do ex. 6–8 again, at 6% compounded semiannually. $1625.50; $2120.33; $660.66

14. Mr. North deposited $1500 with a savings and loan association for his young son to use in college 15 yr. later. Find the amount to which that $1500 will grow in 15 yr. if it earns interest at 4% compounded semiannually. $2717.04 How much interest would be lost if that money were kept in a secret place at home? $1217.04

Discuss the explanation in ex. 1 step by step. Be sure the students understand this work before they do other exercises.

Provide a general review of problem solving and computational skills.

Mixed Practice

Find quotients to the nearest tenth:

1. $2965 \div 46$ 64.5	$73.38 \div 4.61$ 15.9	$12.94 \div .037$ 349.7
2. $1000 \div 27$ 37.0	$297.5 \div 58.7$ 5.1	$36.65 \div .126$ 290.9
3. $4837 \div 19$ 254.6	$7.737 \div .225$ 34.4	$1.931 \div .022$ 87.8

Find the answers to the nearest whole cent:

4. 5% of \$9.50 \$.48	2.4% of \$278 \$6.67	$\frac{1}{2}$% of \$96.60 \$.48
5. 2% of \$4.80 \$.10	6.5% of \$155 \$10.08	$\frac{1}{4}$% of \$75.75 \$.19
6. 3% of \$8.70 \$.26	1.7% of \$480 \$8.16	$\frac{3}{4}$% of \$95.00 \$.71

Study each exercise carefully before working it: Emphasize.

7. Add: $6\frac{1}{2}$, $10\frac{3}{4}$, $9\frac{1}{8}$, $15\frac{7}{8}$. Then change the fractions to decimals and add. Are the two results equal? $42\frac{1}{4}$ and 42.25; yes

8. Find the area of a trapezoid with bases of $8\frac{1}{2}$ ft. and 11 ft. and a height of 8 ft. 78 sq. ft.

9. Find the interest on \$450 at 6% from June 15 to July 9. \$1.80

10. Find the amount to which \$4000 will increase in one year at 2% compounded semiannually. \$4080.40

11. A share of stock is bought at $47\frac{1}{2}$; during the year dividends amounting to \$2.50 are paid on it. Find, to the nearest tenth of 1%, the rate of income on this stock. 5.3%

12. Which team has the better standing, one that wins 35 games and loses 19, (.648) or one that wins 37 games and loses 21? (.638) Give each standing to three decimal places. First team

13. Find, to the nearest whole per cent, the per cent of discount when a desk marked \$38 is sold for \$29. 24%

14. Find the amount to which \$1000 will grow in 20 yr. at 4% compounded annually; \$2191.12 at 4% compounded semiannually. \$2208.04 Explain why these two amounts are different and find their difference. \$16.92 Use the table on page 212.

Through conferences with the students, try to determine if errors are caused by computations, lack of understanding, lack of information, and so on. Be sure students are aware of their own needs. Plan specialized review or reteaching as needed.

Present a set of improvement tests in multiplication. Check the papers carefully and note the kinds of errors. Have the students determine and record their scores (page 48).

Improving by Practice

Multiplication Test 4a. Time: 4 min. after copying.

1.	7538 × 5803	4962 × 3958	5817 × 7430	4069 × 4762
	43,743,014	19,639,596	43,220,310	19,376,578

4

Multiplication Test 4b. Time: 4 min. after copying.

2.	5708 × 1926	3523 × 4279	5297 × 7180	1573 × 5036
	10,993,608	15,074,917	38,032,460	7,921,628

4

Multiplication Test 4c. Time: 4 min. after copying.

3.	9648 × 9276	7308 × 6485	9576 × 7340	1492 × 6024
	89,494,848	47,392,380	70,287,840	8,987,808

4

The Language of Arithmetic

Show that you know what each word or group of words means by using it in a sentence or by giving an example of it:

4. dividend
5. coupon
6. bond
7. interest
8. principal
9. mortgage
10. brokerage
11. corporation
12. below par
13. market price
14. investment
15. stockholder
16. par value
17. small loan
18. share of stock
19. stock broker
20. semiannually
21. preferred stock
22. United States Savings Bonds, Series E
23. redemption value of a Series E bond
24. repaying a loan on a house
25. savings and loan association
26. budgeting an income
27. finding the number of days between dates

If students show a lack of understanding of terms in ex. 4-27, use the developmental material in the text, relating to these terms, in re-teaching. Have them make own lists of terms they need to study.

Present and discuss plans for savings. Point out that the per cent of income saved depends on a person's responsibilities as well as upon income. The per cents in ex. 1 should be presented as goals.

Planned Savings

1. Since it is important for a person to save a part of his income each year, savings should be one of the items in every budget plan. The table at the left shows, under average conditions, what per cent of the annual income, after income taxes, is suggested for various forms of saving. Stress. For example, if Mr. Gale earns $3000 a year after taxes, he should save about 8% of that amount. $240 How many dollars should he save a year? How much should be saved a year by persons having each of the other incomes in the table? (1) $360; (2) $504; (3) $720; (4) $810

	Annual Income	Annual Savings
	$3000	8%
(1)	$3600	10%
(2)	$4200	12%
(3)	$4800	15%
(4)	$5400	15%

2. Mr. Ford earns $85 a week, after taxes. Find his annual income (1) and the amount of money he should save a year, (2) according to the table in ex. 1. (1) $4420; (2) about $530

3. Out of his savings, Mr. Ford deposits $25 a month in a savings and loan association. Besides this, how much more should he save a year in other ways? $230

4. Premiums paid for life insurance are considered to be savings. Out of his annual savings, Mr. Ford pays $115 a year for life insurance; the rest of his savings are invested in United States Savings Bonds at $37.50 each. How many savings bonds can he buy during the year? 3, $2.50 over

5. Mr. Baker has an annual income of $5000. Find the amount the Baker family should plan to save each year. $750

6. Mr. Baker uses $\frac{1}{4}$ of his savings to buy United States Savings Bonds (1) and $\frac{1}{3}$ for life insurance premiums; (2) he puts the rest in a savings bank. (3) How much does Mr. Baker save in each of these ways? (1) $187.50; (2) $250; (3) $312.50

7. Make a circle graph of Mr. Baker's savings, showing how much is allotted to each type of savings. See Guide.

8. What part of the monthly payment on the loan for buying a house would be considered savings? Amount used to reduce debt.

Have the students notice also that the per cents given in ex. 1 might be too large or too small for different people. Explain how a specific person can set his own goal. After ex. 1-8 are discussed, have the students notice the various ways money can be saved.

Provide practice in solving problems which involve previous work.

Review Problems

1. Mr. Mason sold 300 shares of stock at $72\frac{3}{4}$. The brokerage per 100 shares was \$46.28. How much did Mr. Mason receive from the sale of the stock? Disregard taxes. \$21,686.16

2. Find, to the nearest tenth of 1%, the rate of income on stock purchased at 77 and paying an annual dividend of \$4.75 per share. Do not include brokerage. 6.2%

3. Mr. Swift bought a house for \$15,500. A few months later his business office was moved to another city and he sold the house for \$16,800. What was the per cent of gain, to the nearest tenth of 1%, that Mr. Swift made when he sold the house? 8.4%

4. Find the amount to which \$1500 will grow in 10 yr. at 3% compounded semiannually. Use the table on page 212. \$2020.29

5. Mr. Ware bought spring coats for \$33 each. The cost of the coats was 60% of the selling price. When the season was over Mr. Ware had 1 coat of this kind left which he sold at a discount of 30%. Did he sell the coat for more or less than it cost? \$5.50 more If his expenses represent 32% of the regular selling price, did Mr. Ware gain or lose when he sold the coat at a discount? Lost How much? \$12.10

6. The cost of sending a telegram to Westfield is \$1.60 for the first 15 words and 7.5¢ for each extra word. There is also a 10% tax on the cost of the telegram. Find the total cost of sending a telegram of 30 words to Westfield. \$3.00

7. Before Miss Bird left for Mexico last summer she bought travelers checks so that her money would be safe. She had eight 50-dollar checks, ten 20-dollar checks, and fifteen 10-dollar checks. The fee charged for issuing these checks was 1% of the value of the checks. Find the total amount Miss Bird paid when she bought the checks. \$757.50

8. A broker bought 200 shares of stock for Mr. March at $43\frac{1}{2}$. The brokerage per 100 shares was \$40.75. What did the stock cost Mr. March? Omit taxes. \$8781.50

After checking the papers and noting the kinds of errors, return them to the students so that they can find and correct their mistakes. Let volunteers show solutions at the board. Some students may need reteaching or further review of a particular topic.

See the Guide for more detailed answers to ex. 1-5.

Chapter Review

1. The Star Corporation manufactures airplanes in large numbers. Can you tell why a corporation rather than an individual owns this airplane business? Amount of capital required is more than single individual would care to risk.
2. What advantages has preferred stock over common stock? (1) Why is the market value of the preferred stock of a company sometimes less than that of the common stock of the company? (2) (1) Dividends must be paid first; fixed rate of interest; preference in liquidation payments; (2) see Guide (page 194, ex. 24).
3. Why are United States Savings Bonds, Series E, a good investment for the average wage earner? They are backed by the government and pay higher rate of interest than paid by most banks.
4. Which bond would you consider the safer investment, a bond of a first-class corporation paying 4% or a bond of a less dependable corporation paying 6%? Bond of first-class corporation paying 4%
5. Which is usually the safer investment, a bond or the common stock of the same company? A bond, since it carries definite promise to pay fixed amount of interest
6. Name some advantages of investing money in the purchase of a home. Also name some disadvantages. Answers will vary.
7. Tell the difference in meaning between simple interest and compound interest. Simple interest is computed only on the principal; compound interest is computed on sum of principal and interest.
8. What is the maturity value of a United States Savings Bond, Series E, that can be purchased for $75? $100
9. A personal loan company lends money which can be paid back in monthly installments with interest at 2% a month on the unpaid balance. What does this mean?
10. Without using the compound interest table, find the amount to which $500 will grow in 3 yr. at 6% if the interest is compounded semiannually. Disregard the cents in the principal when figuring the interest. $596.96; see Guide for computations.
11. Mr. Adams bought 100 shares of General Rubber at $92\frac{1}{8}$ with a brokerage charge of $48.21. Find the cost of these shares of stock, including brokerage. $9260.71
12. How many times a year is the interest on a bond paid? Usually twice How many times a year are dividends on stock usually paid? Usually four times

Students' answers to ex. 1-7 and ex. 9 will show how well they understand the concepts and terms developed in this chapter. Check these and the solutions for ex. 8, 10-12 carefully. Reteach as needed, stressing meanings.

A Problem Test

1. Sam wants to save \$37.50 in 15 wk. What is the average amount he should save per week? \$2.50 The first week he saved \$2.75, the next three weeks he saved \$2.50 a week, the fifth week he saved \$2.95. So far, are Sam's savings (\$13.20) equivalent to the average amount he needs to save? \$2.64; yes What average amount per week must he save for the next 10 wk.? \$2.43

2. Find the interest on \$90 at 6% from Apr. 9 to Aug. 1. \$1.71

3. Mr. Martin sold 62 doz. eggs at 45¢ a dozen, 42 doz. at 48¢ a dozen, and 46 doz. at 51¢ a dozen. Find, to the nearest cent, the average price per dozen he received for 150 doz. \$.48

4. The population of Parkersfield increased from 3580 to 4296 last year. Find the per cent of increase. 20%

5. An airplane travels 285 mi. in 45 min. At that rate, find the speed of the airplane in miles per hour. 380

6. The installment price of a radio is \$19 down and \$5.00 a month for 9 mo. The cash price is \$56.95. What per cent higher than the cash price is the installment price? 12% Find the answer to the nearest whole per cent.

7. One year the total weight of fish canned in this country was 792,194,000 lb., of which 187,769,000 lb. were salmon. Round off these numbers to the nearest million (1) and find, to the nearest whole per cent, what per cent of the total was salmon. (2)
(1) 792 million, 188 million; (2) 24%

8. The weight of a gallon of crude petroleum is 7.3 lb. Find the weight of a barrel of petroleum containing 42 gal. 306.6 lb.

9. At 8 oranges for 45¢, find the cost of 6 doz. oranges. \$4.05

10. Busses leave Newton for Silver Lake every hour at 43 min. after the hour. If Bill and Joe reach Newton by train at 6:55, how long will they have to wait for the next bus? 48 min.

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Instruct the students to read the problems carefully before beginning their computations. Lead a class discussion of solutions. Confer with students who had errors to help them determine causes.

Present a diagnostic test of skills taught in Chapter 6, with practice-page references.

How Much Have You Learned?

If you miss more than one example in a group, turn to the Practice Pages for that group. Be sure difficulties are cleared up before assigning practice pages.

Find the total cost of buying these shares of stock. The brokerage for 100 shares is given:

	Shares	Stock	Cost	Brokerage	Practice Pages
1.	100	Atlas Gas	$13\frac{1}{2}$	$20.50 $1370.50	191
2.	300	Belt R.R.	43	40.50 $13,021.50	191
3.	500	Penn Bank	$56\frac{1}{4}$	44.63 $28,348.15	191
4.	400	Ogden Oil	108	49.80 $43,399.20	191

Find, to the nearest tenth of 1%, the rate of income on each stock. Omit brokerage:

	Cost	Dividend	Cost	Dividend	Practice Pages
5.	80	$5.00 6.3%	44	$2.75 6.3%	194
6.	$24\frac{1}{2}$	$1.40 5.7%	$85\frac{1}{4}$	$7.00 8.2%	194
7.	115	$9.00 7.8%	155	$10.00 6.5%	194

Using the table on page 208, find the monthly payment for each of the following loans:

8. $13,000 to be repaid in 20 yr., interest at $5\frac{1}{2}$% $89.44 209
9. $19,500 to be repaid in 25 yr., interest at 6% $125.78 209

Using the compound interest table on page 212, find the amount to which each sum will grow if compounded semiannually at the given rate:

			Practice Pages
10.	$500, 5 yr., 3% $580.27	$600, 8 yr., 6% $962.83	213
11.	$850, 4 yr., $1\frac{1}{2}$% $902.36	$900, 12 yr., 4% $1447.60	213
12.	$400, 8 yr., 4% $549.12	$750, 10 yr., 3% $1010.15	213

Check the papers carefully and analyze any errors. Return the papers so that the students can find and correct their mistakes. If desired, review the computation of brokerage (page 190) by have the students show how it is found for 100 shares in ex. 1-4.

See the Guide for the specific aims of Chapter 7.

Chapter 7

Geometric Figures with Three Dimensions

1. You have studied rectangles, triangles, and other flat geometric figures that have only **two** dimensions, **length** and **width.** You have learned how to find the area enclosed by such figures and how to find the distance around them. For example, you know how to find the area and the perimeter of a rectangle.

2. In this chapter you will study **prisms, cylinders, pyramids, cones,** and **spheres.** These geometric figures have **three** dimensions: **length, width,** and **height.** The third dimension, height, is sometimes called **depth** or **thickness.**

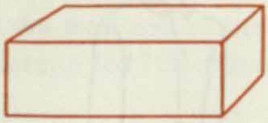
3. You will learn how to find the **volume** enclosed by geometric figures with three dimensions, and how to find the **surface area** of such figures. The volume of a box tells you how much it will hold. The surface area of a box tells you how much material, approximately, was needed to make it.

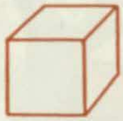
When discussing ex. 2, try to have available models of the different figures mentioned. Also have students make models of some of them from heavy paper.

Reteach finding the volume of a rectangular prism, including the use of the formula for volume (pages 222-224).

Rectangular Prisms

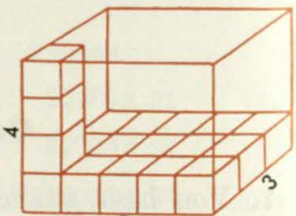
1. The figures shown below are shaped like boxes or blocks. They are called **rectangular prisms.** Each one has 6 **surfaces,** or **faces,** and all the faces are rectangles or squares. Stress. If all 6 faces are squares, that rectangular prism is called a **cube.** Emphasize.


Rectangular Prism


Cube

2. A small cube 1 in. on each edge contains 1 **cubic inch;** its **volume** is 1 cu. in. What is the volume of a cube that is 1 ft. on each edge? 1 cu. ft. of a cube that is 1 yd. on each edge? 1 cu. yd. The cubic inch, the cubic foot, and the cubic yard are **units of measure** used when finding volumes. Emphasize.

3. To find the volume of a box that is 5 in. long, 3 in. wide, and 4 in. high, you need to know how many 1-inch cubes it will hold. It will take a layer of 15 cubes to cover the bottom of the box and 4 layers like this, making 4×15 cubes, to fill the box. Since each cube is 1 cu. in., the volume is 4×15 cu. in., or 60 cu. in. You can find the volume of the box quickly by multiplying the length, width, and height together, like this: $5 \times 3 \times 4 = 60$.



4. The rule for finding the volume of a rectangular prism is:

Volume of rectangular prism = length × width × height

The formula is: $V = lwh$

Emphasize that lwh means $l \times w \times h$.

Find the volumes of boxes having these dimensions. First express all dimensions in inches, or all in feet:

5. 6 in. × 5 in. × $\frac{1}{2}$ ft. 180 cu. in.

6. 1 ft. × 9 in. × 1 in. 108 cu. in.

7. 6 ft. × 2 ft. × 4 in. 4 cu. ft.

8. 9 in. × 6 in. × 1 ft. 2 in. 756 cu. in.

9. 15 ft. × 12 ft. × 3 ft. 6 in. 630 cu. ft.

10. 12 ft. × 18 ft. × 9 ft. 3 in. 1998 cu. ft.

After doing the experiment in ex. 3, ask the students to suggest a quick way of finding volume. Have them measure dimensions of boxes of various sizes and compute volumes.

How Many Boxes in a Case?

1. The Ball Company makes slippers. In the shipping room of this company, large boxes, called shipping cases, are filled with boxes of slippers. Each shipping case is 3 ft. long, Stress. 2 ft. wide, and $2\frac{1}{4}$ ft. high. These measurements are made on the **inside** of the case. Find the **volume,** or **capacity,** of each shipping case in cubic feet. $13\frac{1}{2}$

2. The small boxes that are put in a shipping case each contain a pair of slippers. The **outside** dimensions of these small boxes are 9 in. by 6 in. by 3 in. How many cubic feet of space does each small box need? $\frac{3}{32}$

▶ First change each dimension to feet and then find the volume of each small box in cubic feet.

3. How many boxes of slippers does a shipping case hold? 144 What numbers do you divide to find out? $13\frac{1}{2}$ cu. ft. $\div \frac{3}{32}$ cu. ft.

4. Make a drawing to show how the boxes are placed in the shipping case, to form the bottom layer, so that the boxes will entirely cover the bottom of the case. How many boxes are there in this bottom layer? 16 How many layers like this does it take to fill the case? 9 How many boxes are there in the case? 144 Does this answer agree with that for ex. 3? Yes

5. Change the dimensions of the shipping case in ex. 1 to inches and find its volume in cubic inches. 23,328 Next find the volume of each box in ex. 2 in cubic inches. 162 Then divide to find the number of boxes. 144 Does this answer agree with those in ex. 3 and 4? Yes

6. What is the volume of a room that is 15 ft. long, 12 ft. wide, and 10 ft. high? 1800 cu. ft. that is 18 ft. by 12 ft. by 10 ft.? 2160 cu. ft.

Find the volume of each of these boxes in two ways:

	576 cu. in., $\frac{1}{3}$ cu. ft.	1584 cu. in., $\frac{11}{12}$ cu. ft.	23,328 cu. in., $13\frac{1}{2}$ cu. ft.
7.	$1' \times 8'' \times 6''$	$9'' \times 11'' \times 1\frac{1}{3}'$	$27'' \times 18'' \times 4'$
	720 cu. in., $\frac{5}{12}$ cu. ft.	1050 cu. in., $\frac{175}{288}$ cu. ft.	5040 cu. in., $2\frac{11}{12}$ cu. ft.
8.	$3' \times 5'' \times 4''$	$7'' \times 10'' \times 1\frac{1}{4}'$	$15'' \times 16'' \times 1\frac{3}{4}'$
	3456 cu. in., 2 cu. ft.	4320 cu. in., $2\frac{1}{2}$ cu. ft.	4800 cu. in., $2\frac{7}{9}$ cu. ft.
9.	$4' \times 9'' \times 8''$	$9'' \times 16'' \times 2\frac{1}{2}'$	$10'' \times 15'' \times 2\frac{2}{3}'$

Do ex. 1–6 with the class. Have the students read each problem, discuss the solution, and then solve the problem. In ex. 4, be sure they understand how to find the number of boxes in the bottom layer. Assign ex. 7–9 as independent work if there are no difficulties.

Finding Volumes

1. A cube measuring 1 ft. on each edge is how many inches long? 12 how many inches wide? 12 how many inches high? 12 Show that 1 cu. ft. contains 1728 cu. in.

2. A cube measuring 1 yd. on each edge is how many feet long? 3 how many feet wide? 3 how many feet high? 3 How many cubic feet are there in 1 cu. yd.? 27

3. Learn this table of cubic measure: Following should be memorized after understanding is assured.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cu. ft. = 1 cubic yard (cu. yd.)

4. Find the number of cubic feet in 4 cu. yd.; 108 in $\frac{2}{3}$ cu. yd. 18

5. Find the number of cubic inches in 2 cu. ft. 3456

6. How many cubic yards of earth must be removed in excavating a cellar 27 ft. long, 18 ft. wide, and 6 ft. deep? 108

7. The inside dimensions of a cedar chest are 4 ft., $1\frac{1}{2}$ ft., and 2 ft. Find the volume of the chest. 12 cu. ft. Is this volume more or less than that of a box measuring $2\frac{1}{4}$ ft. on each edge? $11\frac{25}{64}$ cu. ft.; volume of chest is more.

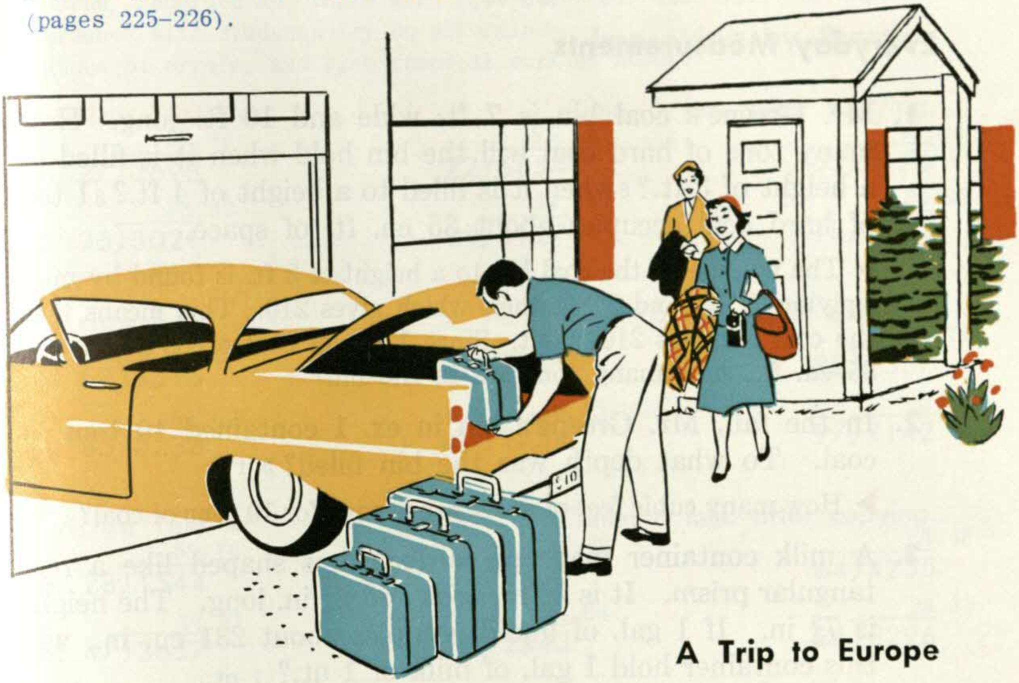
8. A cord of hardwood was stacked in a rectangular pile measuring 8 ft. × 4 ft. × 4 ft. If 1 cu. ft. of this wood weighed 60 lb., how many pounds did the whole cord weigh? 7680

9. Cookies are sold in cardboard boxes 8 in. long, $2\frac{1}{2}$ in. wide, and 2 in. high. How many inches long and wide will a packing case have to be if 12 of these boxes cover the bottom of the case? See Guide. How many inches high will the case have to be to hold 6 layers of boxes? 12

10. The library at Hillside School is 60 ft. long, 50 ft. wide, and 14 ft. high. (42,00 cu. ft.) If 200 cu. ft. of air are allowed for each pupil, what is the largest number of pupils that should use the library at one time? 210

Discuss ex. 1-3 with the class. You might ask some students to make a cubic inch, and a cubic foot, from stiff paper.

Present problems that apply work on finding volumes of rectangular prisms (pages 225–226).



A Trip to Europe

1. Mr. Allen is loading luggage in his car and will take it to the steamship. Mrs. Allen and Mary are sailing for Europe at noon. On such trips, Mrs. Allen used to pack clothing in a steamer trunk measuring $32'' \times 18'' \times 12''$ but now she finds it is much more convenient to use hand luggage instead of a trunk. She says she can take more in the 4 pieces of luggage shown above than she can in the trunk. Find the volume of the trunk in cubic inches. 6912 Then work ex. 2 and tell whether Mrs. Allen is right.

2. The 4 pieces of luggage that Mrs. Allen and Mary are taking on their trip have the following dimensions:

$24'' \times 14'' \times 6''$ 2016 cu. in.	$15'' \times 12'' \times 6''$ 1080 cu. in.
$26'' \times 15'' \times 8''$ 3120 cu. in.	$12'' \times 10'' \times 8''$ 960 cu. in.

Find the volume of each piece in cubic inches. Also find the total volume of all 4 pieces. 7176 cu. in. How does this total volume compare with that of the trunk in ex. 1? Suitcases contain 264 cu. in. more.

3. In ex. 1, change the dimensions of the trunk to feet. Then find the volume of the trunk in cubic feet. 4

Students should be encouraged to do these problems independently. Have volunteers explain solutions at the board. Have others ask questions if they are unsure of any part of this work.

Everyday Measurements

1. Mr. Greene's coal bin is 7 ft. wide and 10 ft. long. How many tons of hard coal will the bin hold when it is filled to a height of 3 ft.? 6 when it is filled to a height of 4 ft.? 8 1 ton of hard coal occupies about 35 cu. ft. of space.

▶ The volume of the coal bin to a height of 3 ft. is found by multiplying 7, 10, and 3 together, which gives 210. This means that the coal occupies 210 cu. ft. Since 1 ton of coal needs a space of 35 cu. ft., how many tons are in the bin?

2. In the fall, Mr. Greene's bin in ex. 1 contained 10 tons of coal. To what depth was the bin filled? 5 ft.

▶ How many cubic feet of space are needed for 10 tons of coal?

3. A milk container made of cardboard is shaped like a rectangular prism. It is $2\frac{3}{4}$ in. wide and $2\frac{3}{4}$ in. long. The height is $7\frac{3}{4}$ in. If 1 gal. of liquid occupies about 231 cu. in., will this container hold 1 gal. of milk or 1 qt.? 1 qt.

4. If a milk container is $2\frac{1}{8}$ in. by $2\frac{1}{8}$ in. by $6\frac{1}{2}$ in., how many cubic inches are there in its volume? $29\frac{45}{128}$ Will this container hold $\frac{1}{2}$ pt. or 1 pt. of milk when it is full? 1 pt.

5. The hippopotamus at the zoo swims in a rectangular tank that is 25 ft. long and 20 ft. wide. The average depth of the water in the tank is $3\frac{1}{2}$ ft. How many gallons of water are there in the tank, if $7\frac{1}{2}$ gal. of water occupy 1 cu. ft. of space? 13,125 Find the cost of the water at 20¢ per 100 cu. ft. \$3.50

In ex. 5 point out that 1 cu. ft. = about $7\frac{1}{2}$ gal. See Guide.

6. Peter has a fish tank that is 18 in. long, 14 in. wide, and 13 in. deep. If the tank is filled with water to a depth of 11 in., how many gallons of water does it contain? 12 Use 1 gal. = 231 cu. in.

7. In ex. 6, find the weight of the water in the tank if it is filled to a depth of 12 in. The weight of 1 cu. ft. of water is $62\frac{1}{2}$ lb. $109\frac{3}{8}$ lb.

8. To work everyday problems like those above, you will find it convenient to use the equivalents given on page 374 of this book. Discuss these with the class.

Have the students read ex. 1 and then let volunteers suggest how it should be solved. Ex. 2–3 may be done in this way also. Then have the students complete ex. 4–7. Lead a class discussion of solutions and clear up difficulties now.

Present a set of improvement tests in division. Tests are to be given, scored, recorded as others were (see pages 87, 125, 165). Through conferences with students try to determine causes of errors, and plan remedial work as needed.

Improving by Practice

Division Test 4a. Time: 6 min. after copying.

1. $27\overline{)2698}$ 99.93　　$72\overline{)6854}$ 95.19　　$53\overline{)2175}$ 41.04

2. $36\overline{)3021}$ 83.92　　$94\overline{)5127}$ 54.54　　$18\overline{)1563}$ 86.83　　6

Division Test 4b. Time: 6 min. after copying.

3. $59\overline{)4315}$ 73.14　　$46\overline{)1983}$ 43.11　　$32\overline{)1337}$ 41.78

4. $85\overline{)5238}$ 61.62　　$61\overline{)2754}$ 45.15　　$97\overline{)2142}$ 22.08　　6

Division Test 4c. Time: 6 min. after copying.

5. $28\overline{)1544}$ 55.14　　$73\overline{)2765}$ 37.88　　$64\overline{)4255}$ 66.48

6. $47\overline{)3657}$ 77.81　　$92\overline{)2543}$ 27.64　　$86\overline{)3296}$ 38.33　　6

To the Pupil. Find quotients to the nearest hundredth.

Column Addition

Present a review of column addition.

Add and check the work: Emphasize the importance of checking.

7.	337	125	232	195	913	615	372
	452	463	419	726	653	186	540
	310	512	385	458	184	857	285
	278	181	243	143	465	279	399
	345	245	754	467	520	367	214
	113	472	188	569	131	755	140
	460	233	229	147	312	286	373
	257	633	495	322	768	158	214
	326	144	353	240	175	436	197
	129	389	293	113	448	239	199
	3007	3397	3591	3380	4569	4178	2933

8. 28 + 13 + 19 + 40 + 83 + 72 + 73 + 56 + 70 + 79　533

9. 56 + 31 + 13 + 72 + 92 + 73 + 12 + 89 + 49 + 67　554

10. 17 + 42 + 61 + 12 + 11 + 65 + 37 + 83 + 12 + 62　402

If students have difficulty with ex. 7-10, have them do computations aloud so you can judge if adding by endings, facts, carrying, and so on, cause difficulties. Plan remedial work and/or drill as needed.

Present problems that apply work on finding the volume of a rectangular prism.

The Swimming Pool

1. In the Elm City park there is a swimming pool where the boys and girls swim during summer vacation. The pool is 60 ft. long and 50 ft. wide. The average depth of the water in the pool is $5\frac{1}{2}$ ft. Find the number of cubic feet of water in the pool. 16,500

2. In Elm City water costs 12¢ per 100 cu. ft. What does it cost to fill the pool in ex. 1 to the depth of $5\frac{1}{2}$ ft.? $19.80

3. How much more would the water cost if the pool in ex. 1 were filled to a depth of 5 ft. 9 in. instead of $5\frac{1}{2}$ ft.? $.90

4. One day during a heavy rainstorm 2 in. of rain fell and increased the depth of the water in the pool by 2 in. What would these extra 2 in. of water cost if purchased at the city rate of 12¢ per 100 cu. ft.? $.60

5. When the pool is filled to a depth of $5\frac{1}{2}$ ft., as it was in ex. 1, how many gallons of water does it contain? 1 cu. ft. of water equals $7\frac{1}{2}$ gal. 123,750

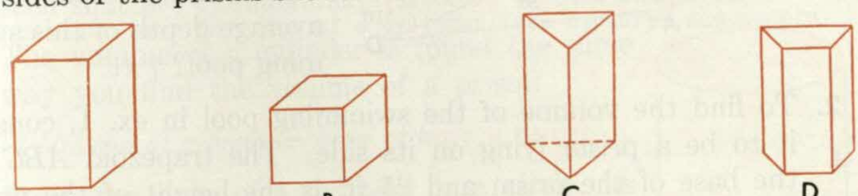
6. When the pool is emptied, the water flows out at the rate of 150 gal. per minute. How long does it take to empty the pool if it contains the number of gallons in ex. 5? $13\frac{3}{4}$ hr.

In ex. 1 lead the students to see that the amount of water in a pool equals the volume of the rectangular prism which has the same length and width as the pool, and a height equal to the average depth of the water. Then have them complete the solution and do ex. 2–6.

Present a description of prisms. Teach and use the formula for finding the volume of any prism (pages 229-230).

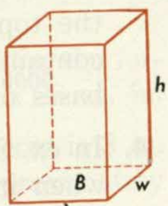
Volume of a Prism

1. You have learned that rectangular figures with three dimensions are called **rectangular prisms.** In addition to rectangular prisms, there are many other kinds of prisms. In a **prism,** the two **bases** are exactly alike and are parallel. The sides of the prisms shown below are rectangles. Emphasize.



2. Prisms take their names from the shape of their bases. Thus a prism whose bases are rectangles is called a **rectangular prism;** one whose bases are triangles is called a **triangular prism.** What kind of prism is your schoolroom? Stress.

3. To find the volume of the rectangular prism at the right, you use the rule:



Volume of prism = length × width × height

You can see that the length times the width is the same as the area of the base of the prism. So you have:

Volume of prism = area of base × height

Emphasize that B represents the area of the base.

If you let B represent the area of the base, you can write this as the formula

$$V = Bh$$

You may use this formula to find the volume of **any** prism, even when the bases are not rectangles. Emphasize.

4. Find the volume of a triangular prism if the area of its base is 20 sq. in. and its height is 7 in. 140 cu. in.

▶ Using the formula $V = Bh$, you get $V = 20 \times 7$, or 140. What is the volume of the prism?

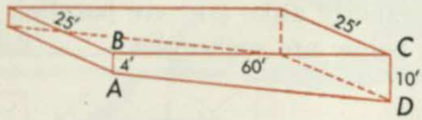
5. Find the volume of a hexagonal prism if the area of its base is 45 sq. ft. and its height is 12 ft. 540 cu. ft.

Ask the students to give the name of prism D in ex. 1. Using various models, demonstrate that bases of prisms are exactly alike and parallel. Point out and show that the side on which a prism lies or stands is not always considered to be the base.

Supplement the work on this page with similar problems involving various prisms.

Finding the Volume of a Pool

1. Most swimming pools are deeper at one end than they are at the other end. The picture below shows a swimming pool that is 10 ft. deep at one end and 4 ft. deep at the other end. What is the average depth of this swimming pool? 7 ft.



2. To find the volume of the swimming pool in ex. 1, consider it to be a prism lying on its side. The trapezoid *ABCD* is the base of the prism and 25 ft. is the height of the prism. First find the area of the trapezoid. 420 sq. ft. What will you use for the bases and the height of the trapezoid? Bases = 4′ and 10′, height = 60′ After finding the area of the trapezoid, how will you find the volume of the prism (pool)? $V = Bh$ How many cubic feet does the pool contain? 10,500

3. If the swimming pool in ex. 2 is filled to within 1 ft. of the top of the pool, how many cubic feet of water will it contain? 9000 In this case, what dimensions will you use for the bases of the trapezoid that forms the base of the prism? Bases = 3′ and 9′, height = 60′

4. In ex. 3, what is the average depth of the water in this pool when it is filled to within 1 ft. of the top? 6 ft.

5. In ex. 3, how many gallons of water does the swimming pool contain? 67,500 1 cu. ft. = $7\frac{1}{2}$ gal.

6. A swimming pool is 120 ft. long and 80 ft. wide. The water in the pool is 8 ft. deep at one end and 2 ft. deep at the other end. How many cubic feet of water does this pool contain? 48,000 How many gallons of water is that? 360,000

7. One winter the ice on Blue Lake was 10 in. thick. The ice on the lake was cut over a rectangular surface area 80 ft. by 60 ft. What was the total volume of the ice that was cut? 4000 cu. ft. How much did this ice weigh if 1 cu. ft. of ice weighs $57\frac{1}{2}$ lb.? 230,000 lb. How many tons of ice were obtained? 115

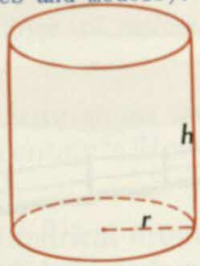
8. In ex. 7, how many tons of ice would have been obtained if the ice had been cut over a surface area 120 ft. by 100 ft.? $287\frac{1}{2}$

Ex. 1–2 need careful explanation and discussion. Be sure the students understand average depth and which side is considered the base of the prism. Discuss the solutions of ex. 3–8 carefully.

Develop the formula for the volume of a cylinder and apply it in problem solving (pages 231-233).

Volume of a Cylinder

1. The geometric figure shown below is called a **cylinder.** The bases of a cylinder are equal circles and the height is perpendicular to the bases. Canned tomatoes are packed in cans that are cylinders. Name some other objects you have seen that are cylinders. Have students prepare an exhibit of everyday articles which have the form of a cylinder (use pictures and models).



2. The volume of a cylinder is found the same way you find the volume of a prism:

   Volume of cylinder = area of base × height

   Since the base of a cylinder is a circle, its area is πr^2. If you substitute πr^2 for the **area of the base** in the above rule, you get this formula:

   $$V = \pi r^2 h$$

3. What is the volume of a cylinder with a height of 7 in. and a diameter of 4 in.? Use $\pi = 3\frac{1}{7}$.

   ▶ Since the diameter is 4 in., the radius is 2 in. Using the formula, you get $V = 3\frac{1}{7} \times 2 \times 2 \times 7$. The volume is 88 cu. in.

4. Find the volume of a cylindrical jar whose diameter is 3 in. and whose height is 4 in.? Would this jar hold approximately 1 pt., 1 qt., or 1 gal.? 1 pt 1 gal. = 231 cu. in.
   $28\frac{2}{7}$ cu. in.

5. A cylindrical hot water tank with a diameter of 16 in. is 60 in. tall. How many gallons of water, to the nearest whole gallon, will this tank hold? 52 Use $\pi = 3.14$.

Find the volume of these cylinders. Use $\pi = 3.14$:

	Area of Base	Height			Radius of Base	Height	
6.	9 sq. in.	3 in.	27 cu. in.	11.	4 in.	6 in.	301.44 cu. in.
7.	14 sq. in.	10 in.	140 cu. in.	12.	2 ft.	9 ft.	113.04 cu. ft.
8.	15 sq. ft.	4 ft.	60 cu. ft.	13.	2 in.	3 in.	37.68 cu. in.
9.	12 sq. ft.	$3\frac{1}{2}$ ft.	42 cu. ft.	14.	7 ft.	$5\frac{1}{2}$ ft.	846.23 cu. ft.
10.	20 sq. ft.	6 ft.	120 cu. ft.	15.	10 in.	7 in.	2198 cu. in.

Have the students notice that the radius is given in ex. 11-15. Give them practice in rounding off answers to a given precision.

Silos

1. On modern farms finely cut green corn and other green fodder is kept in tall cylinders like the one shown above. These cylinders are called **silos** and the food they contain is called **silage.** This food is fed to cows, sheep, and other livestock during the winter in place of grass. Silos are made in the shape of a cylinder to avoid corners in which the silage is apt to spoil.

2. The silo above has an inside diameter of 14 ft. and a height of 44 ft. How many cubic feet of silage will this silo hold when full? 6776 How many cubic feet of silage will it hold when filled to a height of 30 ft.? 4620 of 20 ft.? 3080 10 ft.? 1540 Use $\pi = 3\frac{1}{7}$.

3. When the silo in ex. 2 is full of silage, for how many days will this amount of silage feed 28 cows if each cow gets an average of 1 cu. ft. of silage per day? 242

4. Suppose a silo is built that has an inside diameter of 7 ft. and a height of 22 ft. These dimensions are half as long as those of the silo in ex. 2. Will this smaller silo hold half as much as the one in ex. 2? 847 cu. ft.; no The volume of this smaller silo is what part of the volume of the silo in ex. 2? $\frac{1}{8}$

Solutions of these problems should be discussed and explained thoroughly.

Let volunteers explain their answers for ex. 1-6. Urge pupils to ask questions if they have any difficulties. All should have good understanding before proceeding further.

Other Cylinders

Find each answer to the nearest whole number. Use $\pi = 3\frac{1}{7}$:

1. A cylindrical well 40 ft. deep is $\frac{3}{4}$ filled with water. If the inside diameter of the well is 3 ft., how many gallons of water are there in the well? 1 cu. ft. = $7\frac{1}{2}$ gal. About 1591

2. A cylindrical water tank on top of a large building is 8 ft. in diameter and 14 ft. high. How many gallons of water will it hold when it is completely filled? 5280

3. A cylindrical pan $10\frac{1}{2}$ in. in diameter is filled with water to a depth of 4 in. How many gallons does it contain? $1\frac{1}{2}$ How many quarts? 6 1 gal. = 231 cu. in.

4. How many gallons of oil can be stored in a cylindrical drum whose diameter is 21 in. and whose height is 2 ft. 4 in.? 42 in a drum whose diameter is 21 in. and whose height is 4 ft.? 72

5. A milk truck has a cylindrical tank 4 ft. in diameter and 10 ft. long. How many gallons of milk will it hold? About 943

6. A railway tank car has a cylindrical tank that is 30 ft. long and 5 ft. in diameter. How many gallons of oil will it hold? About 4420

Develop the formula for finding the area of a curved surface of a cylinder and give problems in finding such areas.

Area of Surface of Cylinder

1. A tomato can is a cylinder. The **curved surface** of the can is the surface that is covered by the label. If you cut the label vertically and carefully remove it from the can, the label can be spread out to form a rectangle that has the same area as the curved surface of the can. The height of the rectangle equals the height of the can; the base of the rectangle equals the circumference of the can. Thus, the area of the curved surface (which equals the area of the rectangle) equals the circumference of the base of the can times the height.

Area of curved surface = circumference of base × height

Letting S stand for the **area of the curved surface** and substituting $2\pi r$ for the circumference, you get the formula

$$S = 2\pi rh$$

2. A can of pears 5 in. high has a radius of 2 in. Find the area of the curved surface of the can. $62\frac{6}{7}$ sq. in. Use $\pi = 3\frac{1}{7}$.

3. To get the **total** area of the surface of the can in ex. 2, you must add the areas of the circular bases of the can to the area of the curved surface. Do this. What is the total area of the can? 88 sq. in.

4. How many gallons of paint will it take to put 2 coats of paint on the curved surface of the silo in ex. 2 on page 232 if 1 gal. of paint covers 400 sq. ft. with two coats? About 5 Give the answer in whole gallons.

5. Find the cost of the paint in ex. 4. The paint sells at \$2.98 a gallon, but if 5 or more gallons are purchased at the same time, the price is \$2.88 a gallon. \$14.40

By the formula, find the curved surfaces of these cylinders:

6. $c = 14$ ft., $h = 6$ ft. 84 sq. ft.

7. $c = 73$ in., $h = 30$ in. 2190 sq. in.

8. $d = 35$ in., $h = 9$ in. 990 sq. in.

9. $r = 4$ in., $h = 25$ in. $628\frac{4}{7}$ sq. in.

10. $d = 8$ ft., $h = 20$ ft. $502\frac{6}{7}$ sq. ft.

11. $r = 7$ in., $h = 31$ in. 1364 sq. in.

Perform the experiment described in ex. 1 (or a similar one). Lead the students to tell you what the height and the base of the rectangle equal. In ex. 3 be sure they understand the difference between the area of a curved surface and the total area of the surface of a cylinder.

Provide practice on the four fundamental operations with whole numbers, fractions, and decimals, and on percentage.

Mixed Practice

Add and check the work: Emphasize the importance of checking all work.

1.	63468	46829	$23\frac{3}{4}$	$12\frac{7}{10}$	6.125
	25245	84354	$17\frac{7}{8}$	$10\frac{1}{2}$	8.750
	98979	29578	$10\frac{9}{16}$	$2\frac{3}{5}$	2.500
	11696	54987	$38\frac{1}{2}$	16	9.875
	37864	14626	$25\frac{5}{8}$	$8\frac{9}{10}$	4.375
	237,252	230,374	$116\frac{5}{16}$	$50\frac{7}{10}$	31.625

Subtract and check the work:

2.	92024	70000	$38\frac{3}{4}$	$50\frac{9}{10}$	\$65.00
	63078	42758	$24\frac{5}{6}$	$45\frac{1}{2}$	48.75
	28,946	27,242	$13\frac{11}{12}$	$5\frac{2}{5}$	\$16.25

Multiply or divide:

3. $10 \times 6\frac{3}{4}$ $67\frac{1}{2}$ $\frac{5}{6} \times \frac{3}{10}$ $\frac{1}{4}$ $7\frac{1}{2} \div 2\frac{1}{4}$ $3\frac{1}{3}$ $\frac{7}{8} \times 11$ $9\frac{5}{8}$

4. $12 \div 2\frac{1}{4}$ $5\frac{1}{3}$ $\frac{3}{4} \div \frac{1}{8}$ 6 $\frac{15}{16} \times 6\frac{2}{3}$ $6\frac{1}{4}$ $9 \times \frac{7}{12}$ $5\frac{1}{4}$

5. $3\frac{1}{2} \times 1\frac{1}{4}$ $4\frac{3}{8}$ $9 \div \frac{2}{3}$ $13\frac{1}{2}$ $9\frac{1}{3} \div 1\frac{1}{6}$ 8 $\frac{5}{8} \div 2$ $\frac{5}{16}$

Divide. Find answers to the nearest tenth:

6. $2853 \div 35$ 81.5 $1774 \div 2.16$ 821.3 $1.527 \div .07$ 21.8

7. $19.8 \div 29$ .7 $386.9 \div 1.57$ 246.4 $46.82 \div .83$ 56.4

Find the answers to the nearest cent:

8. 28% of \$9.80 \$2.74 215% of \$285 \$612.75 $\frac{1}{2}$ of 1% of \$125 \$.63

9. .6% of \$825 \$4.95 4.5% of \$195 \$8.78 $\frac{3}{4}$ of 1% of \$287 \$2.15

Find, to the nearest tenth of 1%, what per cent the first number is of the second number:

10. 12, 87 13.8% 12, 285 4.2% 6, 796 .8% 328, 476 68.9%

11. 28, 93 30.1% 91, 846 10.8% 4, 821 .5% 125, 193 64.8%

12. 72, 88 81.8% 53, 184 28.8% 9, 367 2.5% 254, 628 40.4%

Before assigning this practice, review the concepts and procedures related to these examples. These include: finding the least common denominator, using a trial divisor in long division, placing the decimal point in the quotient, and so on.

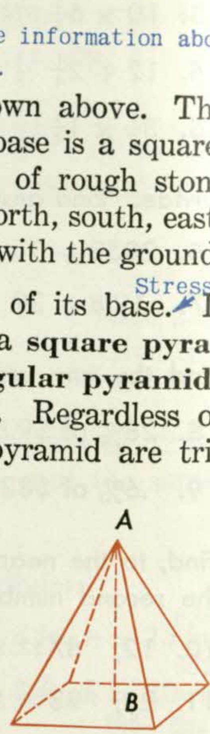
Define the various kinds of pyramids. Develop the formula for the volume of a pyramid and use the formula in problem solving (pages 236–237).

Studying Pyramids Have the students look up more information about Egyptian pyramids and report their findings to class.

1. One of the great pyramids of Egypt is shown above. The sides of this pyramid are triangles and its base is a square. The Egyptian pyramids were usually built of rough stone blocks. Their triangular sides face directly north, south, east, and west, and slope at an angle of about 50° with the ground.

2. A pyramid takes its name from the shape of its base. Stress. If the base is a square, the pyramid is called a **square pyramid;** if the base is a triangle, it is a **triangular pyramid;** if a hexagon, it is a **hexagonal pyramid.** Regardless of the shape of its base, the sides of every pyramid are triangles. Emphasize. Tell where you have seen pyramids.

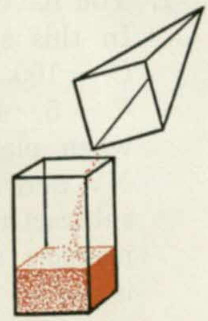
3. In the figure at the right, point A is called Stress. the **vertex** of the pyramid. AB, which is the **perpendicular** distance from the vertex to the base, is the **height** of the pyramid. Is the height AB longer than or shorter than one of the slanting edges of the pyramid? Shorter



In discussing ex. 1–2, emphasize that the sides of every pyramid are triangles. Review the meaning of vertex before doing ex. 3. Then emphasize that the height is the perpendicular distance from the vertex to the base.

Perform the experiment in ex. 1. Make figures out of cardboard, with the same square bases and the same heights. Make a pyramid first in order to make the heights equal.

1. Try this experiment using two containers having the shapes of a pyramid and of a prism with identical bases and heights. Fill the pyramid with sand and pour the sand into the prism. When you have emptied the sand into the prism, the prism will be just $\frac{1}{3}$ full. The volume of the pyramid is $\frac{1}{3}$ of the volume of the prism. Since the volume of the prism equals the area of the base times the height, you have:



Volume of pyramid $= \frac{1}{3} \times$ *area of base* $\times$ *height*

or $V = \frac{1}{3}Bh$

2. Find the volume of a pyramid whose base has an area of 24 sq. in. and whose height is 8 in. 64 cu. in.
3. The Egyptian pyramid shown on page 236 is 461 ft. high. Its square base measures 746 ft. on a side. What is the volume of this pyramid in cubic feet? in cubic yards? $85,517,958\frac{2}{3}$ $3,167,331\frac{65}{81}$
4. The roof of the square house on Elm Street is shaped like a pyramid. The base of this pyramid is a square measuring 30 ft. on each side. The height of the attic from the floor to the vertex of the pyramid is 15 ft. Find the volume of the attic. 4500 cu. ft.
5. In the fall a farmer sold his apples at a road stand. To attract attention, some of his best apples were put in a pile shaped like a pyramid. The pyramid was 4 ft. high and had a square base measuring 5 ft. on each side. ($33\frac{1}{3}$ cu. ft.) About how many bushels were there in the pile if 1 bu. = $1\frac{1}{4}$ cu. ft.? $26\frac{2}{3}$

Find the volume of each pyramid:

	Base of Pyramid	Height	
6.	Rectangle: 14 in. $\times$ 11 in.	24 in.	1232 cu. in.
7.	Triangle: base, 8 in.; height, 14 in.	24 in.	448 cu. in.
8.	Hexagon: area of base, 132 sq. in.	24 in.	1056 cu. in.

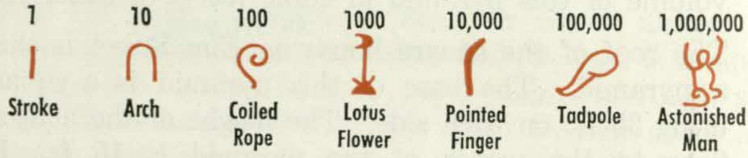
After performing the experiment, lead the students to suggest the rule and the formula in ex. 1. Have them look for prisms, cylinders, pyramids, and cones on various buildings.

Describe how the ancient Egyptians wrote numbers.

Egyptian Numerals

1. You have learned to write numbers with Roman numerals. In this system the primary symbols are: I = 1, X = 10, C = 100, and M = 1000; and the secondary symbols are: V = 5, L = 50, and D = 500. The symbols are additive when placed from left to right and when repeated as in XV and XXXIII. A primary symbol can be placed in a subtractive position before the next higher secondary or primary symbol as in IV and IX. A secondary symbol is never placed in a subtractive position, or repeated in one number. Write in Roman numerals: 49, 99, 499, and 999.
XLIX, XCIX, CDXCIX, CMXCIX

2. The ancient Egyptians' knowledge of geometry is shown today by their great pyramids. The number system which they developed, although no longer in use, is very interesting. This system was based on groups of ten, but it did not have place value. It was similar to the Roman system except that there were no secondary symbols. The hieroglyphic, or picture, symbols in the Egyptian system were:



The symbols were always additive and were written left to right, right to left, horizontally, and vertically in various arrangements. The meaning of a symbol did not depend upon its position. 1347 could be written these two ways:

3. Write in Egyptian numerals: See Guide.
243 213 1421 2003 10101 MDC MCVI 123_8

4. Change to Egyptian numerals and then do the work: See Guide.
$524 + 679$ $707 - 505$ 21×1234

5. How many symbols are needed to write 999 in Egyptian numerals? 27

Point out the lack of place value in the Egyptian system and the need for new symbols as numbers increase in size. Compare the Egyptian system with the place-value number systems.

Present a set of improvement tests in addition. Have the students determine and record their own scores (page 48).

Improving by Practice

Addition Test 5a.

Time: 4 min.

1. 6023	1867	3863	3459	2014	
7943	2326	6659	5792	3400	
5552	5138	5082	7038	5246	
7874	7053	4668	3615	3793	
1475	6412	2713	2439	6765	
4726	4736	4587	6708	5582	
5182	4109	9406	3719	2246	
9316	9535	4726	9866	1177	5
48,091	41,176	41,704	42,636	30,223	

Addition Test 5b.

Time: 4 min.

2. 2295	3104	6053	5814	7485	
2926	5775	2944	7584	1691	
7572	4988	3268	6139	4869	
4815	5732	8356	2238	6410	
8908	9184	5617	9101	8276	
1085	5243	6858	7079	4787	
6241	7046	8109	6288	8496	
7198	3268	9636	5969	3359	5
41,040	44,340	50,841	50,212	45,373	

Addition Test 5c.

Time: 4 min.

3. 7948	5886	5827	6858	9939	
8896	8987	3386	8618	5755	
4943	4632	4663	2542	7268	
9553	2196	3955	5739	5137	
3424	7754	1698	8874	8447	
8708	9502	4529	6267	1513	
6079	7018	9806	9054	9976	
3749	8975	7597	2569	4529	5
53,300	54,950	41,461	50,521	52,564	

To the Teacher. If individual pupils get low scores on Improvement Tests in addition, subtraction, multiplication, and division, such pupils may be assigned extra practice which will be found on pages 358–361.

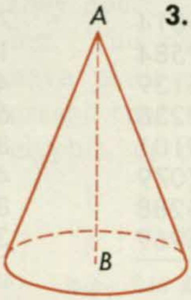
Students who do poorly on tests may need a drill on facts and on adding by endings. Clear up difficulties before assigning practice pages.

Present the formula for finding the volume of a cone and use it in problem solving.

Volume of a Cone

1. Everybody knows the name of the object that Jim is holding in his hand. If an empty ice cream cone is turned upside down, it will look like the figure shown in ex. 3, which is called a **cone.** Name some objects that are cones.

2. The **base** of a cone is a circle. In the figure below, point A is the Emphasize. **vertex** of the cone; AB, which is perpendicular to the base, is the **height** of the cone.



3. The method for finding the volume of a cone is the same as that of a pyramid:

Volume of cone = $\frac{1}{3}$ × area of base × height

Since the base of the cone is a circle, its area is πr^2. If you substitute πr^2 for the area of the base, you get this formula:

$$V = \frac{1}{3} \pi r^2 h$$

Use $\pi = 3\frac{1}{7}$ in the following problems:

4. The diameter of the base of a cone is 5 in. and the height is 14 in. Find the volume of the cone. $91\frac{2}{3}$ cu. in.

▶ The volume is $\frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{14}{1}$. In cases like this, cancel when you can. Stress.

5. A pile of sand has the shape of a cone. The pile is $4\frac{2}{3}$ ft. high and its base is a circle with a diameter of 9 ft. How many cubic yards of sand does the pile contain? $3\frac{2}{3}$

6. Mr. Day sells a scoop of ice cream for 15¢. The scoop is made in the shape of a cone with a diameter of $2\frac{1}{2}$ in. and a height of 3 in. Find, to the nearest whole cubic inch, the cubic contents of the cone. 5 How many scoops of ice cream can he get from 1 gal. of ice cream? 46 231 cu. in. = 1 gal.

Repeat the experiment on page 237, with a cone and a cylinder having the same base and the same height. Have students state the rule and develop the formula.

Provide a general review of problem solving.

Review Problems

1. Of the 76,036,400 telephones in use in North America one year, 93.5% were dial telephones. How many dial telephones were there in North America that year? 71,094,034

2. Mr. Baker sold 200 shares of Union Copper stock at $32\frac{5}{8}$. Find the brokerage per 100 shares by using the table on page 190. How much did Mr. Baker receive from the sale of this stock? Do not consider taxes. $6454.38

3. In a recent year different types of lenders provided the following per cents of the total money loaned for buying houses: savings and loan associations, 44%; (158.4°) commercial banks, 16%; (57.6°) individuals, 12%; (43.2°) insurance companies, 4%; (14.4°) savings banks, 5%; (18.0°) others, 19%. (68.4°) Make a circle graph to show where loans for buying homes were obtained.

4. A kilometer is about .6 mi. How many miles is the distance between San Pedro and Rio if it measures 870 km.? 522

5. Using the table on page 212, find the amount to which $1500 will grow in 10 yr. at 3% if compounded annually. $2015.88

6. Motor vehicle accidents caused 37,998 deaths one year and 38,309 deaths the next year. By what per cent, to the nearest tenth of a per cent, did the number of deaths by motor vehicles increase? .8%

7. The Northside Company had sales last year amounting to $575,000. In a report for the year, they stated that 76.5% of this money was used in costs and operating expenses,(1) 16.5% for the payroll,(2) 3.6% for taxes,(3) and the rest was net earnings. How many dollars were used in each way? (1) $439,875; (2) $94,875; (3) $20,700; (4) $19,550

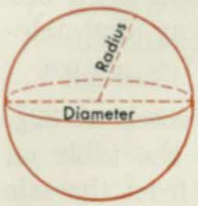
8. One winter in northern Michigan the monthly snowfall for five months was as follows: November, 10.4 in.; December, 19.9 in.; January, 33.6 in.; February, 31.7 in.; March, 13.2 in. Find the average monthly snowfall for those months. 21.76 in.

9. At 50¢ a dozen, find the cost of enough lemons to make 250 glasses of lemonade, if $\frac{1}{2}$ lemon is used for each glass and if lemons are sold only in dozens or half dozens. $5.25

Check the papers carefully and note the kinds of errors. Return the papers so that the students can find and correct their mistakes. Have volunteers explain the work at the board. If group errors are present, re-teaching of a particular topic may be necessary.

Present the formula for the area of the surface of a sphere and use it in solving problems (pages 242-243).

Area of a Sphere



1. One of the most familiar geometric figures is a **sphere.** Balls, marbles, and soap bubbles have the shape of a sphere. Name five other spherical objects.

2. A line segment drawn from the center of a sphere to any point on its surface is a **radius** of the sphere. A **diameter** of the sphere, which passes through the center, is twice as long as a radius. It is often important to know the surface area and the volume of a sphere. It would be difficult to measure them directly; but they can be found by using formulas if you know the length of the radius or the diameter of the sphere.

3. Mathematicians have shown that the formula for the surface area of a sphere is

$$A = 4\,\pi r^2$$

where A represents the surface area and r the radius.

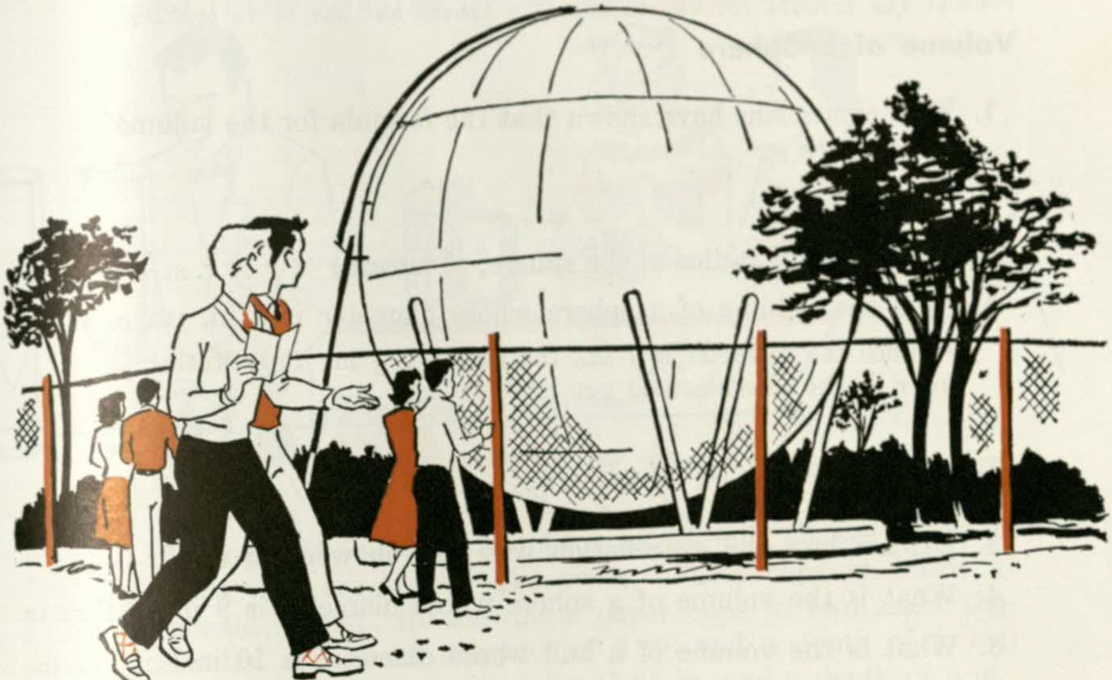
4. Show that the formula $A = 4\,\pi r^2$ may be changed to $A = \pi d^2$, where d is the diameter of the sphere. See Guide. When the length of the diameter is given, it is easier to use the formula $A = \pi d^2$; when the radius is given, use $A = 4\,\pi r^2$.

Find the surface area of each sphere. In ex. 5–7 use $\pi = 3.14$. In ex. 8–9 use $\pi = 3\frac{1}{7}$.

5.	$r = 10$ in. 1256 sq. in.	$d = 200$ ft. 125,600 sq. ft.	$r = 18$ in. 4069.44 sq. in.
6.	$r = 50$ ft. 31,400 sq. ft.	$d = 8$ in. 200.96 sq. in.	$d = 10$ ft. 314 sq. ft.
7.	$r = 37$ ft. 17,194.64 sq. ft.	$r = \frac{1}{2}$ in. 3.14 sq. in.	$d = 11$ ft. 379.94 sq. ft.
8.	$r = 7$ ft. 616 sq. ft.	$r = 14$ in. 2464 sq. in.	$d = 140$ ft. 61,600 sq. ft.
9.	$r = 35$ in. 15,400 sq. in.	$r = 28$ yd. 9856 sq. yd.	$d = 420$ in. 554,400 sq. in.

10. The dome on the top of an observatory has the shape of a hemisphere with a diameter of 21 ft. Find the area of the surface of this dome. 693 sq. ft.

Remind the students to notice whether the radius or the diameter is given in ex. 5-9. Before assigning ex. 10, discuss the meaning of the term hemisphere. Compare the area of the surface of a sphere with the area of a circle having the same radius.



11. The large sphere shown above is a high pressure gas tank. The outside diameter of this tank is 42 ft. What is the area of the surface of this tank? How many gallons of aluminum paint will it take to paint the outside of this spherical tank with one coat of paint if 1 gal. of paint covers 600 sq. ft.? What will this paint cost at \$4.37 a gallon? 5544 sq. ft 10 \$43.70

12. Suppose you have a smaller spherical tank whose outside diameter is 21 ft. What is the area of the surface of this tank? Fred says it is not necessary to work this problem out in full. He says that common sense tells him that the area of this smaller tank will be exactly half the area of the larger tank since the diameter of the smaller tank is half that of the larger tank. To work this problem, Fred takes $\frac{1}{2}$ of the area that he found in ex. 12. Is Fred right? If not, how does the area of this smaller tank compare with that in ex. 12? 1386 sq. ft. No; see Guide. It is $\frac{1}{4}$ as large.

13. The dome on the top of a planetarium has the shape of a hemisphere with a diameter of 75 ft. Find the surface area of this dome. Use $\pi = 3.14$. 8831.25 sq. ft.

In ex. 12 be sure all the students discover and understand that if the radius is doubled, the area of the surface of the sphere is 4 times as great. Illustrate in other problems if necessary.

Present the formula for the volume of a sphere and use it in problem solving.

Volume of a Sphere

1. Mathematicians have shown that the formula for the volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere; r^3 means $r \times r \times r$. Stress.

2. Find the volume of a sphere whose diameter is 7 in. $179\frac{2}{3}$ cu. in.

▶ Since the diameter is 7 in., the radius is $\frac{7}{2}$ in. Substituting $\frac{7}{2}$ for r in the formula, you get

$$V = \frac{\cancel{4}}{3} \times \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{\cancel{2}} \times \frac{7}{\cancel{2}} = \frac{539}{3} = 179\frac{2}{3}$$

3. Explain how the cancellation was done in working ex. 2.

4. What is the volume of a sphere whose diameter is 9 in.? $381\frac{6}{7}$ cu. in.

5. What is the volume of a ball whose diameter is 10 in.? $523\frac{17}{21}$ cu. in.

6. Oil is sometimes stored in spherical tanks like the one shown on page 243. Find the volume of this spherical tank, which has a diameter of 42 ft. 38,808 cu. ft. How many gallons of oil will this tank hold if completely filled? 1 cu. ft. = $7\frac{1}{2}$ gal. 291,060 gal.

7. At a party, lemonade was served from a large glass bowl shaped like a hemisphere. The diameter of the bowl was 14 in. ($718\frac{2}{3}$ cu. in.) About how many gallons of lemonade would the bowl hold if filled to the top? 3

8. Joe rolled a large snowball that had a diameter of 2 ft.; ($4\frac{4}{21}$ cu. ft.) Betty rolled one with a diameter of $1\frac{1}{2}$ ft. ($1\frac{43}{56}$ cu. ft.) If 1 cu. ft. of closely packed snow weighs about 42 lb., how many pounds did each of these snowballs weigh? 176, Joe's; $74\frac{1}{4}$, Betty's

9. A spherical balloon has a diameter of 28 ft. When it is fully inflated, how many cubic feet of gas will it hold? $11,498\frac{2}{3}$

Using $\pi = 3\frac{1}{7}$, find the volume of the following spheres:

10. $r = 7$ in. $1437\frac{1}{3}$ cu. in. **12.** $d = 20$ in. $4190\frac{10}{21}$ cu. in. **14.** $r = 4.2$ ft. 310.464 cu. ft.

11. $d = 30$ ft. $14,142\frac{6}{7}$ cu. ft. **13.** $d = 16$ ft. $2145\frac{11}{21}$ cu. ft. **15.** $r = 2.1$ in. 38.808 cu. in.

Emphasize that the radius is used in the formula for the volume of a sphere. Remind the students to give units in which volumes are expressed.

Present problems involving spheres, cones, and cylinders.

Spheres, Cones, Cylinders

1. At his soda fountain, Mr. King serves milk shakes in paper cups that are shaped like cones. These cups fit inside a plastic holder. He has these paper cups in two sizes: the smaller cup has a diameter of 3 in. and a depth of 4 in.; the larger cup has a diameter of $3\frac{3}{4}$ in. and a depth of 5 in. How do the volumes of these 2 cups compare? (1) Mr. King charges 15¢ for a milk shake in the smaller cup and 20¢ for one in the larger cup. Which size gives more for the money? (2)

(1) S: $9\frac{3}{7}$ cu. in., L: $18\frac{93}{224}$ cu. in.; (2) larger; it is almost twice size of smaller.

2. Mr. King also sells milk shakes in glasses shaped like a cylinder. These glasses have a diameter of $2\frac{3}{4}$ in. and a depth of 5 in. Mr. King charges 25¢ for a glass of milk shake. Which is the best value for the money, a glass of milk shake or one of the cups in ex. 1? The glass ($29\frac{159}{224}$ cu. in.), since it is about 3 times size of smaller cup and $1\frac{1}{2}$ times size of larger).

3. The earth is almost a perfect sphere with a diameter of approximately 8000 mi. Find the total area of the surface of the earth to the nearest hundred million square miles. 200,000,000

4. Find the volume of a sphere whose diameter is 7 ft.; $179\frac{2}{3}$ cu. ft. of one whose diameter is 14 ft. $1437\frac{1}{3}$ cu. ft. What happens to the volume of a sphere when you double the diameter? It is multiplied 8 times.

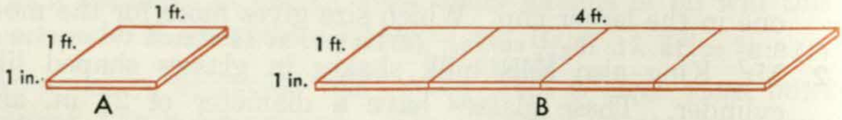
5. In the park in Reed City there is a marble sphere. Find the volume of the sphere if it has a diameter of 2 ft. $4\frac{4}{21}$ cu. ft. How many pounds does the sphere weigh if 1 cu. ft. of marble weighs about 170 lb.? $712\frac{8}{21}$

Let the students see how well they can apply their knowledge of formulas to problem solving. Discuss solutions.

Teach how to measure lumber and how to find its cost (pages 246-248). Try to have available several pieces of lumber 1 ft. long, 1 ft. wide, and 1 in. thick.

Measuring Lumber

1. Lumber is measured by a unit called the **board foot,** which is 1 ft. long, 1 ft. wide, and 1 in. **or less** thick, as shown Stress.



in A. If a board is 4 ft. long, 1 ft. wide, and 1 in. thick, as shown in B, it contains 4 board feet. If the board in B were only $\frac{3}{4}$ in. thick, it would also contain 4 board feet.

2. A board that is 1 in. thick, $\frac{3}{4}$ ft. wide, and 8 ft. long contains $1 \times \frac{3}{4} \times 8$, or 6, bd. ft. Bd. ft. means board feet.

To find the number of board feet, multiply the thickness in inches by the width in feet by the length in feet.

3. Charles has a board that is $\frac{7}{8}$ in. thick, 8 in. wide, and 6 ft. long. How many board feet does it contain? 4

▶ Count $\frac{7}{8}$ in. as 1 in. and change 8 in., which is the width, to $\frac{2}{3}$ ft. Then the number of board feet is $1 \times \frac{2}{3} \times 6$. How many board feet is that?

Use pieces of lumber to illustrate the meaning of board foot. Emphasize that the thickness must be given in inches, while the length and the width must be given in, or changed to, feet; and that any thickness less than 1 in. counts as 1 in.

Stress the fact that if the thickness is more than 1 in. it is counted in full. Notice the first sentence in ex. 5.

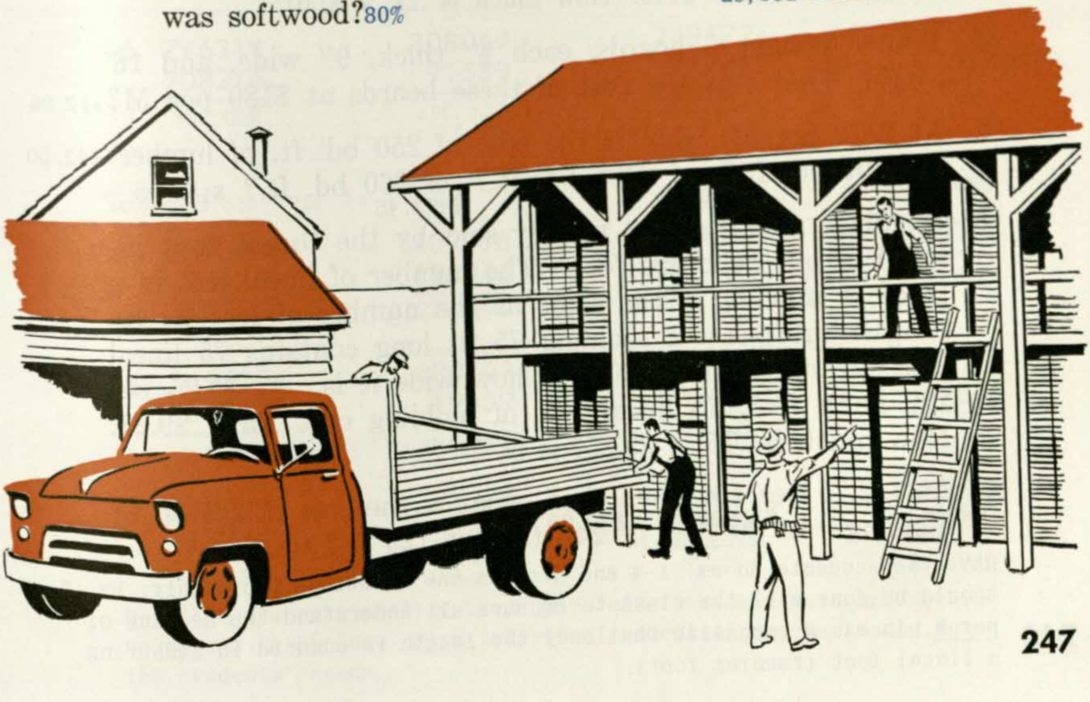
4. If the thickness of a board is 1 in. or more, use the actual number of inches in finding the number of board feet. Thus, if a plank is $3\frac{1}{2}$ in. thick, 8 in. wide, and 12 ft. long, it contains $3\frac{1}{2} \times \frac{2}{3} \times 12$, or 28, bd. ft. How many board feet are there in a beam that is 4 in. thick, 8 in. wide, and 15 ft. long? 40 in a plank that is $2\frac{1}{2}$ in. thick, 9 in. wide, and 16 ft. long? 30 in one that is 3 in. by $1\frac{1}{4}$ ft. by 12 ft.? 45

5. In buying lumber, a fraction of a board foot counts as another board foot. Thus a board 2 in. $\times$ 10 in. $\times$ 8 ft. contains $2 \times \frac{5}{6} \times 8$, or $13\frac{1}{3}$, bd. ft. If just one board is bought, this is counted as 14 bd. ft., but if 25 of these boards are bought, you should multiply by 25 before changing the fraction of a board foot to another board foot, thus: $25 \times 2 \times \frac{5}{6} \times 8 = 333\frac{1}{3}$, which is counted as 334 bd. ft.

6. How many board feet are there in 30 pieces of lumber, each 3 in. thick, 10 in. wide, and 15 ft. long? 1125

7. In a recent year 36,742 million board feet of lumber were produced in the United States. Of this lumber 7180 million board feet were hardwood. What per cent, to the nearest 1%, was hardwood? 20% The remainder of the lumber was softwood. How much was softwood, and what per cent was softwood? 80%

29,562 million board feet



Buying Lumber

1. Bob bought 7 pieces of lumber, each $1\frac{1}{2}''$ thick, $8''$ wide, and $10'$ long. How many board feet did he buy? 70

2. To make a chest, Sam bought 6 boards each $1''$ thick, $9''$ wide, and $10'$ long. How many board feet did he buy? 45

3. A carpenter ordered 42 pieces of lumber $2\frac{1}{2}''$ thick, $10''$ wide, and $10'$ long. How many board feet in all were there in this order? 875

4. Find the total number of board feet in this order:

60 boards $\frac{3}{4}''$ thick, $8''$ wide, $12'$ long 480
144 boards $\frac{7}{8}''$ thick, $10''$ wide, $12'$ long 1440
48 rafters $2''$ thick, $6''$ wide, $16'$ long 768
30 beams $4''$ thick, $8''$ wide, $16'$ long 1280
Total: 3968 bd. ft.

(240 bd. ft.) **5.** Mr. Hall bought 30 boards, each $\frac{3}{4}''$ thick, $6''$ wide, and $33.60 $16'$ long. How much in all did this lumber cost him at $140 per M? Per M means per 1000 bd. ft. M is the Roman numeral for 1000.

▶ Show that Mr. Hall bought 240 bd. ft. of lumber. Since the lumber costs $140 per 1000 bd. ft., 240 bd. ft. will cost $\frac{240}{1000}$, × $140, or .24 × $140. How much is .24 × $140?

(72 bd. ft.) **6.** Walter bought 6 boards, each $\frac{3}{4}''$ thick, $9''$ wide, and $16'$ long. What was the cost of these boards at $180 per M? $12.96

7. At $170 per M, what is the cost of 250 bd. ft. of lumber? $42.50 of 1300 bd. ft.? $221 of 2785 bd. ft.? $473.45 of 860 bd. ft.? $146.20

8. Moldings and trim are usually sold by the **lineal foot** instead of by the board foot. The number of lineal feet in a piece of molding is the same as the number of feet **in its length.** A piece of molding 75 ft. long contains 75 lineal feet, no matter how thick or how wide it is. At $9.25 per 100 lineal feet, 75 lineal feet of molding cost .75 × $9.25. How much is .75 × $9.25? $6.94

9. At $12.75 per 100 lineal feet, what is the cost of 150 ft. of molding? $19.13 of 80 ft.? $10.20 of 240 ft.? $30.60 of 125 ft.? $15.94

Have the students do ex. 1-4 and discuss the solutions thoroughly. Ex. 5 should be done with the class to be sure all understand the meaning of per M. In ex. 8 emphasize that only the length is counted in measuring a lineal foot (running foot).

Present improvement tests in subtraction. Tests are to be given, scored, and recorded as were others (see pages 65, 113, 151, 203).

Improving by Practice

Subtraction Test 5a.

Time: 4 min.

1.	671822	820541	100008	807017	
	194259	527042	38249	652858	
	477,563	293,499	61,759	154,159	
2.	902046	149592	664697	400083	
	187529	77625	39859	91948	
	714,517	71,967	624,838	308,135	
3.	800090	651274	906051	924257	
	16295	528318	492255	54296	12
	783,795	122,956	413,796	869,961	

Subtraction Test 5b.

Time: 4 min.

4.	583783	900072	676495	706805	
	195358	149785	99238	17416	
	388,425	750,287	577,257	689,389	
5.	560000	933174	809031	273868	
	434197	17249	124588	37369	
	125,803	915,925	684,443	236,499	
6.	746314	308069	248172	901930	
	208216	91539	77748	97537	12
	538,098	216,530	170,424	804,393	

Subtraction Test 5c.

Time: 4 min.

7.	271093	409016	853672	200005	
	25025	186046	717479	97035	
	246,068	222,970	136,193	102,970	
8.	345515	480702	720000	885695	
	139287	19339	388925	69836	
	206,228	461,363	331,075	815,859	
9.	610000	571344	765010	843684	
	115743	57606	707226	73739	12
	494,257	513,738	57,784	769,945	

Help the students analyze the causes of errors and note arithmetic needs and weaknesses. Plan individual remedial programs based on the students' needs.

Chapter Review

1. Name a container shaped like a rectangular prism; (1) also name one shaped like a cylinder; (2) like a sphere; (3) like a cone. (4)
(1) Shoe box; (2) tomato can; (3) spherical gas tank; (4) ice cream cone

2. Name an object that is solid throughout and shaped like a rectangular prism. (1) Name one that is shaped like a pyramid; (2) one like a cylinder; (3) one like a sphere. (4) (1) Brick; (2) Egyptian pyramid; (3) stone column; (4) wooden ball

3. Give the names of three different kinds of prisms and draw a picture of each one. Cube, rectangular solid, triangular prism

4. Name three shapes that the base of a pyramid might have. Square, triangle, hexagon
How many shapes can the base of a cone have? Only one (circle)

5. Write the formula for each of the following: volume of a cone; $V = \frac{1}{3}\pi r^2 h$ volume of a cylinder; $V = \pi r^2 h$ volume of a pyramid; $V = \frac{1}{3}Bh$ volume of a sphere; $V = \frac{4}{3}\pi r^3$ area of the surface of a sphere. $A = 4\pi r^2$

6. Find the volume of a pyramid whose base is a rectangle with length 6 ft. and width 5 ft., and whose height is 12 ft. 120 cu. ft.

7. Find the volume (1) and the surface area (2) of a sphere that has a radius of 30 in. Use $\pi = 3.14$. (1) 113,040 cu. in; (2) 11,304 sq. in.

8. Find the total area of the surface of a cylindrical can that is 7 in. high and has a diameter of 4 in. Be sure to include both end surfaces. $113\frac{1}{7}$ sq. in.

9. Find the number of cubic inches in the volume of the can in ex. 7. 88 If a quart of liquid occupies about 58 cu. in., does this can hold about 1 qt., $1\frac{1}{2}$ qt., or 2 qt. when it is full of liquid? $1\frac{1}{2}$

10. How many small boxes, each 4 in. by $2\frac{1}{2}$ in. by $1\frac{1}{2}$ in. on the outside, can be packed in a large box that is 1 ft. by 10 in. by 9 in. on the inside? 72 Tell how many boxes there will be in each layer and how many layers there will be.
12 boxes in each layer, 6 layers

11. A rectangular swimming pool is 90 ft. long and 60 ft. wide. The depth of the pool is greater at one end than at the other but the average depth of the water is $4\frac{1}{2}$ ft. How many gallons of water are there in the pool when the water is at the average depth? 182,250 1 cu. ft. = $7\frac{1}{2}$ gal.

Check the papers carefully and note the kinds of errors. Through conferences with the students, determine the causes of errors (lack of information, incorrect use of formulas, inability to apply formulas, and so on).

1. A train travels $2\frac{3}{4}$ mi. in 3 min. At this rate how many feet will the train go per second? $80\frac{2}{3}$

2. The price of a ring is \$13.20. This includes a tax of 10% on the actual price of the ring. How much is the tax? \$1.20 What is the actual price of the ring? \$12.00

3. Of the 827 pupils in the Page School, 579 are members of the Athletic Association. Find, to the nearest tenth of 1%, what per cent are members of the Athletic Association. 70.0%

4. How much more interest will you get in 2 yr. on \$8000 at 4% compounded semiannually (\$659.44) than you will have at 4% simple interest? (\$640) Do not use the interest table in your work. \$19.44 more compounded semiannually

5. Find, to the nearest whole cubic foot, the volume of a spherical storage tank with a radius of 20 ft. 33,524 Use $\pi = 3\frac{1}{7}$.

6. Lemonade is served in a cone-shaped paper cup $3\frac{1}{2}$ in. in diameter and $4\frac{3}{4}$ in. deep. (15.2 cu. in.) If the cup is full, does it hold more or less than $\frac{1}{2}$ pt.? More A pint of liquid occupies about 29 cu. in.

7. Find the total amount of a bill for $2\frac{1}{2}$ lb. of cheese at \$.55 a pound, $1\frac{1}{2}$ lb. of beans at \$.19 a pound, $5\frac{1}{4}$ lb. of beef at \$.65 a pound, and 9 lemons at 3 for \$.11. \$5.42

8. At what price should Mr. Bell sell a chair that cost \$33 if he figures that his expenses are 32% of his selling price and he wishes to make a profit of 8% of the selling price? \$55 If he sells the chair for \$48, does he have a profit or a loss? \$2.60 loss

9. Miss Lee received a commission of \$75 on her sales last week which were \$1000. Find the rate of commission. $7\frac{1}{2}\%$

10. Three boys hiked 13 mi. in $3\frac{1}{2}$ hr. Find, to the nearest tenth of a mile per hour, their rate of walking. 3.7

SCORE	0–5	6–7	8–9	10
	You need help	Fair	Good	Excellent

Let volunteers explain the solutions at the board so that others may find and correct their mistakes. Have students compare the results of the test with previous ones, as shown on their needs charts, and note improvements and/or weaknesses. Plan remedial work as needed.

Present a diagnostic test of the skills taught in Chapter 7, with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Page for that row. Use $\pi = 3\frac{1}{7}$.

Find the volume of these prisms:

	Base	Height		Base	Height		Practice Pages
1.	7 sq. ft.	$4\frac{1}{2}$ ft.	$31\frac{1}{2}$ cu. ft.	84 sq. in.	$3\frac{1}{2}$ ft.	3528 cu. in.	229
2.	8 sq. in.	$9\frac{1}{4}$ in.	74 cu. in.	$4\frac{1}{2}$ sq. ft.	66 in.	$24\frac{3}{4}$ cu. ft.	229

Find the volume of these rectangular prisms:

3.	8 in. × 7 in. × 3 in.	168 cu. in.	$1\frac{1}{2}'' \times 1\frac{1}{2}'' \times 1\frac{1}{2}''$	$3\frac{3}{8}$ cu. in.	222
4.	6 ft. × 3 ft. × 1 ft.	18 cu. ft.	$8\frac{1}{4}'' \times 2\frac{2}{3}'' \times 3\frac{1}{2}''$	77 cu. in.	222

Find the volume of these cylinders:

	Radius	Height		Diameter	Height		
5.	$3\frac{1}{2}$ in.	8 in.	308 cu. in.	10 in.	21 in.	1650 cu. in.	231
6.	14 ft.	9 ft.	5544 cu. ft.	42 ft.	25 ft.	34,650 cu. ft.	231

Find the volume of these cones:

	Radius	Height		Diameter	Height		
7.	6 in.	14 in.	528 cu. in.	7 ft.	16 ft.	$205\frac{1}{3}$ cu. ft.	240

Find the area of spheres with these radii:

8.	3 ft.	$113\frac{1}{7}$ sq. ft.	14 in.	2464 sq. in.	$10\frac{1}{2}$ ft.	1386 sq. ft.	35 ft.	15,400 sq. ft.	242

Find the volume of spheres with these radii:

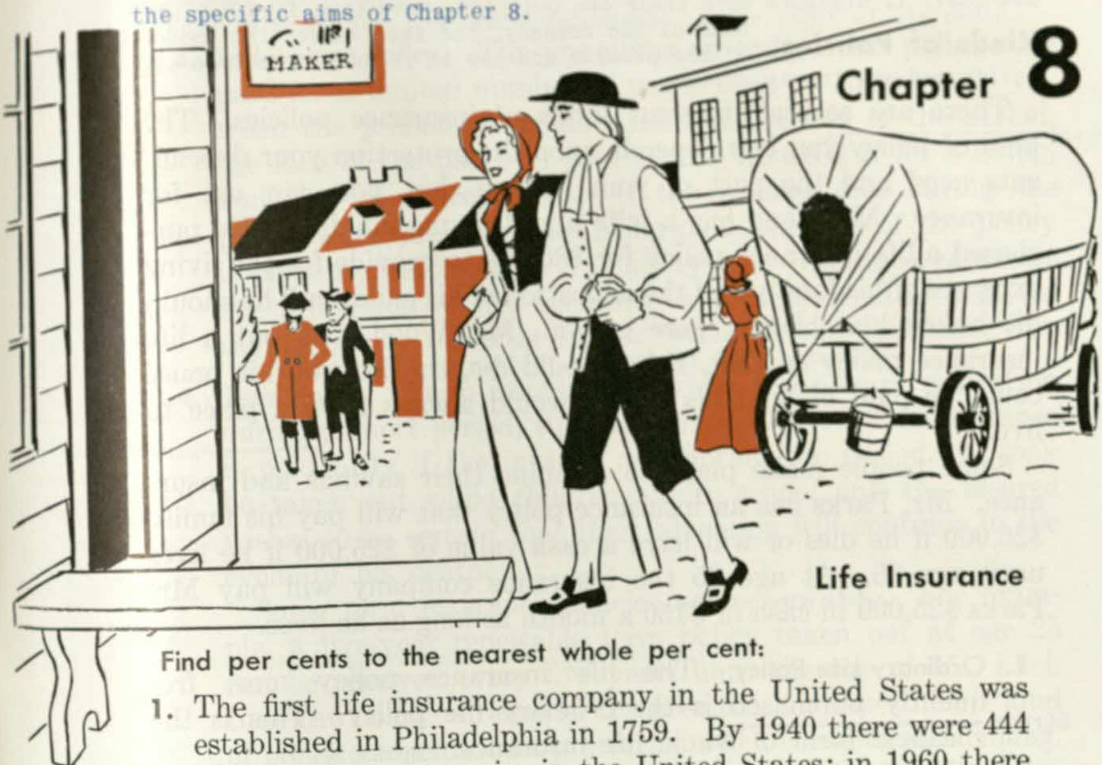
9.	3 in.	$113\frac{1}{7}$ cu. in.	7 in.	$1437\frac{1}{3}$ cu. in.	$3\frac{1}{2}$ ft.	$179\frac{2}{3}$ cu. ft.	21 ft.	38,808 cu. ft.	244

Find the curved surface of these cylinders:

	Radius	Height		Diameter	Height		
10.	7 in.	12 in.	528 sq. in.	21 ft.	30 ft.	1980 sq. ft.	234

After correcting papers and noting errors, return them to the students so they can find and correct their mistakes. Give individual help as needed. Be sure all difficulties are cleared up before assigning remedial work.

Show and discuss the importance of life insurance. See the Guide for the specific aims of Chapter 8.



Chapter 8

Life Insurance

Find per cents to the nearest whole per cent:

1. The first life insurance company in the United States was established in Philadelphia in 1759. By 1940 there were 444 life insurance companies in the United States; in 1960 there were 1444 companies. By what per cent did the number of companies increase during this 20-year period? 225%

2. In a recent year 103 million people in the United States had their lives insured with our insurance companies. Of these people 40 million were men, 33 million were women, and 30 million were children under 18 years of age. What per cent of the people who had life insurance were men? were women? were children under 18? About 29% About 39% About 32%

3. The life insurance in force in the United States recently totaled \$629,493,000,000. Ten years before that the amount of life insurance in force was \$253,140,000,000. (1) Round off these amounts to the nearest billion. What was the per cent of increase during this period of ten years? (2) (1) \$629 billion, \$253 billion; (2) 149%

4. One year 24,145,000 life insurance policies were purchased. The amount of these policies was \$48,427,000,000. What was the average size of these life insurance policies? About \$2000

Notice that the life insurance discussed in this chapter is "legal reserve" life insurance, in which a reserve fund is set aside for each policy and part of the annual premium is used to build it up. Ex. 1–4 may be discussed and done with the class.

Present a description of four kinds of life insurance policies (pages 254-255). If students have their own policies, have them find out the name of the company, the face value, the amount of the premium and how it is paid, the type of policy and why it was chosen.

Kinds of Policies

There are several different kinds of insurance policies. The kind of policy you buy depends upon the protection your dependents need and the part of your income that you can use for insurance. Mr. Long has a wife and 4 young children. He purchased a life insurance policy for $30,000 to provide for the living expenses of his family and the education of his children if he should die before his children were grown. Mr. Wood took out a life insurance policy so that, if he should die, the loan on his house could be paid off and his family would always have a place to live.

Some people make plans to combine their savings and insurance. Mr. Parks has an insurance policy that will pay his family $25,000 if he dies or will have a cash value of $25,000 if he lives until age 65. At age 65 the insurance company will pay Mr. Parks $25,000 in cash or $150 a month as long as he lives.

1. Ordinary Life Policy. The life insurance policy most frequently purchased is the ordinary life policy. This is the Stress. cheapest form of whole life insurance. Emphasize.

If Mr. Gray buys an ordinary life policy for $10,000, the **face** of the policy is $10,000. If Mr. Gray dies, Mrs. Gray will receive $10,000 from the insurance company. Mr. Gray is called the **insured** and Mrs. Gray is called the **beneficiary.** Each year **as long as he lives,** Mr. Gray pays a **premium** to the insurance company. Discuss meaning of terms thoroughly.

2. Limited Payment Policy. The ordinary life policy requires that premiums be paid every year as long as the insured lives. Since it may be difficult for the insured to make these payments when he gets older, he may prefer to buy a 20-payment life policy on which premiums are paid for only 20 yr. After that, there are no more premiums to be paid but the insur- Emphasize ance **remains in force** until the insured dies. Other types of policies require premium payments for only 10 yr., or 15 yr., or 25 yr. The annual premiums on limited payment policies are higher than those on ordinary life policies. See the table on page 256. Ask pupils why this is true.

Ask questions to be sure all students understand the different features of each kind of policy and the meaning of the terms face of policy, insured, beneficiary, premium, in force, renewable.

Be sure the students notice the major purpose of life insurance: to provide funds for a person's dependents in the event of his death.

3. Endowment Policy. The premiums on endowment policies are paid for a limited number of years, such as 15 yr. or 20 yr., when the premiums are all paid. At the end of the period, the face of the policy is paid to **the insured** if he is living, and the insurance ceases. If the insured dies during the period when premiums are being paid, the face of the policy is paid to the beneficiary. This type of insurance combines the protection of life insurance with a systematic plan for saving money. Emphasize savings feature of this insurance. Point out fact that this is another purpose of life insurance.

Stress.

4. Term Policy. Term insurance provides **temporary** protection during a short period, such as 5 yr. to 15 yr. The insurance is paid **only** if the insured dies during the period. It may be taken out while children are in school and the insured wishes to be certain that their education will continue in the event of his death.

Emphasize.

Some term insurance policies are **renewable.** For example, a five-year renewable term policy taken out at age 25 can be renewed every five years until the age of 65. Each time the policy is renewed the premium increases. This kind of policy gives low-cost protection when the insured is young. It may be preferred if the insured expects his income to increase as his age increases. Discuss why premium increases. Point out and explain that term policies are often "convertible."

5. It is possible to arrange for the beneficiary of a policy to receive, instead of a single sum of money, equal monthly payments for a certain number of years or for life. When do you think this would be desirable? Lead class discussion of this point.

Teach the students how to find the premium on any amount of life insurance and give problems in calculating premiums (pages 256-257).

Paying Insurance Premiums

1. Life insurance premiums are usually paid annually; they may also be paid semiannually or quarterly. For most types of policies, the premium remains the same each year. The amount of the annual premium depends upon the face of the policy and the age of the insured when the policy is taken out. The premium also depends upon the type of policy, as shown in the table below. If Mr. West buys a $5000 ordinary life policy at age 25, his premium is $13.16 for each $1000 of insurance. The premium on a $5000 policy will be 5 × $13.16, or $65.80.

Annual Premium for Each $1000 of Insurance				
Age	**Ordinary Life**	**20-Payment Life**	**20-Year Endowment**	**10-Year Term**
20	$11.31	$21.02	$43.20	$ 4.37
25	13.16	23.36	43.33	4.51
30	15.58	26.15	43.64	5.05
35	18.76	29.49	44.27	6.26
40	22.78	33.35	45.24	8.15
45	27.68	37.94	46.94	11.31
50	33.90	43.20	49.51	16.40

2. Mr. Miles was 25 yr. old when he took out a 20-payment life policy for $5000. Find the amount of his annual premium on this policy. $116.80

3. What is the annual premium for a 10-year term policy for $15,000 if it is issued at age 25? at age 30? at age 40? $122.25
$67.65 $75.75

4. Find the difference between the annual premium for a 20-payment life policy for $10,000 and an ordinary life policy for the same amount if issued at age 35. $294.90 - $187.60 = $107.30

5. Why is the premium on a 20-payment life policy for $10,000 higher than the premium on the ordinary life policy for the same amount? See ex. 4. These payments end in 20 yr., whereas ordinary life policy continues longer during a normal life span.

6. What is the annual premium on a 20-year endowment policy for $3000 if it is issued at age 20? at age 25? at age 35? at age 40? $135.72
$129.60 $129.99 $132.81

Using the rates on page 256, find the premium for each policy:

1. An ordinary life policy for $20,000 issued at age 35 $375.20
2. A 10-year term policy for $30,000 issued at age 40 $244.50
3. A 20-payment life policy for $6000 issued at age 30 $156.90
4. A 20-year endowment policy for $2000 issued at age 20 $86.40
5. An ordinary life policy for $200,000 issued at age 50 $6780
6. What kind of policy should each man buy?
   (a) Mr. Ray has ordinary life insurance but he needs some extra protection for the next 10 yr. to make certain that his children will be able to complete their education if he should die while they are still in school. 10-year term
   (b) Mr. Peck wants to protect his family while his children are in college but he also wants to provide himself with an income 20 yr. later when he retires. 20-year endowment
   (c) Mr. Hunt now has a good job as an expert mechanic but 20 yr. from now he will have to give up that work and it will not be easy then to pay insurance premiums. 20-payment life
7. Mr. Ball is 25 yr. old. He can afford to spend about $100 a year for life insurance. For that sum, about how much ordinary life insurance can he buy? $7000 how much 20-payment life insurance? $4000 how much 20-year endowment? $2000
8. A person applying for life insurance must pass a medical examination. Unless his health is good, he cannot be insured. Can you tell why this is necessary? Why is it desirable to take out insurance while one is young? See Guide.
9. Mr. Ward, general manager of Art Clothes, is 40 yr. old. If he were to die suddenly the firm would lose $100,000 worth of business before they would be able to find someone who could replace him. What yearly premium will they pay if they insure his life for $100,000 with an ordinary life policy? $2278 with a 10-year term insurance policy? $815 Who would be the beneficiary in this case? The company

Have the students do ex. 1–9 and let volunteers explain the solutions. Discuss ex. 8–9 thoroughly. Supplement the work by having students give a situation in which each type of insurance policy would be suitable.

Explain the reserve fund of a life insurance policy and show its relation to the cash value and the loan value of the policy

Cash Value of a Policy (pages 258–259).

1. Mr. White pays an annual premium of $18.76 on an ordinary life policy for $1000. The company uses this $18.76 each year as follows: (a) Part of this amount is used to pay the cost of insuring Mr. White for the year and to pay his share of the expenses of the company; (b) the rest of the premium is invested to the credit of Mr. White just like a savings bank deposit. The amount so invested builds up a **reserve fund** which grows larger and larger each year, as shown at the left in the column headed "Cash Value." At the end of 5 yr. this fund amounts to $56. How large is this fund at the end of 10 yr.? $147 15 yr.? $244 20 yr.? $345 30 yr.? $529

Age 35. Ordinary Life Policy per $1000	
End of Year	Cash Value
5	$ 56
10	147
15	244
20	345
30	529

2. The cash values given in the above table are the amounts the company will pay back to the policyholder if he stops paying premiums and ends his insurance. For example, if Mr. White drops his policy at the end of 10 yr., the company will pay him its **cash value,** which is $147 at that time. If Mr. White does not drop the policy but finds himself in need of money, he can borrow as much as $147 from the company. In this case $147 is called the **loan value** of the policy. It is seen that the loan value equals the cash value. The cash values of limited payment and endowment policies are larger than those of ordinary life policies. Policies issued at older ages have larger cash values than those issued at younger ages. Discuss why.

Stress.
Stress.
Stress.
Discuss why.

3. Mr. Clay has an ordinary life policy for $3000 which was issued at age 35. What is the cash value of this policy at the end of 5 yr.? $168 at the end of 15 yr.? $732 at the end of 30 yr.? $1587 The cash values of a $3000 policy are 3 times as large as those shown in the above table. At the end of 15 yr. what is the cash value of an ordinary life policy for $10,000 which was issued at age 35? $2440

Be sure the meaning of reserve fund, cash value, and loan value is clear to the students. Point out that the cost of insuring a person's life increases as his age increases, and that the reserve fund enables the company to continue to insure the policyholder without increasing the premium.

Emphasize that the loan value must be equal to (may be more than) the amount the person wants to borrow. Point out that premiums must still be paid to keep the policy in force.

In these problems use the table on page 258:

1. Mr. Wells purchased a $5000 ordinary life policy at age 35. After he had paid premiums for 15 yr., he was obliged to drop the policy. The company paid him the cash value of the policy. How much did he receive? $1220 The cash values in the table on page 258 are for $1000 of insurance, so these values are 5 times as much on a $5000 policy. Stress.

2. Mr. Smith had an ordinary life policy for $7000 issued at age 35. At the end of 20 yr. he needed to borrow $2000. Was the loan value of his policy large enough so that he could borrow that amount from the company? Yes If the company loaned him $2000 at 5%, how much interest did he have to pay each year that he kept the money? $100

3. In ex. 2, suppose Mr. Smith died before paying off his loan of $2000. In such cases the company pays the $7000 insurance less the amount of the loan and the interest due on it. Stress. If Mr. Smith died 6 mo. after making the loan, how much interest was due on the loan for those 6 mo.? $50 How much did the company pay to the beneficiary, Mrs. Smith? $4950

Find the cash values of ordinary life policies taken out at age 35 for these face amounts. Use the table on page 258:

4. $6000 at end of 5 yr. after policy was issued $336
5. $8000 at end of 10 yr. after policy was issued $1176
6. $7000 at end of 30 yr. after policy was issued $3703
7. $10,000 at end of 20 yr. after policy was issued $3450

Find the loan values of ordinary life policies taken out at age 35 for these face amounts. Use the table on page 258:

8. $5000 if loan is made at end of 5 yr. $280
9. $15,000 if loan is made at end of 15 yr. $3660
10. $25,000 if loan is made at end of 10 yr. $3675

In ex. 3 be sure the students understand that the amount of the loan and the interest on it are subtracted from the amount paid to the beneficiary. Point out also that the cash value of the policy protects the company from losing the amount lent, if the person does not pay the premiums.

Explain life insurance dividends and show how they may be used. Only mutual companies are discussed here. The premiums on page 256 are typical of a stock company. Obtain sample rates from both kinds of companies.

Life Insurance Dividends

1. The annual premiums paid to a life insurance company are used to pay the insurance of policyholders who die during the year, to pay the expenses of running the company, and to build up a fund which represents the cash value of each policy. At the end of the year the company often finds that fewer people have died during the year than was expected and that the expenses for the year have been less than was estimated. The company may also have earned a higher rate of interest on its reserve funds than was required. Mutual life insurance companies divide these savings and extra interest earnings among the policyholders at **the end of the year** in the form of **dividends.**

2. Mr. Green took out a 20-payment life policy for \$1000 with a mutual life insurance company. The annual premium is \$36.03. At the end of the first year he received a dividend of \$5.66. How much did his insurance actually cost him for the year? \$30.37

3. Dividends may become larger each year as the policy grows older. Mr. Reed's dividends on his ordinary life policy for the first 5 years were \$3.49, \$3.83, \$4.18, \$4.53, and \$4.87. What was his average dividend per year? \$4.18 If his annual premium was \$22.47, what did his insurance actually cost him, on the average, per year? \$18.29

4. Mr. Brooks purchased a \$1000 ordinary life policy from a mutual life insurance company. During the first 10 yr. that he had the policy he received a total of \$54.29 in dividends. At the end of 10 yr. he decided to drop his policy. The insurance company gave him the cash value of the policy, which at that time was \$74.00. How much did Mr. Brooks receive in all from the insurance company in dividends and in cash value? \$128.29 How much did he pay the company in premiums during the 10 yr. if his annual premium was \$19.62? \$196.20 How much did his life insurance actually cost him for the 10 yr.? \$67.91 This was an average cost of how much per year? \$6.79

Explain that if the dividends are left with the company, they accumulate at compound interest. This fund is paid upon request or, in the event of death, added to the face value paid to the beneficiary.

Present a set of improvement tests in multiplication and in division. Have the students record their scores on the graphs in their Record Books (page 48).

Improving by Practice

Multiplication Test 5a.

Time: $3\frac{1}{2}$ min. after copying.

1.	\$682.47 × 96	\$740.95 × 83	\$196.85 × 27	\$692.47 × 74	\$742.58 × 52
	\$65,517.12	\$61,498.85	\$5314.95	\$51,242.78	\$38,614.16

5

Multiplication Test 5b.

Time: $3\frac{1}{2}$ min. after copying.

2.	\$319.28 × 83	\$908.63 × 45	\$819.35 × 96	\$468.37 × 82	\$275.46 × 67
	\$26,500.24	\$40,888.35	\$78,657.60	\$38,406.34	\$18,455.82

5

Multiplication Test 5c.

Time: $3\frac{1}{2}$ min. after copying.

3.	\$701.83 × 47	\$792.46 × 59	\$163.85 × 94	\$625.74 × 37	\$293.75 × 26
	\$32,986.01	\$46,755.14	\$15,401.90	\$23,152.38	\$7637.50

5

Division Test 5a.

Time: 5 min. after copying.

4. $456\overline{)30275}$ 66.4 $\qquad$ $719\overline{)25623}$ 35.6

5. $913\overline{)86347}$ 94.6 $\qquad$ $285\overline{)13521}$ 47.4

4

Division Test 5b.

Time: 5 min. after copying.

6. $812\overline{)65596}$ 80.8 $\qquad$ $635\overline{)26189}$ 41.2

7. $258\overline{)17794}$ 69.0 $\qquad$ $657\overline{)29182}$ 44.4

4

Division Test 5c.

Time: 5 min. after copying.

8. $986\overline{)24613}$ 25.0 $\qquad$ $321\overline{)25784}$ 80.3

9. $734\overline{)35418}$ 48.3 $\qquad$ $218\overline{)10235}$ 46.9

4

To the Pupil. In ex. 4–9, find quotients to the nearest tenth.

Instruct the students to copy carefully and do all work neatly and legibly. After correcting papers and analyzing errors, return the papers so the students can find and correct their mistakes. Use "More Practice" pages for remedial work.

Describe industrial insurance, group life insurance, and United States Government insurance.

Other Types of Life Insurance

1. Industrial Insurance. This type of life insurance is sometimes called **weekly premium insurance** since the premiums are often paid weekly, though monthly premium payments are also made. The face values of industrial policies are usually limited to a few hundred dollars, with weekly premiums of 5¢, 10¢, or 25¢; these premiums are usually collected by an agent of the company. Industrial insurance was originally intended for industrial workers whose incomes are low and who wish the protection of insurance to pay funeral expenses at death. Perhaps some member of your family carries an industrial insurance policy; if so, ask about it and tell the class. Use this information as the basis for further discussion the next day.

   At age 30, a premium of 10¢ weekly will buy \$186 worth of industrial insurance; in this case, what does the insurance cost per year? \$5.20

2. Group Insurance. This form of life insurance is written to cover a group of employees working in the same business or industry. There must be at least 25 employees in the group in order to make the company eligible for group insurance. No medical examination is required; this feature helps the older employees since they might not be able to pass a physical examination. The employer often pays part or all of the premium. The part paid by the employee is deducted each week from his wages. In a recent year 46 million persons in this country were protected by group life insurance. If the total amount of group life insurance in force on these 46 million lives was about \$192 billion, what was the average amount of group life insurance carried per person? \$4173.91

   Emphasize.

   Stress.

   Emphasize.

3. Government Insurance. The United States Government issued life insurance to members of the armed forces in World War I and World War II, and issues life insurance to present members of the armed forces. Perhaps a member of your family has one of these Government policies and can tell you about it. If so, get as much information as you can about the policy and make a report to the class. Use for further discussion.

In ex. 2 point out that the amount of group insurance an employee can get usually depends on his salary, or length of service, or a combination of both. Group insurance is sometimes not available after a person retires or becomes unable to work.

Present a review of important technical terms used in arithmetic. Students who do poorly should be tested in a week or so, after further review, to be sure they have clear understanding of terms.

The Language of Arithmetic

Read the statements below and tell the correct word to put in each space. Do not write in the spaces:

1. A written promise to pay a sum of money at a given time is called a promissory note. The date on which the money is to be paid is called the date of maturity.

2. Money paid for the use of money is called interest.

3. The money that an agent is paid for selling a house is called his commission

4. When an article is purchased by making weekly or monthly payments, it is bought on the installment plan.

5. One way in which banks earn the interest they pay on savings accounts is by lending money.

6. An interest rate of 2% monthly is a yearly rate of 24 %.

7. The profits of a corporation which are divided among stockholders are called dividends.

8. The selling price of an article should be equal to the sum of cost, expenses, and profit.

9. Interest paid on the principal and on interest left in the bank is called compound interest

10. It costs less to buy an article for cash than to buy it on the installment plan.

11. A rule written in brief form by using letters instead of words is called a formula.

12. Lines that never meet no matter how far they are extended are called parallel lines.

13. A line drawn from the vertex of a triangle perpendicular to the base is called the height, or altitude of the triangle.

14. The fee paid to a stock broker is called brokerage.

15. Triangles that have exactly the same size and shape are called congruent triangles.

Have the students write only the answers for ex. 1-15. Let partners check each other's work as volunteers give answers. Have students note which terms they are unsure of. Redevelop meanings before the students study the terms independently.

Explain the Social Security Act and describe some benefits available to workers under it.

Old Age and Survivors Insurance

1. The Social Security Act was passed by Congress in 1935. It established a federal program of old age insurance. This act, with its amendments, covers most wage earners and a great proportion of the self-employed. Some of the important benefits for workers who have been employed long enough to be insured under this system are:

(1) A worker receives a monthly pension for life when he retires. A worker may retire at age 65, or with a reduced pension at age 62.

(2) The wife of a retired worker receives a pension at age 65, or a reduced pension at age 62.

(3) The widow of an insured worker receives a pension if her age is 62 or over, or at any age if she is caring for a child under age 18.

(4) The children of an insured worker who dies receive monthly benefits as long as they are under age 18.

2. The monthly benefits of most workers insured under Social Security depend upon the average monthly wages of the worker since 1950. The table below gives the monthly benefits for different wages in several circumstances.

Average Monthly Earnings	Retired Worker 65	Worker 65 and Wife 65	Widow 62	Widow and 1 Child	Widow and 2 Children
$ 67	$ 40.00	$ 60.00	$ 40.00	$ 60.00	$ 60.00
100	59.00	88.50	48.70	88.50	88.50
150	73.00	109.50	60.30	109.60	120.00
200	84.00	126.00	69.30	126.00	161.60
250	95.00	142.50	78.40	142.60	202.40
300	105.00	157.50	86.70	157.60	236.40
350	116.00	174.00	95.70	174.00	254.00
400	127.00	190.50	104.80	190.60	254.00

The smallest monthly payment to a retired worker is $40 and the largest is $127. The total monthly benefits on one worker's earnings cannot exceed $254.

3. Tell from the table how much a retired worker would receive at age 65 if his average monthly earnings were $250. $95.00

Check to see that all the students understand and can use the table in ex. 2. Note that the Social Security Act is changed or amended frequently. See the Guide for sources of latest information.

4. If an insured worker has an average monthly wage of $200, what pension will he receive when he retires? $84.00 What total pension will he and his wife receive? $126.00

5. In the table, the monthly benefits for a man and his wife are what per cent greater than those of a single worker? 50%

6. An insured worker whose average wage was $250 died suddenly at age 40. What are the total monthly benefits to his widow and 2 children as long as the children are under 18? $202.40

7. In the beginning the rate of contribution for Social Security was 1%, but it has gradually been increasing. In 1963 each worker covered by Social Security contributed $3\frac{5}{8}\%$ of the first $4800 of his wages. Each worker's employer contributes an equal amount to Social Security.

8. Mr. Case earned $4000 during the year. What was the total amount of his Social Security tax? $145 How much did Mr. Case's employer also have to pay to the government? $145

9. Miss Burns earned $5300 during the year. What was her Social Security tax? $174 Remember that Miss Burns pays tax only on the first $4800 that she earns during the year. Stress.

10. Try to find out more details about Social Security.

Emphasize that the employer contributes an equal amount. In ex. 10 students can use The World Almanac as one source of reference.

Give information on fire insurance, including the premium rates charged, and give problems in calculating premiums (pages 266–269).

Fire Insurance

1. Fire losses in the United States reach approximately \$1,209,000,000 a year. Only a part of this loss is covered by insurance. Wise persons who wish protection against losses by fire insure their houses, factories, stores, and valuables by taking out policies with fire insurance companies.

2. In a recent year there were 1,024,000 fires in our cities and towns. Of these fires 482,000 were in buildings and the others were in grass, brush, or other outside places. What per cent of the fires, to the nearest whole per cent, were in buildings? 47%

3. As in life insurance, the amount for which the property is insured is the **face** of the policy, the amount paid each year for protection is the annual **premium**, and the person taking out the insurance is the **policyholder.** If the face of a policy is \$4000, this means that the greatest amount the insurance company will pay the policyholder in case of total loss of the building insured against fire is \$4000.

Ex. 1–3 should be discussed with the class. Emphasize the importance of having fire insurance. Note that the latest figures on fire losses in the United States are given in The World Almanac. Have the students check these figures.

1. The annual premium for a fire insurance policy depends upon the face of the policy and the rate that is charged for each $100 of insurance. The rate differs with the type of building, its construction, and the locality in which it stands. Stress. For example, the rate for a frame barn filled with hay and located on a farm is much higher than the rate for a house with a fireproof roof in a city having a good fire department. Why is this so?

2. Try to get the fire insurance rates for some buildings in your city or town and report to the class.

3. Mr. Williams insures his new house for $12,000 at the rate of $.24 per $100. Find his annual fire insurance premium.

▶ The face of the policy is $12,000. The rate is $.24 per $100, which means that the rate must be multiplied by 120 because $12,000 is 120 × $100. The annual premium is 120 × $.24, or $28.80. In case of total loss of the house by fire, the company will pay Mr. Williams $12,000.

4. Mr. Banks has his house insured against fire for $6000. The rate is $.34 per $100. Find the premium that Mr. Banks must pay on this policy for 1 yr. $20.40

Find the annual premium on these insurance policies: Have volunteers explain their work.

	Face	Rate per $100			Face	Rate per $100	
5.	$3000	$.28	$8.40	13.	$20,000	$.64	$128
6.	$3500	$.22	$7.70	14.	$15,800	$.15	$23.70
7.	$8400	$.18	$15.12	15.	$12,500	$.36	$45
8.	$4000	$.37	$14.80	16.	$14,000	$.88	$123.20
9.	$6000	$.19	$11.40	17.	$80,000	$.69	$552
10.	$5900	$.40	$23.60	18.	$72,000	$.35	$252
11.	$2800	$.76	$21.28	19.	$11,600	$.16	$18.56
12.	$5600	$.64	$35.84	20.	$19,000	$.40	$76

Use ex. 1 to initiate class discussion of reasons why rates vary according to type of building, its construction, and its location. In ex. 3 be sure the students understand why $.24 is multiplied by 120 (since there are 120 hundreds in 12,000).

Saving on Fire Insurance

1. The rate on a fire insurance policy can be reduced by buying a 3-year policy or a 5-year policy instead of several 1-year policies. The rate for a 3-year policy is only 2.7 times the 1-year rate, while the rate for a 5-year policy is 4.4 times the 1-year rate. If the premium for insuring a house on a 1-year policy is \$20, the premium for a 3-year policy will be 2.7 × \$20, or \$54. How much is saved in this case by buying a 3-year policy instead of three 1-year policies? \$6 If the premium for a 1-year policy is \$18, how much is saved by buying a 5-year policy instead of five 1-year policies? \$10.80

Stress. Stress.

2. If a house is insured for \$13,000 at the annual rate of \$.32 per \$100, what is the premium on a 1-year policy? \$41.60 a 3-year policy? \$112.32 a 5-year policy? \$183.04

3. Mr. North's store has been insured for 15 yr. for \$29,000 at the annual rate of \$.40 per \$100. Each year he has taken out a 1-year policy. During these 15 yr., how much would he have saved if he had taken out five 3-year policies instead of 1-year policies? \$174 if he had taken out three 5-year policies? \$208.80

Find the premiums at the given yearly rates per \$100:

	Face	Rate	Term			Face	Rate	Term	
4.	\$3000	\$.20	1 yr.	\$6	**12.**	\$15,000	\$.30	3 yr.	\$121.50
5.	\$8700	\$.42	3 yr.	\$98.66	**13.**	\$11,500	\$.62	3 yr.	\$192.51
6.	\$4750	\$.60	3 yr.	\$76.95	**14.**	\$24,500	\$.27	5 yr.	\$291.06
7.	\$9000	\$.85	5 yr.	\$336.60	**15.**	\$60,000	\$.73	5 yr.	\$1927.20
8.	\$2350	\$.38	5 yr.	\$39.29	**16.**	\$17,850	\$.19	5 yr.	\$149.23
9.	\$2800	\$.85	3 yr.	\$64.26	**17.**	\$12,200	\$.34	3 yr.	\$112.00
10.	\$4200	\$.58	1 yr.	\$24.36	**18.**	\$15,000	\$.26	3 yr.	\$105.30
11.	\$2500	\$.14	3 yr.	\$9.45	**19.**	\$44,500	\$.87	5 yr.	\$1703.46

20. In ex. 18, find the amount saved in 3 yr. by taking a 3-year policy instead of three 1-year policies. \$11.70

The method of reducing the cost of insurance, as explained in ex. 1, should be discussed carefully with the students. Do ex. 2-3 with the class to be sure all understand how premiums on 3-year and 5-year policies are computed.

1. In case of a loss by fire, the fire insurance company never pays more than the amount of the actual loss and never more than the face of the policy. If the loss is greater than the face of the policy, the company pays only the face amount. Suppose Mr. Brown carried $5000 insurance on his house and a fire damaged the house to the extent of $1500. $1500 How much did the company pay Mr. Brown? Suppose the damage in this case had been $6500. How much would the company have paid then? $5000

2. Careful records show that out of every 100 fires only 1 to 5 fires result in **total loss** of the property. In most cities, with good fire departments, most of the fires are put out before much damage is done. In a great many fires, the damage amounts to about 10% of the value of the property. For this reason, some people insure their property for a small per cent of its value, which greatly reduces their premiums. In order to induce people to insure their property more adequately, most insurance companies offer lower premium rates when property is insured for a higher per cent of its value, such as 80%. In this case, if a house is valued at $15,000, it is insured for 80% of $15,000, or $12,000. What is the face of the policy, if a house worth $14,000 is insured for 80% of its value? $11,200

3. Mr. Ware's store is worth $45,000. He insures it for 80% of its value, the annual rate being $.24 per $100. What is the premium on a 3-year policy? $233.28 a 5-year policy? $380.16

4. Miss Best insured her house for 90% of its value. Find the premium for a 3-year policy if the annual rate is $.20 per $100; the house has a value of $12,000. $58.32

5. A house is insured for 6 mo. for $7000, the annual rate being $.32 per $100. When insurance is carried for only 6 mo., the company charges a premium equal to 60% of the annual premium. Find the premium in this case. $13.44 Give a reason for insuring a house for only 6 mo. You may plan to sell it in 6 months.

In ex. 1–2 discuss the reasons why people insure property for a small per cent of its value and why insurance companies try to induce them to insure it more adequately.

Discuss automobile insurance and the reasons for carrying it.

Automobile Insurance

1. During a recent year about 38,000 people were killed and over 3,057,000 were injured in automobile accidents in the United States. That year our traffic accidents cost more than $7,900,000,000 to pay for property damage, medical and hospital costs, loss of income due to absence from work, and other costs.

2. If a person is injured by a car, the owner of the car may have to pay damages as high as $50,000 or more. If the car owner has no automobile insurance, he may have to sell his home in order to pay these damages, but if he carries automobile insurance covering **bodily injury liability,** the insurance company will pay all the damages.

3. A car owner may also be sued for damages if his car injures the car or property of another person. He also runs the risk of losing his car by fire or theft. To protect against all such losses, wise automobile owners purchase automobile insurance of several kinds.

Have the students give reasons why automobile insurance is important.

Present the last set of improvement tests in addition. Tests are to be given, scored, recorded as others were (see pages 49, 99, 139, 193, 239).

Improving by Practice

Addition Test 6a.

Time: 4 min.

1.	$888.27	$758.80	$703.82	$601.67	$280.58	
	695.49	688.23	689.93	894.55	6.90	
	736.95	67.11	392.90	112.70	667.29	
	753.98	834.90	169.78	19.54	639.26	
	3.17	312.22	3.59	4.30	37.67	
	39.69	130.86	211.26	147.61	.06	
	805.72	319.99	411.09	428.94	589.49	5
	$3923.27	$3112.11	$2582.37	$2209.31	$2221.25	

Addition Test 6b.

Time: 4 min.

2.	$ 72.68	$981.78	$ 75.69	$830.21	$ 83.66	
	480.57	44.20	164.71	293.78	284.08	
	285.43	765.42	929.44	18.59	368.23	
	525.24	94.72	65.90	306.75	168.50	
	358.30	159.78	988.83	486.48	537.27	
	345.93	94.07	4.37	147.18	999.67	
	73.26	556.38	320.49	5.29	756.79	5
	$2141.41	$2696.35	$2549.43	$2088.28	$3198.20	

Addition Test 6c.

Time: 4 min.

3.	$ 83.56	$ 50.59	$981.75	$445.25	$775.18	
	363.76	88.31	321.12	932.59	929.67	
	938.22	348.65	930.21	.23	5.98	
	273.20	894.71	4.18	379.55	56.97	
	10.65	897.36	295.58	26.44	604.95	
	509.99	653.60	665.33	540.88	449.44	
	512.87	471.16	452.78	599.69	440.78	5
	$2692.25	$3404.38	$3650.95	$2924.63	$3262.97	

Since this is the last set, note improvements and/or weaknesses.

The Language of Arithmetic

Show that you know what each word or group of words means:

4. premium
5. dividend
6. common stock
7. congruent
8. perimeter
9. proceeds
10. vertex
11. octagon
12. down payment
13. stub of check
14. unpaid balance
15. bank discount

Conduct the review orally so you can spot-check for class or individual weaknesses. Have the students explain the terms and let the class decide if the definitions are correct.

Present information on the more important kinds of automobile insurance, premium rates for such insurance, and automobile insurance problems (pages 272-275).

Kinds of Automobile Insurance

1. When Mr. Baker purchased a new car he bought the following kinds of automobile insurance in order to protect himself against damages and losses:

(1) **Bodily Injury Liability.** If anyone is injured or killed by Mr. Baker's car, the insurance company will pay the damages up to a certain amount. The premium for this insurance depends upon the amount that the company agrees to pay, the way Mr. Baker uses his car, the locality in which he lives, and the ages of the drivers of the car. In a large city where the traffic is heavy, the premium is higher than it is in a small town. In Mr. Baker's locality, for a premium of $43 a year, the company Stress. will pay damages up to $5000 for each person injured and up to a total of $10,000 for each accident. Mr. Baker wished protection up to $20,000 for injury to each person and up to $40,000 for each accident. The premium for these larger amounts of protection is 132% of $43. How much was Mr. Baker's premium? $56.76 Do you think that he was wise to carry this additional protection for the small extra cost? Yes Why? See Guide. If Mr. Baker had a son under age 25 who drove the car, or if he used his car for business purposes, his premium would be higher. Can you give reasons for this? See Guide. On the other hand, if Mr. Baker did not drive his car to work his premium would be somewhat smaller. Emphasize that the first limit applies to each individual, and the second to total damages that will be paid.

(2) **Property Damage Liability.** If a car or any other property is damaged by Mr. Baker's car, the insurance company will pay the damages up to $5000. The premium for this kind of insurance also depends upon the locality in which Mr. Baker lives and the way the car is used. For this protection he pays the company a premium of $18 a year.

(3) **Comprehensive Material Damage.** If Mr. Baker's car is stolen or damaged by fire, windstorm, hail, flood, or other causes except collision, the insurance company will pay

Have students explain why premiums vary, why you should be adequately insured, and so on. Be sure they understand the different features of each kind of insurance.

the actual cash value of the loss. For comprehensive insurance he pays $12 a year.

(4) **Collision.** The property damage liability insurance, described in paragraph (2), covers only the damage done by Mr. Baker's car to another car or to other property. For protection against damage to his own car in a collision, Mr. Baker purchased $50 deductible collision insurance. If his car is damaged in a collision, Mr. Baker must pay the first $50 of the cost of repairs and the company will pay the rest. The premium for this insurance, which depends upon the value of the car, was $46. When the insurance company agrees to pay all of the costs of repair, the premium is very much higher. Why? The company will be risking greater amounts of money.

2. Find the total amount that Mr. Baker pays for all his automobile insurance; see paragraphs (1) to (4) above. $132.76

3. Mr. Gates, who lives in the same locality as Mr. Baker, wished to buy bodily injury liability insurance with limits of $50,000 for injury to one person and $100,000 for injury to two or more persons. He found that the premium for this greater protection was only 150% of the $43 premium that is charged when the limits are $5000 and $10,000. How much did Mr. Gates pay for this insurance? $64.50

Emphasize the meaning of limits in ex. 3.

4. Mr. Gates also purchased $50 deductible collision insurance on his car. If the cost of repairing his car after an accident was $89.78, how much did the insurance company pay? $39.78 How much did Mr. Gates pay? $50

5. Automobile insurance policies have a term of one year. If one sells his car or puts it in storage before the policy expires, he no longer needs such insurance. In this case the company will cancel the policy and return part of the premium. If a bodily injury liability insurance policy is canceled after it has been in force 30 da., 81% of the premium is returned.(1) If it is canceled after 165 da., 44% of the premium is returned.(2) How much will be returned in each of these cases of canceling the insurance policy if the annual premium is $23? $43.40? $74.12? (1) $60.04; (2) $32.61

(1) $18.63; (2) $10.12 (1) $35.15; (2) $19.10

Be sure the students understand what is meant by "$50 deductible collision insurance." Illustrate the term through examples. Do ex. 3-5 with the class, letting different students explain the work.

Have the students find out about the insurance their parents carry on their cars and bring this information to class.

Buying Automobile Insurance

1. Find out what the premiums are in your city for bodily injury liability insurance with limits of \$5000 and \$10,000 and for property damage liability insurance with a limit of \$5000. Why aren't the premiums for these kinds of insurance the same in all cities? In your answer give as many reasons as you can. Discuss these fully with the class.

2. The premium for bodily injury liability insurance with limits of \$5000 and \$10,000 is called the **basic rate.** You can find the premiums for higher limits by taking the following per cents of the basic rate:

Limits of Liability	Per Cent of Basic Rate
\$ 5,000 and \$ 10,000	100%
\$ 10,000 and \$ 20,000	115%
\$ 20,000 and \$ 40,000	132%
\$ 25,000 and \$ 50,000	137%
\$ 50,000 and \$100,000	150%
\$100,000 and \$300,000	162%

Find the premiums for bodily injury liability insurance with each of the limits given above when the basic rate is \$29; \$34; \$41; \$56; \$69; \$84. See Guide.

3. The basic rate for property damage liability insurance is the premium a person must pay for property damage insurance with a limit of \$5000. The premiums for higher limits are found by taking the following per cents of the basic rate:

Limit of Liability	Per Cent of Basic Rate	Limit of Liability	Per Cent of Basic Rate
\$ 5,000	100%	\$ 30,000	121%
\$10,000	110%	\$ 40,000	123%
\$15,000	115%	\$ 50,000	125%
\$20,000	118%	\$100,000	130%

Find the premiums for property damage liability insurance with each of the limits given above when the basic rate is: \$15; \$18; \$22; \$25; \$30; \$36. See Guide.

In ex. 2 be sure the students understand what is meant by basic rate. Do part of this example with them. Let them complete it and ex. 3 if there are no questions.

Ex. 4, 5, and 9 should be used to initiate further discussion of this topic.

4. Find out what the premiums are in your city for comprehensive insurance and for \$50 deductible collision insurance for a new car of some particular make. Explain why the premiums for these kinds of insurance are not the same for all makes of cars.

5. If you buy an automobile on the installment plan, what kinds of insurance are you required to buy? Explain.

6. Damages of \$15,000 were awarded a man injured by Mr. Smith's car. If Mr. Smith carried bodily injury liability insurance with limits of \$10,000 and \$20,000, how much of these damages did the insurance company pay? Up to \$10,000

7. Damages of \$12,000 were awarded Mr. Hill, who was injured by a car whose owner carried bodily injury liability insurance with limits of \$10,000 and \$20,000. Mrs. Hill, who was also injured in the same accident, was awarded damages of \$5000. How much of these damages did the insurance company pay? \$15,000

8. Mr. Parker's car was in a collision with a truck whose contents were ruined by the accident. Mr. Parker was held responsible for \$6500 in damages. He carried property damage liability insurance with a limit of \$5000. How much of these damages did Mr. Parker have to pay himself? \$1500

9. Give reasons for purchasing each kind of insurance mentioned in ex. 2–4. Which kind of insurance do you think is the most important for an automobile owner to carry?

In ex. 6–8 remind the students of the meaning of limits.

Give information on accident and health insurance and hospitalization plans (pages 276-278).

Accident and Health Insurance

1. If a man is disabled by accident or sickness, he may lose his income for part or all of the time that he is unable to work. This loss may be a serious problem for his family and himself.

2. For protection against losses due to accidents, Mr. Peters purchased **accident** insurance for which he pays a premium of $61.35 a year. This insurance provides these benefits:

(1) If he dies due to an accident, his family will receive $5000.

(2) If he is continuously disabled by accident so that he cannot work, the insurance company will pay him $50 a week as long as the disability continues.

(3) If in an accident he loses both hands, or both feet, or one hand and one foot, or the sight of both eyes, or one hand or one foot and the sight of one eye, he will receive a payment equal in amount to 200 of the weekly payments in (2).

(4) If in an accident he loses one hand or one foot, he will receive a payment equal in amount to 100 weekly payments.

(5) If in an accident he loses the sight of one eye, he will receive a payment equal in amount to 50 weekly payments.

(6) The company will pay doctor bills and hospital bills due to an accident up to a total amount of $1000, this payment being in addition to benefits described above to which he is entitled.

3. Mr. Peters also purchased **health** insurance with an annual premium of $49, which has the following benefits:

If he is disabled by disease, the insurance company will, after the first 2 weeks of such illness, pay him $50 a week as long as he is disabled up to 52 wk.

4. The employees of a factory or a business are often protected against accidents and illness by **group accident and health insurance** which corresponds to group life insurance. Find out all you can about this insurance and tell the class.

5. Ask an insurance agent to tell you about the accident and health insurance policies that he sells. Also find the yearly premium on such policies.

The information in ex. 1-3 should be read carefully by the students. Then ask them questions to see that all have a good understanding.

1. Read carefully the conditions of Mr. Peters' accident and health insurance policies given in ex. 2 and 3 on page 276.

2. Mr. Peters became ill with pneumonia and had to be absent from work for 7 wk. For how many weeks did he receive benefits? 5 How much in all did Mr. Peters receive from the insurance company? $250

3. Mr. Peters earns $93 a week when he is well. How much in wages did he lose because of his illness? $651 How much of that loss was not made up by his health insurance? $401

4. The health insurance policy described in ex. 3 on page 276 provides no benefits for the first 2 wk. of any illness. Why is this? See Guide. It is possible to buy a health insurance policy which pays benefits for the first 2 wk., but the premium on such policies is much higher than Mr. Peters' premium. Why? See Guide.

5. If Mr. Peters is killed in an automobile accident, how much will his family receive? $5000

6. If Mr. Peters loses both feet in a serious accident, how much will the insurance company pay him for this loss? $10,000 If he loses one foot, how much will he be paid? $5000

7. Mr. Black carries accident insurance exactly like Mr. Peters'. Mr. Black was injured in a fall on a cracked sidewalk and was disabled by this accident for 9 wk. His doctor and hospital bills amounted to $785. How much in all did he receive from the insurance company? $1235

8. The premiums for accident and health insurance depend upon the amounts the insurance policy agrees to pay. Insurance can be purchased with smaller or larger benefits than those in Mr. Peters' policies. Premiums also depend upon the occupation of the insured. Why? Some occupations are more dangerous than others.

Let volunteers explain the answers to the problems. Be sure the students understand the concepts in ex. 4 and 8.

Supplement the work on these pages with information about a newer kind of insurance; that is, catastrophe or major medical expense.

Insuring Against Hospital Bills

1. Since hospital services are costly, about 60 million people subscribe to **hospitalization plans.** The membership fees of these plans vary and depend upon the kinds and amounts of services provided. Such plans usually cover part of the hospital costs of the subscribing member and, if desired, of the members of his family.

2. Mr. Snow, who is unmarried, belongs to a hospitalization plan for which he pays a membership fee of $44.40 a year. This plan entitles him to a hospital room, meals, and general nursing care up to a cost of $15 per day for 120 days during each hospitalization. In addition, he is entitled to special services, such as laboratory tests, operating room, X-ray service, drugs, serums, and dressings without charge during these 120 days.

3. During a recent year, Mr. Snow was ill in a hospital for 17 days. If he had not been a member of the hospitalization plan, he would have had to pay $28 a day for room and board, $32 for laboratory tests, and $37 for drugs, besides his doctor's bill. How much was Mr. Snow's hospital bill? $221
$279.60 How much did he save by being a member of the hospitalization plan? Remember that he paid $44.40 for his membership in a hospitalization plan. Emphasize.

4. Miss Lane is a member of the same hospitalization plan as Mr. Snow. She had a serious accident last year and had to be in the hospital for 20 weeks. The rate for her room was $21 a day, and her additional charges, all during the first 120 days, were: X-rays, $140; operating room, $80; and drugs, $57. What was her hospital bill? How much did she save by being a member of the plan? $2032.60 $1140

5. Inquire about hospitalization plans in your city. Find out what hospital services are provided to members. Also find out what membership fees are charged for single persons, for a man and his wife, and for families with children. Find out whether your family is a member of such a plan.

As suggested in ex. 5, have the students investigate hospitalization plans in your community and report their findings to the class.

Provide a review of the work previously presented.

Mixed Practice

1. Change $\frac{11}{38}$ to a decimal to the nearest thousandth. .289
2. Find, to the nearest cent, $\frac{3}{4}$ of 1% of \$875. \$6.56
3. Find the length of the circumference of a circle with a radius of $3\frac{1}{2}$ ft. 22 ft.
4. Find, to the nearest whole per cent, what per cent 128 is of 171. 75% Also find what per cent 190 is of 171. 111%
5. Which is greater, $2\frac{3}{4}$% of \$60 or $3\frac{1}{2}$% of \$50? $3\frac{1}{2}$% of \$50 How much greater is it? \$.10
6. Find the exact number of days between September 4, 1963, and January 8, 1964. 126
7. Using the compound interest table on page 212, find the amount to which \$900 will grow in 20 yr. at 3% compounded semiannually. \$1632.62
8. A rectangular box is to be constructed so that its volume is 35 cu. ft. The inside of the box will have a length of 4 ft. and a height of $3\frac{3}{4}$ ft. How wide must it be? $2\frac{1}{3}$ ft.
9. What part of 4 yd. 2 ft. is 3 ft. 6 in.? $\frac{1}{4}$
10. Find the average of $16\frac{1}{2}$, $18\frac{3}{4}$, $15\frac{1}{2}$, and $19\frac{1}{4}$. $17\frac{1}{2}$
11. How much more is a discount of 20%, 10% on \$1000 than a discount of 10%, 20% on the same amount? They are the same: \$720.
12. Write the formula for the area of a trapezoid. $A = \frac{1}{2}h(a+b)$ Find the area of a trapezoid having bases of 45 ft. and 50 ft. and a height of 38 ft. 1805 sq. ft.
13. Find, to the nearest hundredth, 17.7% of 18,597. 3291.67
14. By any method, find the interest on \$800 for 72 da. at 6%; \$9.60 on \$600 for 33 da. at 4%. \$2.20
15. A loan of \$100 was paid back in 9 monthly payments of \$12.25 each. Find the annual rate of interest. 24.6%
16. Which is larger, $\frac{192}{360}$ or $\frac{192}{365}$? Check by changing the fractions to decimals correct to the nearest thousandth. .533; .526

Students should do this work independently. Then let different students explain their answers. Use review to spot-check for class or individual weaknesses. Reteaching or further review of some topics may be needed.

Discuss prime and composite numbers. Teach the use of prime factors to find the largest common factors and the least common multiples.

Prime Numbers

1. A whole number greater than 1 which cannot be expressed as a product of two smaller whole numbers is called a **prime** number. All other whole numbers greater than 1 are called **composite** numbers. Examine all the whole numbers between 1 and 25 and tell whether each is prime or composite. See Guide.

▶ 2 is prime; $6 = 2 \times 3$, so 6 is composite; 11 is prime; $18 = 3 \times 6$, so 18 is composite.

2. 24 can be expressed as 2×12, 3×8, and 4×6. Only two factors, 2 and 3, are prime. Study the following:

$$24 = 2 \times 12 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3$$

$$24 = 3 \times 8 = 3 \times 2 \times 4 = 3 \times 2 \times 2 \times 2$$

$$24 = 4 \times 6 = 2 \times 2 \times 2 \times 3$$

When **any** composite number is expressed as the product of prime factors you get identical factors, except for the order in which they occur. It is convenient to arrange the factors by size, starting with the smallest: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$, $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$.

Express each of these numbers as a product of prime factors:

8 9 12 14 15 18 21 25 28 32 40

See Guide.

3. Any composite number greater than 6 can be expressed as a sum of different prime numbers. $8 = 3 + 5$; $9 = 2 + 7$; $12 = 2 + 3 + 7$ or $5 + 7$. Express each composite number from 13 to 30 as a sum of different prime numbers. See Guide.

4. When 24 and 30 are expressed as products of prime factors you get: $24 = 2 \times 2 \times 2 \times 3$, $30 = 2 \times 3 \times 5$.

The factor 2 and the factor 3 are factors of both 24 and 30; so 2 and 3 are called **common** factors of 24 and 30. The **largest common factor** of 24 and 30 is 2×3 or 6. Why?

Find the largest common factor of each pair of numbers:

30, 75 15 20, 30 10 8, 18 2 15, 60 15 9, 24 3 14, 35 7 45, 75 15

▶ $30 = 2 \times 3 \times 5$ and $75 = 3 \times 5 \times 5$. The product of the common prime factors is 3×5 or 15. The largest common factor of 30 and 75 is 15.

Emphasize that a whole number can be expressed as a product of prime factors in only one way, except for order. Note that the number 1 is not considered to be a prime number.

5. You know that $12 = 6 \times 2$. The product 12 is often called a **multiple** of 2, since 2 is a factor of 12. Multiples of 2 are 2, 4, 6, 8, 10, 12, ... (The three dots are read "and so on.") Multiples of 5 are 5, 10, 15, 20, 25, 30, ... Write six multiples of 3; of 4; of 6; of 12. Answers will vary.

6. Multiples of 2 are 2, 4, 6, 8, . . ., and multiples of 3 are 3, 6, 9, 12, ... Some numbers are multiples of both 2 and 3. Numbers that are multiples of more than one number are called **common** multiples of these numbers. Two common multiples of 2 and 3 are 6 and 12. Why? Give three more common multiples of 2 and 3. Give two common multiples of 3 and 5; of 4 and 6. Answers will vary.

7. Multiples of 6 are 6, 12, 18, 24, . . ., and multiples of 9 are 9, 18, 27, 36, ... A common multiple of 6 and 9 must contain as factors both 6 and 9. Three common multiples of 6 and 9 are 18, 36, and 54. Since $6 = 2 \times 3$, each common multiple of 6 and 9 must contain 2 and 3 as factors. Since $9 = 3 \times 3$, each common multiple of 6 and 9 must contain 3 as a factor twice. The smallest number that contains 2 and 3 as factors and also contains 3 as a factor twice is $2 \times 3 \times 3$ or 18. This number is called the **least common multiple** of 6 and 9. The least common multiple of two or more numbers must contain as a factor **each prime factor** of the numbers; it must also contain each prime factor the **greatest number of times** that it is contained as a factor in any one number.

8. Find the least common multiple of each group of numbers:
12, 16 48 9, 15 45 12, 18 36 30, 40 120 8, 12, 15 120

▶ $12 = 2 \times 2 \times 3$ and $16 = 2 \times 2 \times 2 \times 2$. The least common multiple of 12 and 16 must contain 2 as a factor four times and 3 as a factor once; therefore it is $2 \times 2 \times 2 \times 2 \times 3$ or 48.

9. Least common multiples are very useful in mathematics. The least common denominator for two or more fractions is the least common multiple of their denominators. Find the least common denominator for these fractions:
$\frac{1}{3}$, $\frac{3}{8}$, and $\frac{7}{12}$ 24 $\frac{4}{5}$, $\frac{1}{10}$, and $\frac{5}{6}$ 30 $\frac{2}{9}$, $\frac{7}{16}$, and $\frac{5}{24}$ 144

In ex. 9 have the students find the least common denominators by finding the least common multiples of the denominators, using the method in ex. 8. Then have them find the least common denominators by using the method given on page 16. Compare the two methods and give the advantages of each.

Show how the local tax rate is determined and how taxes on property are computed (pages 282–283).

The Tax Rate

1. Each year every city has to raise large sums of money to pay salaries of teachers, firemen, and other city workers; to maintain streets and public buildings; and to provide health and similar services for its citizens. This money is raised largely by taxing the owners of property. Each year officers of the city fix the value of all property to be taxed; this value is called the **assessed valuation.** City officers also prepare a **budget** for the coming year which is an estimate of the amount of money that will be needed to run the city.

2. This year the assessed valuation of all the property in one town is \$2,500,000. The budget of the town shows that \$78,000 is needed. The tax rate is found in the following way: \$78,000 ÷ \$2,500,000 gives .0312. This means that each property owner must pay a tax of .0312, or 3.12%, of the assessed valuation of his property. Since 3.12% of \$100 is \$3.12, the tax rate is often stated as \$3.12 per \$100, which means \$3.12 on **each** \$100 of assessed valuation.

Stress.

Emphasize after understanding is assured.

To find the tax rate, find what per cent the amount to be raised is of the total assessed valuation. Then multiply \$100 by this per cent to get the rate per \$100.

3. Tell the tax rate per \$100 that each decimal or per cent represents: .0284; .0305; 2.60%; 2.87%; 3.09%; .0415. \$4.15

\$2.84 \$3.05 \$2.60 \$2.87 \$3.09

4. The total assessed valuation of property in New City is \$6,250,000, and \$221,250 is needed for expenses. Find the tax rate and express it as a decimal, as a per cent, and as a rate per \$100. .0354, 3.54%, \$3.54 per \$100

Find the tax rate per \$100 on each of the following:

	Assessed Valuation	Amount Needed			Assessed Valuation	Amount Needed	
5.	\$92,668,000	\$3,845,722	\$4.15	**8.**	\$287,550,000	\$9,546,660	\$3.32
6.	\$78,262,500	\$2,285,265	\$2.92	**9.**	\$750,608,000	\$33,402,056	\$4.45
7.	\$66,400,000	\$1,978,720	\$2.98	**10.**	\$872,588,000	\$31,849,462	\$3.65

Be sure the meaning of assessed valuation is clear to the students. Point out the difference between it and real value (market value) of property. Carefully explain the method of computing the tax rate. Do a few more examples like ex. 3–4 before assigning ex. 5–10.

1. After the tax rate of a city is determined for the coming year, the amount of taxes that each property owner must pay is computed by the tax collector. If Mr. Field's property is assessed at $7500 and the tax rate is $3.42 per $100, his tax is found as follows: Find how many times $100 is contained in $7500, which is 75 times. Then multiply $3.42 by 75 to get the tax. How much is Mr. Field's bill for taxes? $256.50

2. Mr. Walker's property is assessed at $9200. The tax rate is $2.95 per $100. Find the amount of his tax. $271.40

Find the taxes, using these assessed valuations and tax rates:

	Assessed Value	Rate per $100			Assessed Value	Rate per $100	
3.	$8400	$3.85	$323.40	8.	$250,000	$2.84	$7100
4.	$10,500	$4.15	$435.75	9.	$121,500	$3.48	$4228.20
5.	$38,450	$2.88	$1107.36	10.	$456,400	$3.62	$16,521.68
6.	$72,200	$5.15	$3718.30	11.	$318,200	$2.76	$8782.32
7.	$86,750	$3.86	$3348.55	12.	$765,000	$3.03	$23,179.50

13. In Weston, the **actual value** of the property is $65,299,200 and the **assessed valuation** is 75% of the real value. If $2,081,412 is the amount needed for the expenses of the coming year, what is the tax rate? $4.25 per $100 Find the amount of Mr. Bacon's tax if his property is assessed at $8500. $361.25

14. Mr. Andrews has property valued at $20,000 and assessed at 70% of its value. What is the amount of his taxes if the tax rate for a certain year is $3.55 per $100? $497.00

15. Mr. Johnson and Mr. King both own property actually worth $16,000. Mr. Johnson lives where the assessed valuation of property is 60% of its real value and the tax rate is $4.75 per $100. Mr. King lives where the assessed valuation is 80% of the real value and the rate is $3.30 per $100. Who pays more taxes? Mr. Johnson How much more? $33.60

Have different students explain the work in ex. 3–12 at the board. Use ex. 13–15 to emphasize the fact that the assessed valuation is usually less than the actual value and that the rate of taxation depends on the assessed valuation.

Teach pupils how to use a tax table and give practice in using it.

How to Use a Tax Table

In a city where thousands of people pay taxes it would take too much time to compute each tax as you have done on page 283. If the tax rate is \$4.26 per \$100, a table like the one below shortens the work of computing taxes:

Tax Table for Rate of \$4.26 per \$100

	\$00	\$1000	\$2000	\$3000	\$4000	\$5000	\$6000	\$7000
\$00		42.60	85.20	127.80	170.40	213.00	255.60	298.20
50	2.13	44.73	87.33	129.93	172.53	215.13	257.73	300.33
100	4.26	46.86	89.46	132.06	174.66	217.26	259.86	302.46
150	6.39	48.99	91.59	134.19	176.79	219.39	261.99	304.59
200	8.52	51.12	93.72	136.32	178.92	221.52	264.12	306.72
250	10.65	53.25	95.85	138.45	181.05	223.65	266.25	308.85
300	12.78	55.38	97.98	140.58	183.18	225.78	268.38	310.98
350	14.91	57.51	100.11	142.71	185.31	227.91	270.51	313.11
400	17.04	59.64	102.24	144.84	187.44	230.04	272.64	315.24
450	19.17	61.77	104.37	146.97	189.57	232.17	274.77	317.37
500	21.30	63.90	106.50	149.10	191.70	234.30	276.90	319.50

1. Using this table, find the tax on property assessed at \$5250 at the rate of \$4.26 per \$100. \$223.65

▶ To find the tax on property assessed at \$5250, place a ruler horizontally under the number 250 in the left-hand column. Then in the column headed \$5000, find the number above the ruler, which is 223.65. This shows that the tax on \$5250 is \$223.65.

2. Using the table, find the tax on \$6450; \$274.77 on \$7500; \$319.50 on \$2200; \$93.72 on \$3500; \$149.10 \$4350; \$185.31 \$5450; \$232.17 \$7100; \$302.46 \$6050. \$257.73

3. In the table, study the column headed \$00. In this column, how much greater is each number than the number above it? \$2.13 Can you tell how the tax table is made? In the column headed \$4000, extend the table by giving the tax on \$4550, \$4600, etc., to \$4950; also continue the column headed \$5000 in the same way. See Guide.

4. Study the column headed \$7000. Can you tell how \$298.20 was computed? (1) How much greater is each number in this column than the number above it? (2) Extend the table to the right by making a column for \$8000, another for \$9000, and one for \$10,000. (3) (1) \$4.26 x 5 = \$21.30, \$276.90 + \$21.30 = \$298.20; (2) \$2.13; (3) see Guide.

Have the students follow the directions in ex. 1 and check to see that all locate the tax correctly. Do a few more examples and then have the students complete ex. 2. Ex. 3-4 should be discussed first before the students extend the table.

Present a discussion of federal and state taxes.

Federal and State Taxes

1. It costs billions of dollars to run all the branches of the national government and most of the money is raised by taxes. Included in these taxes are **internal revenue taxes,** such as those on gasoline, tobacco, beverages, etc.; **duties** or **customs,** which are taxes on foreign goods brought into this country; and **income taxes.** By far, the largest amount comes from income taxes. The amount of income that is taxable changes from year to year. Persons with large incomes pay a much higher tax proportionately than those with small incomes. Discuss the reasons for the last statement.

2. Since 1943, the income taxes of many wage earners have been collected weekly or monthly by having their employers withhold a certain part of their wages on each pay day. This plan, which is known as the pay-as-you-go system, keeps taxes coming into the national treasury all during the year. For many people the taxes collected in this way cover their entire income tax.

The difference between the total amount withheld and the amount of tax actually owed is settled on April 15 of the following year when the taxpayer files his income tax return. Some taxpayers are also required at this time to file estimates of their income tax and the amount to be withheld from their salaries during the current year. If the amount to be withheld is less than the estimated tax, then $\frac{1}{4}$ of the difference must be paid every three months.

3. Perhaps some member of your family pays an income tax on the pay-as-you-go plan. If so, find out how much tax is withheld from his wages on each pay day.

4. Many of the states of the United States also collect income taxes. So there are many people who pay two income taxes, one to the federal government and one to the state. When a person pays two income taxes, the income tax paid to the state is usually much smaller than that paid to the federal government. Is there an income tax in your state? If so, try to get details about it.

Lead a class discussion of the material on this page. Students should understand the difference between internal revenue taxes, duties, and income taxes. Use ex. 3-4 to initiate a further discussion the next day.

Present a brief discussion of federal income tax returns and give an illustration of how income taxes are computed.

Income Tax Returns

1. Every United States citizen or resident who has income of a certain amount must file an income tax return. One of two forms is used when making a tax return. Some taxpayers report their wages, together with the amount of tax withheld, on the short card form, and the tax collector figures their taxes and sends a bill or a refund. Other taxpayers prepare the short form of the tax return and find the amount of their taxes in a table. If a person has income greater than $10,000, he must prepare the long form of the tax return.

Be sure the meaning of taxable income is clear to the students.

2. The taxable income of each taxpayer depends upon the number of his exemptions and the amount of his deductions. An exemption of $600 is allowed each taxpayer for himself and for each of his dependents, such as his wife, a child, or a parent. He may also deduct from his income certain payments he has made, such as contributions to the church, interest on loans, and most state and city taxes.

3. When the long form of the income tax return is used, the amount of income tax may be calculated by using a tax rate schedule provided by the tax collector. Part of one tax rate schedule for single persons and married persons filing separate returns is shown below:

Taxable Income	Taxes
Not over $2000	20%
From $2000 to $4000	$400, plus 22% of excess over $2000
From $4000 to $6000	$840, plus 26% of excess over $4000
From $6000 to $8000	$1360, plus 30% of excess over $6000
From $8000 to $10,000	$1960, plus 34% of excess over $8000

4. Miss Clark used the long form of the tax return. After subtracting her allowance for exemptions and her deductions, she found her taxable income was $3150. Find her tax. $653

▶ First find how much of her income was over $2000. Have the students explain why this is done.

5. Using the above schedule, find the amount of income tax on the following taxable incomes: $1750; $2800; $4275; $5600; $7500; $8430; $9250. $2385 $1256

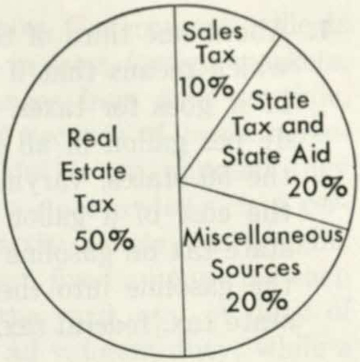
$350 $576 $911.50

$1810 $2106.20

Have in class one of the latest federal tax forms to use to supplement the work in ex. 5. Emphasize that taxes are computed on taxable income. Have the students use the tax rate schedule from the latest form to redo ex. 5. Also show how tax tables are used.

Review circle graphs through a project showing how a city gets and spends its money.

Income and Expenses of a City

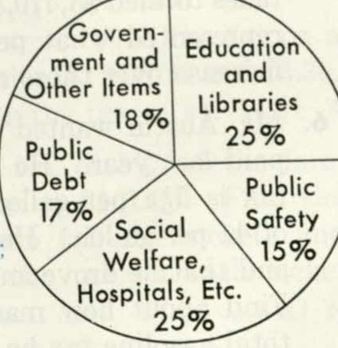


1. Fair City has a budget of \$38,500,000 for next year. To raise this money the city officials must plan carefully. The circle graph at the right shows where they expect to get the money and what part of it comes from each source. Since 50% of the income must come from real estate taxes, what amount of money must be raised by the real estate taxes? \$19,250,000

2. The assessed valuation of the real estate in Fair City is \$550,000,000. What is the tax rate if \$19,250,000 is to be raised by real estate taxes? \$3.50 per \$100

3. Using the other per cents shown on the graph, find how much money will be obtained from the sales tax,(1) how much will come from the state tax and state aid,(2) and how much from miscellaneous sources.(3) (1) \$3,850,000; (2) and (3) \$7,700,000

4. The sales tax in Fair City is approximately 2% on all sales of goods. If \$3,850,000 is to be raised by the tax of 2%, how many dollars' worth of goods must be sold? \$192,500,000



5. The circle graph at the right shows how the budget of \$38,500,000 will be spent. How much money will be used for education and libraries?(1) for the city government and other items?(2) for social welfare, hospitals, etc.?(3) (1) \$9,625,000 (2) \$6,930,000; (3) \$9,625,000

6. The 15% allowance for public safety includes the police and the fire departments. How much money will these services cost? \$5,775,000 If 17% of the budget covers the public debt, how much money is allowed for that item? \$6,545,000

Before the students do the problems, have them study the graphs in ex. 1 and ex. 5. Ask them why the sum of per cents on each is 100%.

Give information and problems on gasoline taxes.

Taxes on Gasoline

1. About one third of the price of a gallon of gasoline is taxes, which means that if a gallon of gasoline costs 27¢, then 9¢ of it goes for taxes. The federal government has a tax of 4¢ per gallon in all states. There is also a state tax in all the 50 states, varying from 5¢ to 8¢ per gallon. Find out the cost of a gallon of gasoline in your city and also the state tax on gasoline in your state. Then divide the cost of the gasoline into these three parts: actual cost of gasoline, state tax, federal tax.

2. In 1 year, a state having a gasoline tax of 6¢ per gallon raised $72,984,000 in this way. How many gallons of gasoline were sold to produce this amount in taxes? 1,216,400,000

3. How much did the federal government get from its tax of 4¢ per gallon on the gasoline sold in ex. 2? $48,656,000

4. In a recent year, the state gasoline taxes collected by all the states reached a total of $3,625,755,000. Of this total California collected the largest part, which was $360,532,000. Round off these numbers to the nearest million(1) and then find, to the nearest tenth of 1%, what per cent of the total was collected in California.(2) (1) $3626 million, $361 million; (2) 10.0%

5. During the year previous to that in ex. 4 state gasoline taxes totaled $3,470,882,000. Gasoline taxes of $3,625,755,000 represented what per cent, to the nearest tenth of 1%, of increase over those collected the previous year? 4.5%

6. Mr. Austin wanted to know how much gasoline tax he had paid last year. He lives in a state where the state gasoline tax is $5\frac{1}{2}$¢ per gallon. He also had to pay the federal tax of 4¢ per gallon. He figured that he drove 7500 mi. last year and that he drove an average of 15 mi. per gallon of gasoline. Find about how many gallons of gasoline he used 500 and the total gasoline tax he paid. $47.50

7. Every motor vehicle must carry a state license which is a tax. What is the cost of this license in your state?

Note that some students may need a review of the type of percentage problem (page 64) in ex. 5. After students complete these problems, let different ones explain their answers. Have the students report their findings for ex. 1 and ex. 7 the next day.

Give information and problems on customs duties and internal revenue.

Customs Duties and Internal Revenue

1. One way in which the United States Government collects taxes is by **duty** on goods brought in from foreign markets. The amount of duty collected changes from time to time, and in recent years has diminished because of trade agreements with many nations. When duties are imposed they are of two kinds: one, called an **ad valorem duty,** is a certain per cent of the price at which the article was bought; the other, called a **specific duty,** is a fixed amount for each unit of measure, such as the ton, the yard, etc. A duty of 30% on the cost of an article is an ad valorem duty, while a duty of $1.25 per ton is a specific duty. Duties are collected by customs officers of the United States Government.

2. Try to find out about the duty on some special article, such as gloves manufactured abroad, bulbs from Holland, or woolen material from Great Britain, and report to the class on what you have learned.

Stress. **3. Internal revenue** means the federal taxes collected on certain items within the United States. The chief source of internal revenue is the income tax, but there are other internal revenue taxes,* such as a 10% tax on jewelry and luggage. Find the tax on a watch costing $21.90; $2.19 on one costing $35.20; $3.52 $65. $6.50 Find the tax on a diamond ring costing $1275; $127.50 on one costing $1850. $185 Stress.

*Point out that these are called excise taxes.

4. There is also a tax of 10% on telegrams. Find the tax on a telegram costing $1.70; $.17 on one costing $1.20; $.12 $2.20; $.22 $3.30. $.33 Find the total cost of each, including the tax. $1.87, $1.32, $2.42, $3.63

5. All airline tickets are subject to a tax of 5% of the fare. The one-way fare between two cities is $161.20. Find the total cost,(1) including the tax,(2) of two full-fare tickets and three half-fare tickets. Calculate the tax on each ticket separately. (1) $592.41; (2) $8.06 on full fare, $4.03 on half fare

6. There is a 10% tax on the cost of renting a safe deposit box in a bank. Find the tax on a box that has an annual rental charge of $7.50. $.75

Emphasize that duties are taxes on goods brought into the United States from foreign countries, while excise taxes are placed on services and articles made and sold in this country.

Chapter Review

1. Tell two differences between ordinary life insurance and 20-payment life insurance. See Guide.
2. What is meant by the cash value of a life insurance policy? by a life insurance premium? a dividend? See Guide.
3. Name a type of building and its location on which the rate for fire insurance is probably low. Also name a type on which the rate is likely to be high. Answers will vary.
4. Name a use to which a building might be put which would cause it to have a high rate of fire insurance. Used to store gasoline, explosives, other inflammable goods
5. At an annual rate of \$.34 per \$100, how much cheaper are three 5-year policies than five 3-year policies if the face of the policy is \$5000? \$5.10
6. In one state the rates on automobile insurance policies covering bodily injury have recently been raised 16.5% and on policies covering property damage, 27.5%. Name some reasons why you think this increase has taken place. Accidents were increasing in number.
7. How is the tax rate on property determined? How is each individual's real estate tax figured? See pages 282-283.
8. Name three different taxes collected by the federal government; name two different taxes collected by most states; name two different taxes collected by your city. Answers will vary.
9. Name some taxes paid by automobile owners. Answers will vary.
10. Many of the states in this country have state income taxes which must be paid by residents in addition to the federal income tax. In one state the rate is 2% on the first \$1000 of net income, 4% on the next \$2000, 6% on the next \$2000, and $7\frac{1}{2}$% on all over \$5000. Find Mr. Lee's tax in this state if his net income is \$4500. \$190
11. In a recent year the total amount of motor fuel used was 57,877,826,000 gal., on which the federal government got a tax of 4¢ a gallon. Find the total taxes on motor fuel collected by the government that year. \$2,315,113,040

After the students complete ex. 1-11, have different ones explain their answers. Be sure concepts, meanings, and terms in ex. 1-4 and ex. 6-9 are clearly understood. Plan further individual or class review as needed.

1. Mr. Bond insured his life for \$20,000. If the annual premium is \$21.30 per \$1000, find the total amount of the annual premium Mr. Bond pays. \$426.00

2. Miss Wood had a life insurance policy for \$5000 on which she paid a premium of \$18.21 per \$1000. After 10 yr. she dropped the policy and received a cash value of \$430. How much did this policy actually cost Miss Wood for 10 yr.? \$480.50

3. Mr. Carter can insure his house for \$8000 at 20¢ per \$100 for 1 yr. A 3-year policy costs 2.7 times as much. How much can Mr. Carter save in 3 yr. by buying the 3-year policy instead of three 1-year policies? \$4.80

4. What is the rate of income in per cent if a stock is bought at 40 and pays a dividend of \$1.60 per year? 4%

5. The city of Beaver needs taxes of \$78,372. If the assessed valuation of the taxable property in Beaver is \$2,488,000, find the tax rate on real estate. \$3.15 per \$100

6. If the cost of a book increases from 55¢ to 89¢, find, to the nearest whole per cent, the per cent of increase. 62%

7. Miss Ray can buy a chair for \$50 cash or on the installment plan by making a down payment of \$10 and 11 monthly payments of \$4 each. How much more does the chair cost if bought on installments than for cash? \$4

8. Tom can row a boat 2 mi. in 24 min. At that rate, how far can he row it in 1 hr.? 5 mi.

9. At 25 sheets for 9¢, find the cost of 300 sheets of paper; \$1.08 find the cost of 175 sheets. 63¢

10. The scale on a map is $\frac{3}{4}$ in. = 15 mi. Find the actual distance between two cities that are $2\frac{1}{4}$ in. apart on this map. 45 mi.

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

After checking the answers and noting the kinds of errors, return the papers so that the students can find and correct their mistakes. Through students' explanations of their work, determine the causes of errors and plan remedial work accordingly.

Present a diagnostic test of percentage skills, with practice-page references.

How Much Have You Learned?

If you miss more than one example in a group, turn to the Practice Pages for that group.

Find the answers to the nearest cent: (Practice Pages)

			Practice Pages
1.	6.5% of \$258 \$16.77	16.4% of \$724 \$118.74	43
2.	$\frac{3}{4}$% of \$425 \$3.19	$\frac{1}{2}$ of 1% of \$550 \$2.75	45
3.	280% of \$640 \$1792	127.6% of \$250 \$319	50

Find the interest:

4.	\$600, $\frac{1}{2}$ yr., 4% \$12	\$750, $\frac{1}{3}$ yr., 5% \$12.50	118
5.	\$300, 1 mo., 3% \$.75	\$450, 7 mo., 2% \$5.25	120
6.	\$480, 63 da., 6% \$5.04	\$800, 24 da., 6% \$3.20	122–124
7.	\$500, 36 da., 3% \$1.50	\$780, 90 da., 5% \$9.75	124

Find the selling price; the expenses and profit are given as per cents of the selling price:

	Cost	Exp.	Profit		Cost	Exp.	Profit		
8.	\$438	30%	10%	\$730	\$8.12	36%	6%	\$14	73
9.	\$336	32%	12%	\$600	\$2.52	35%	9%	\$4.50	73
10.	\$206	40%	$8\frac{1}{2}$%	\$400	\$4.55	33%	15%	\$8.75	73

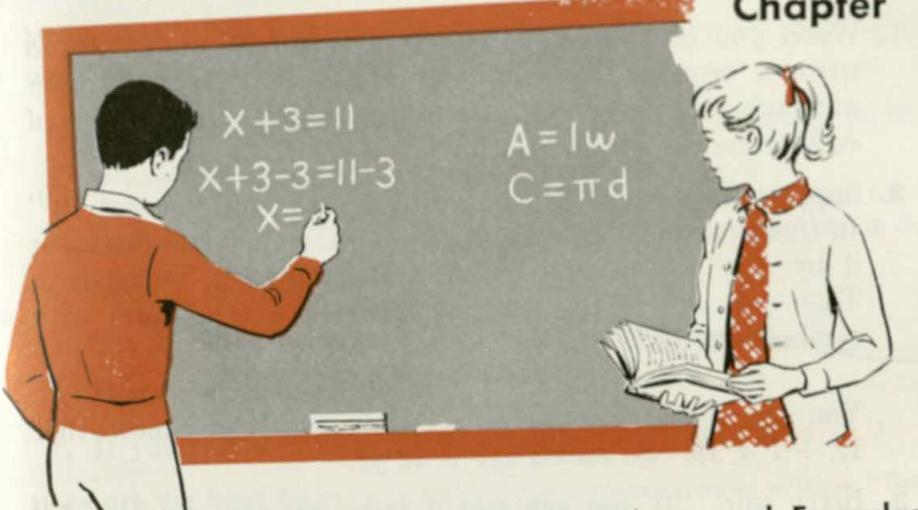
Find the missing number for each space. In ex. 13, find the nearest whole per cent; in ex. 14–15, find the nearest tenth of 1%:

11.	42 is 24% of ... 175	99 is 45% of ... 220	64, 66
12.	38 is 20% of ... 190	87 is 75% of ... 116	64, 66
13.	11 is ... 28 % of 39	50 is ... 69 % of 72	52
14.	67 is ... 79.8 % of 84	53 is ... 55.2 % of 96	53, 54
15.	28 is ... 35.4 % of 79	34 is ... 81.0 % of 42	53, 54

Let volunteers explain their work so others can find and correct mistakes. Be sure the students understand why they made mistakes. Reteach as necessary, emphasizing understanding.

See the Guide for the specific aims of Chapter 9.

Chapter 9



Sentences, Equations, and Formulas

1. A sentence that tells how numbers are related is called a **mathematical sentence.** It is convenient and useful to write such a sentence in a brief form by using mathematical symbols. The sentence "three plus five equals eight" can be written with symbols as "$3 + 5 = 8$." Some mathematical sentences involve one or more unknown numbers. $x + 3 = 11$ is a sentence in which the letter x represents an unknown number. Unknown numbers in mathematical sentences are usually represented by letters, such as x, r, and A.

2. When a mathematical sentence expresses equality, it is called an **equation.** $3 + 5 = 8$ and $x + 3 = 11$ are equations. In this chapter you will study equations and other kinds of mathematical sentences.

3. A **formula** is an important kind of equation. You have used the formula $A = lw$. If the length and width of a rectangle are known, this formula can be used to find the area of the rectangle. You have used other formulas to find the area of triangles and circles and the volume of pyramids and cylinders.

Discuss the meaning of the terms mathematical sentence, equation, and formula. Point out the differences in meaning or use of these terms.

Show how to use letters to represent numbers, how to write mathematical sentences, and how to find unknown numbers mentally (pages 294–295).

Using Letters for Numbers

1. When you use letters to represent unknown numbers and write a sentence showing how these numbers and other numbers are related, you are using algebra. Illustrations of such sentences are given below.

2. Suppose Jack says, "If you add 7 to my age, you get 19." In this situation Jack's age is unknown, so you can represent it by x or by the first letter of the word *age,* which is a. Then you can write Jack's sentence in a brief form with symbols like this:

$$a + 7 = 19$$

You can easily see that $a = 12$ because you know that $12 + 7 = 19$. So Jack's age is 12 yr.

3. Betty said, "If you subtract 4 from my age, you get 7." Written with symbols, this becomes:

$$a - 4 = 7$$

What is the value of a? 11 How many years old is Betty? 11

4. Bill said, "5 times my age is 45." Writing this in a brief form, you get:

$$5\,a = 45$$

Here $5\,a$ means 5 times a. What is the value of a? 9 What is Bill's age? 9 yr.

5. In algebra multiplication signs are usually omitted; ab means $a \times b$. Likewise, $9\,b$ means $9 \times b$ and $\frac{1}{2}\,b$ means $\frac{1}{2} \times b$ or $\frac{1}{2}$ of b. What does $5\,x$ mean? What does $\frac{1}{3}\,bh$ mean?

6. How would you write with symbols "8 times a number"? $8n$ "$\frac{1}{4}$ of a number"? $\frac{1}{4}n$ "100 times a number"? $100n$ "6 less than a number"? $n - 6$ "50 more than a number"? $n + 50$

7. John said, "$\frac{1}{2}$ of my age is 7." Write John's sentence with symbols. $\frac{1}{2}a = 7$ What is John's age? 14 yr.

8. Grandmother said, "$\frac{1}{3}$ of my age is 25." Write her sentence with symbols. $\frac{1}{3}a = 25$ What is her age? 75 yr.

Emphasize that the letter a or x represents a number. Give the students more practice in writing mathematical sentences.

Have volunteers explain their answers at the board. Encourage others to ask questions if they are unsure of this work.

Finding Unknown Numbers

Write each of the following in symbols, as explained on page 294, and find the unknown number:

1. 4 plus a number gives 12. $4 + x = 12$; 8
2. If you add 5 to a number, you get 13. $x + 5 = 13$; 8
3. Half of a number is 6. $\frac{1}{2}x = 6$; 12
4. Add 12 to a number and you get 20. $x + 12 = 20$; 8
5. If you take 8 from a number, you get 14. $x - 8 = 14$; 22
6. If you take $\frac{1}{3}$ of a number, the answer is 5. $\frac{1}{3}x = 5$; 15
7. If you take a number from 10, you have 7 left. $10 - x = 7$; 3

6 times a number is 18. $6x = 18$; 3

10 less a number is 3. $10 - x = 3$; 7

Here are the brief forms in which several sentences were written. Tell what each sentence was and what number each letter stands for:

8. $x + 1 = 3$ 2
9. $8 - x = 2$ 6
10. $5 - n = 4$ 1
11. $4 + n = 11$ 7
12. $k - 5 = 12$ 17
13. $n - 11 = 8$ 19
14. $2n = 8$ 4
15. $9n = 36$ 4
16. $\frac{1}{2}n = 14$ 28

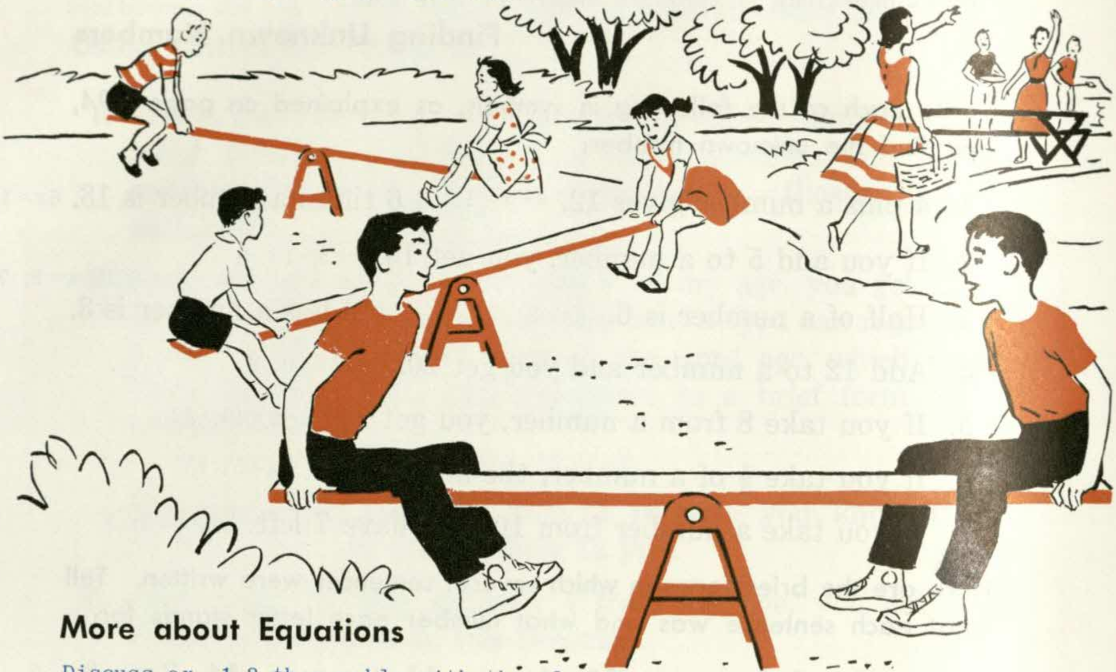
17. The sentence $x + 3 = 5$, in which one expression is equal Stress. to another, is called an **equation.** You **solve** the equation when you find the **value** of x that makes the equation a true sentence. In this equation 2 is the **value** of x that makes it true. To check the work, substitute 2 for x in the equation $x + 3 = 5$. Since $2 + 3 = 5$, the value of x checks.

Solve the following equations by finding the value of x. Check:
Have the students show the check as in ex. 17.

18. $x + 2 = 8$ 6
19. $x - 8 = 5$ 13
20. $7 - x = 2$ 5
21. $x + 5 = 6$ 1
22. $3 + x = 9$ 6
23. $12 = x + 9$ 3
24. $15 = x - 6$ 21
25. $11 = 18 - x$ 7
26. $30 = x + 12$ 18
27. $21 = x + 13$ 8
28. $4x = 20$ 5
29. $15 = 5x$ 3
30. $9x = 27$ 3
31. $\frac{1}{4}x = 20$ 80
32. $\frac{1}{8}x = 3$ 24

Do ex. 8-16 orally. Be sure all the students can give the original sentences. In ex. 17 emphasize that "to solve" an equation means to find the value of the letter that makes the mathematical sentence true.

Explain more about equations and show how to solve them, using the subtraction principle (pages 296-298).



More about Equations

Discuss ex. 1-3 thoroughly with the class.

1. You can easily solve the equation $x + 8 = 11$ mentally, but it is more difficult to find the value of x in the equation $7x + 16 = 191$. However, when you have learned a little more about equations, you will be able to solve equations of this type by applying a few simple rules.

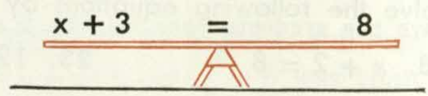
2. Every equation has two sides separated by the sign $=$. Thus the equation $x + 3 = 8$ has a left side, $x + 3$, and a right side, 8, with the sign $=$ between them.

Left side Right side

$$x + 3 = 8$$

3. When you solve an equation you find a value of x which makes the left side exactly equal to the right side. An equation is similar to a seesaw. When the correct value is substituted for x in an equation, the two sides are equal and the equation is balanced. When the weight on one end of a seesaw is exactly equal to the weight on the other end, the seesaw is balanced.

Emphasize.



When discussing the illustration, make clear to the students that when both sides of an equation are equal in value, it is balanced. Therefore, to solve an equation you must find the value of an unknown number that will balance the equation.

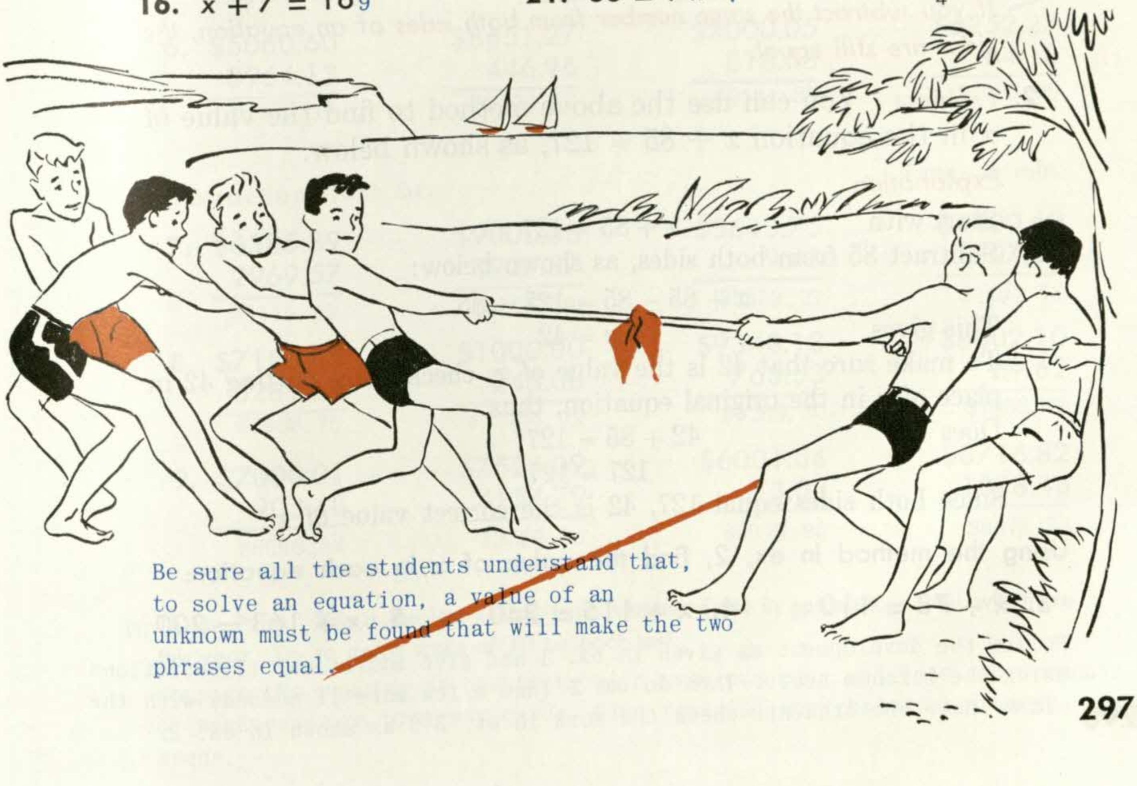
4. The two sides of an equation are sometimes called mathematical or number **phrases.** To solve an equation you find a value of the unknown number that makes these two phrases equal.

Write the following phrases in symbols. Use *n* as the unknown number.

5. A number plus six $n+6$
6. One-third of a number $\frac{1}{3}n$
7. Seven subtracted from a number $n-7$
8. Five times a number $5n$
9. Seven added to three times a number $3n+7$
10. Five subtracted from four times a number $4n-5$
11. Eight times a number and that amount subtracted from 12 $12-8n$

Express each equation as a sentence in words. Then solve each equation mentally.

12. $x + 5 = 12$ 7
13. $d - 3 = 21$ 24
14. $6 + x = 14$ 8
15. $x - 11 = 15$ 26
16. $x + 7 = 16$ 9
17. $2n = 16$ 8
18. $60 = 5a$ 12
19. $\frac{1}{2}n = 7$ 14
20. $4n = 36$ 9
21. $63 = 9n$ 7
22. $\frac{1}{2}n = 35$ 70
23. $3x = 24$ 8
24. $15 = 2x$ $7\frac{1}{2}$
25. $5 = \frac{1}{3}x$ 15
26. $\frac{1}{4}b = 10$ 40



Be sure all the students understand that, to solve an equation, a value of an unknown must be found that will make the two phrases equal.

The Subtraction Principle

1. Problem Jim tried this experiment with the balance scale shown in the drawing. He placed a 16-oz. weight and a 4-oz. weight in each pan of the scale. The scale is in balance because there is the same total weight in each pan.

If Jim takes the 4-oz. weight out of one of the pans of the scale, will it still be in balance? No What must be taken from the other pan of the scale to keep it in balance? 4 oz. weight What weight is then left in each pan? 16 oz.

Explanation The above may be written in equation form, thus:

Start with $16 + 4 = 16 + 4$

Then subtract 4 from both sides, like this:

$$16 + 4 - 4 = 16 + 4 - 4$$

This leaves $16 = 16$

Emphasize after understanding is assured.

If you subtract the same number from both sides of an equation, the sides are still equal.

2. Problem You can use the above method to find the value of x in the equation $x + 85 = 127$, as shown below.

Explanation

Start with $x + 85 = 127$

Subtract 85 from both sides, as shown below:

$$x + 85 - 85 = 127 - 85$$

This gives $x = 42$

To make sure that 42 is the value of x, check it by putting 42 in place of x in the original equation, thus:

Does $42 + 85 = 127$?

$127 = 127$

Since both sides equal 127, 42 is the correct value of x.

Using the method in ex. 2, find the value of x in each equation:

3. $x + 78 = 110$ 32 **4.** $x + 115 = 240$ 125 **5.** $x + 163 = 200$ 37

Follow the development as given in ex. 1 and give additional illustrations using the balance scale. Then do ex. 2 (and a few more if needed) with the class. Have the students check the work in ex. 3–5 as shown in ex. 2.

Present the last set of improvement tests in subtraction.

Improving by Practice

Subtraction Test 6a.

Time: 4 min.

1.	\$8000.82 2185.83 \$5814.99	\$6131.84 4791.99 \$1339.85	\$4221.62 2.86 \$4218.76	\$3007.01 951.16 \$2055.85	
2.	\$9619.03 5954.07 \$3664.96	\$1300.18 979.02 \$ 321.16	\$6000.00 685.15 \$5314.85	\$5893.61 98.74 \$5794.87	
3.	\$9639.37 152.59 \$9486.78	\$5000.06 1479.76 \$3520.30	\$7843.25 33.66 \$7809.59	\$9200.45 7842.59 \$1357.86	12

Subtraction Test 6b.

Time: 4 min.

4.	\$7030.50 675.63 \$6354.87	\$3791.42 2295.48 \$1495.94	\$4000.00 69.51 \$3930.49	\$7235.73 3470.76 \$3764.97	
5.	\$6742.41 3964.83 \$2777.58	\$7428.15 7144.86 \$ 283.29	\$8050.80 2.55 \$8048.25	\$7000.00 6413.90 \$ 586.10	
6.	\$5060.60 3964.13 \$1096.47	\$8651.27 446.96 \$8204.31	\$3000.05 818.68 \$2181.37	\$6252.27 64.18 \$6188.09	12

Subtraction Test 6c.

Time: 4 min.

7.	\$6385.39 2969.57 \$3415.82	\$9000.06 3927.37 \$5072.69	\$5648.75 9.48 \$5639.27	\$1400.60 207.84 \$1192.76	
8.	\$7185.45 5281.69 \$1903.76	\$1000.00 243.08 \$ 756.92	\$9348.13 763.53 \$8584.60	\$8002.10 69.62 \$7932.48	
9.	\$7000.01 903.48 \$6096.53	\$7524.39 4154.79 \$3369.60	\$6001.04 73.08 \$5927.96	\$8716.82 1838.48 \$6878.34	12

To the Pupil. This is the last set of Improvement Tests in subtraction you will have this year. Try to get a score of 10 on each test.

Compare the results of the tests with previous ones and note improvements or weaknesses on progress cards. Plan remedial work based on student needs.

Teach the use of the addition principle in solving equations. The students should show all work as in ex. 2.

The Addition Principle

1. **Problem** Ann and Susan are sisters. They each have \$50 in the savings bank. Susan is now going to put \$12 more in the bank. If Ann wants to keep her bank account equal to Susan's, how much more money must she put in her account? \$12 Will these sisters each have the same amount in the bank then? Yes

Explanation This can be written in equation form, like this:

Start with	\$50 = \$50
Add \$12 to each side:	\$50 + \$12 = \$50 + \$12
This gives	\$62 = \$62

Emphasize after understanding is assured.

If the same number is added to both sides of an equation, the sides are still equal.

2. You may use the above principle to solve the equation $x - 5 = 8$. Here $x - 5$ means that 5 has been taken away from x. To get x again, you must add the number that was taken away; so you add 5 to $x - 5$. This gives $x - 5 + 5$, which equals x. Be sure students understand this.

Explanation

Start with the equation	$x - 5 = 8$
Add 5 to both sides:	$x - 5 + 5 = 8 + 5$
This gives	$x = 13$

To check, put 13 in place of x in the original equation.

Does $13 - 5 = 8$?
$8 = 8$

3. What must you do to $x - 7$ to change it to x? Add 7

4. If you add 3 to the left side of an equation, what must you do to the right side of the equation? Add 3 Why? To keep both sides equal

Solve the following equations and check the work:

5. $x - 4 = 9$ 13	9. $n - 36 = 14$ 50	13. $a - 400 = 531$ 931
6. $x - 1 = 5$ 6	10. $d - 72 = 75$ 147	14. $b - 723 = 100$ 823
7. $x - 7 = 4$ 11	11. $a - 15 = 12$ 27	15. $c - 106 = 320$ 426
8. $n - 5 = 3$ 8	12. $c - 40 = 40$ 80	16. $r - 212 = 240$ 452

Discuss ex. 1 with the class and illustrate further as needed. Ex. 2 must be carefully explained to the students. Emphasize that we add to both sides of an equation the number that will leave only x on one side. Have the students check the answers in ex. 5-16 as is done in ex. 2.

Summarize the addition and subtraction principles and give practice in their use.

Solving Equations

1. You have seen that an equation is similar to a balance scale. If you subtract a number from one side of an equation, you must subtract the same number from the other side also in order to keep the equation in balance. If you add a number to one side of an equation, you must add the same number to the other side of the equation also.

2. When you solve the equation $x + 85 = 127$, you want to find the value of x. Therefore you subtract 85 from $x + 85$ so that x will be left alone on the left side of the equation. Then you must subtract 85 from 127 also to keep the equation in balance. When you subtract 85 from both sides, the sides are still equal, and you find that $x = 42$. So 42 is the value of x that makes the equation a true sentence.

3. When you solve the equation $n - 53 = 75$, you want to find the value of n. So you add 53 to $n - 53$ so that n will be left alone on the left side of the equation. Then you must add 53 to the right side of the equation to keep the equation in balance. You find that $n = 128$, which is the value of n that makes the equation a true sentence.

In the following, find the value of x by subtraction or by addition. Check by putting the answer in place of x in the given equation:

4. $x + 4 = 25$ 21
5. $x - 6 = 54$ 60
6. $x + 4 = 37$ 33
7. $9 + x = 16$ 7
8. $5 + x = 44$ 39
9. $x + 18 = 31$ 13
10. $x + 29 = 56$ 27
11. $x - 12 = 47$ 59
12. $40 + x = 62$ 22
13. $19 + x = 81$ 62
14. $x - 125 = 432$ 557
15. $x - 218 = 321$ 539
16. $x + 465 = 825$ 360
17. $500 + x = 905$ 405
18. $450 + x = 675$ 225

Write each sentence as an equation, letting *n* represent the unknown number. Then find the number and check.

19. A certain number plus 14 equals 36. $n + 14 = 36$; 22
20. A certain number less 125 equals 346. $n - 125 = 346$; 471
21. 45 plus a certain number equals 93. $45 + n = 93$; 48

In connection with the work on this page, discuss the relationship of addition and subtraction as inverse operations.

Teach the use of the multiplication and division principles in solving equations (pages 302-303). The students' written work should be done as in ex. 2.

The Multiplication Principle

Emphasize the importance of checking.

1. Problem Joe was given two puppies, each weighing 3 lb. A year later each puppy weighed 5 times as much. What were their weights then? 15 lb. Were their weights equal? Yes

Explanation You can express these facts in equation form, thus:

Start with	$3 = 3$
Multiply both sides by 5:	$5 \times 3 = 5 \times 3$
This gives	$15 = 15$

Emphasize after understanding is assured.

If both sides of an equation are multiplied by the same number, the sides are still equal.

2. You may use the above principle to solve the equation $\frac{1}{2}n = 7$, in which $\frac{1}{2}n$ means $\frac{1}{2}$ of n. You can change $\frac{1}{2}n$ to n by multiplying $\frac{1}{2}n$ by 2. So, in the work below you must multiply **both sides** of the equation by 2. Emphasize.

Explanation

Start with	$\frac{1}{2}n = 7$
Multiply both sides by 2:	$2 \times \frac{1}{2}n = 2 \times 7$
This gives	$n = 14$
Check: Does	$\frac{1}{2} \times 14 = 7?$

3. Another way to write $\frac{1}{2}n$ is $\frac{n}{2}$. What is another way to write $\frac{1}{3}x$? $\frac{x}{3}$ $\frac{1}{4}c$? $\frac{c}{4}$

4. By what number do you multiply $\frac{1}{3}x$ to change it to x? 3

Solve the following equations. Check your results:

5. $\frac{1}{2}x = 8$ 16	**11.** $\frac{1}{5}x = 6$ 30	**17.** $\frac{1}{4}d = 16$ 64
6. $\frac{1}{3}x = 9$ 27	**12.** $5 = \frac{1}{2}x$ 10	**18.** $12 = \frac{1}{3}r$ 36
7. $\frac{1}{5}x = 3$ 15	**13.** $\frac{1}{7}x = 6$ 42	**19.** $\frac{1}{8}x = 10$ 80
8. $\frac{1}{2}x = 6$ 12	**14.** $\frac{1}{4}x = 3$ 12	**20.** $15 = \frac{1}{5}n$ 75
9. $\frac{1}{3}a = 7$ 21	**15.** $\frac{1}{6}x = 4$ 24	**21.** $\frac{1}{2}x = 13$ 26
10. $\frac{b}{5} = 2$ 10	**16.** $\frac{x}{4} = 24$ 96	**22.** $\frac{r}{3} = 18$ 54

In ex. 2 try to lead the students to suggest how to change $\frac{1}{2}n$ to n. Do a few more examples like ex. 2 with the class. Have students explain why they multiplied by certain numbers in ex. 5-22.

1. **Problem** The Carson twins received \$10 apiece on their birthday. They decided that they would each put $\frac{1}{5}$ of their money in the bank. Did each twin put the same amount in the bank? \$2, yes

Explanation You can write the above in equation form, thus:

Start with $10 = 10$

Divide each side by 5: $10 \div 5 = 10 \div 5$

This gives $2 = 2$

Emphasize after understanding is assured.

If both sides of an equation are divided by the same number, the sides are still equal.

2. You can use the above principle to solve the equation $3x = 24$, in which $3x$ means 3 times x. To change $3x$ to x, divide $3x$ by 3; also divide 24 by 3 to keep both sides of the equation equal. Emphasize that you must divide both sides by 3.

Explanation

Start with $3x = 24$

Divide both sides by 3: $\frac{3x}{3} = \frac{24}{3}$

This gives $x = 8$

Check: Does $3 \times 8 = 24$?

3. By what number do you divide $8x$ to change it to x? 8

Solve the following and check your results:

4. $5x = 30$ 6
5. $7b = 35$ 5
6. $4d = 60$ 15
7. $28 = 7a$ 4
8. $8n = 8$ 1
9. $3x = 1$ $\frac{1}{3}$
10. $6x = 3$ $\frac{1}{2}$
11. $5b = 10$ 2
12. $20 = 8x$ $2\frac{1}{2}$
13. $9x = 36$ 4
14. $2x = 500$ 250
15. $150 = 3n$ 50
16. $4b = 200$ 50
17. $6a = 150$ 25
18. $7x = 210$ 30

19. If Tom has two 8-pound pumpkins and puts one on each side of a scale, will the scale balance? Yes If he cuts each pumpkin into 4 equal parts and puts one part on each side of the scale, will the scale balance? Yes Write the equation. $8 = 8$; $\frac{8}{4} = \frac{8}{4}$; $2 = 2$

Point out that we divide both sides of an equation by a number that will leave only x on one side. Discuss the relationship of multiplication and division as inverse operations.

Summarize the four principles of equations and use them in solving equations.

Four Principles of Equations

1. You have studied these four different kinds of equations:

$x + 4 = 8$ (2) $\quad x - 4 = 8$ (1) $\quad \frac{1}{4}x = 8$ (3) $\quad 4x = 8$ (4)

To solve each of these equations, tell whether you would

(1) add the same number to both sides, or
(2) subtract the same number from both sides, or
(3) multiply both sides by the same number, or
(4) divide both sides by the same number.

2. When you solve an equation, you use the principle that leaves only x (or the letter that stands for the unknown number) on one side of the equation and only known numbers on the other side of the equation.

For example, in solving the equation $x + 4 = 8$, you subtract 4 from both sides in order to have only x left on one side. What must you do to $x - 4 = 8$ to have only x left on one side? What must you do to $\frac{1}{4}x = 5$ to have only x left on one side?

What would you do to each phrase to have only x left?

3.	$x + 7$ Subtract 7	$x - 12$ Add 12	$11x$ Divide by 11	$\frac{1}{4}x$ Multiply by 4	$x + 11$ Subtract 11
4.	$x - 4$ Add 4	$x + 14$ Subtract 14	$90x$ Divide by 90	$\frac{1}{5}x$ Multiply by 5	$8 + x$ Subtract 8

Decide which principle to use, and solve the following:

5. $x + 1 = 9$ 8	**13.** $\frac{1}{2}x = 13$ 26	**21.** $10 = x + 4$ 6
6. $x - 3 = 6$ 9	**14.** $\frac{1}{4}x = 9$ 36	**22.** $15 = 3 + n$ 12
7. $r + 21 = 22$ 1	**15.** $\frac{1}{5}x = 11$ 55	**23.** $28 = 14x$ 2
8. $x - 11 = 64$ 75	**16.** $4m = 32$ 8	**24.** $28 = \frac{1}{2}x$ 56
9. $b + 15 = 33$ 18	**17.** $9x = 72$ 8	**25.** $28 = x + 27$ 1
10. $x - 18 = 27$ 45	**18.** $11x = 33$ 3	**26.** $28 = x - 27$ 55
11. $10 + d = 48$ 38	**19.** $\frac{1}{3}d = 50$ 150	**27.** $28 = 28a$ 1
12. $11 + c = 25$ 14	**20.** $6x = 138$ 23	**28.** $14 = 28a$ $\frac{1}{2}$

Read and discuss ex. 1-2 with the class. Do ex. 3-4 as an oral activity. Have the students complete ex. 5-28 and point out that the unknown is not always on the left side. Ask the students to explain which principle they used and why.

Teach students how to understand and write the language of algebra (mathematical phrases).

Getting Ready to Solve Problems

1. Sam had walked x miles. If he walked 7 miles more, how many miles did he walk in all? $(x + 7)$ miles

▶ In the language of algebra, you say that the number of miles Sam walked was $(x + 7)$ miles.

2. Mr. Weeks had d puppies in his shop. After he had sold 3 puppies, how many puppies were there left? $(d - 3)$ puppies

▶ In algebraic language, you say that $(d - 3)$ puppies were left.

3. Last week, John sold x tickets for the school play. If Bill sold twice as many tickets as John, how many tickets did Bill sell? $2x$ tickets If Robert sold half as many tickets as John, how many tickets did Robert sell? $\frac{1}{2}x$ tickets

4. Mr. Martin bought a boat for s dollars. How many dollars did he receive for the boat if he sold it at a profit of \$8? $(s + 8)$ dollars if he sold it at a loss of \$3? $(s - 3)$ dollars if he sold it for twice as much as it cost? $2s$ dollars if he sold it for half as much as it cost? $\frac{1}{2}s$ dollars

5. Peggy bought a pair of roller skates for d dollars. Jane waited for a sale and bought similar skates for \$1 less than Peggy paid. How many dollars did Jane's skates cost? $(d - 1)$ dollars

6. Jerry hiked x miles from his camp to Silver Lake and 1 mile from Silver Lake to Indian Cave. How many miles did he hike from his camp to the cave? $(x + 1)$ miles

7. If x represents Mary's age, what represents your age if you are twice as old as Mary? $(2x)$ years if you are 5 years older than Mary? $(x + 5)$ years if you are 3 years younger than Mary? $(x - 3)$ years

8. Sue baked x cookies. The family ate 19 cookies for supper. How many cookies were left? $(x - 19)$ cookies

9. After 123 cones of ice cream were sold in the school lunch room, there were x cones left. How many cones had there been at first? $(x + 123)$ cones

10. If $x = 12$, find the value of $x + 2$; 14 of $x - 7$; 5 of $\frac{1}{4}x$. 3

11. If $x = 7$, find the value of $2x$; 14 of $3x$; 21 of $3x + 4$. 25

If students have difficulty with ex. 1-9, ask them to write similar phrases which involve only numbers.

Teach the students the use of equations in solving problems (pages 306-307).

A New Way to Solve Problems

1. You can solve many simple problems by arithmetic, but you will find mathematical sentences very helpful in solving more difficult problems. The problem in ex. 2 is solved by the equation method. While you could do this problem by arithmetic, it is important to study the work in ex. 2 to see how such a problem is solved by the equation method.

2. Problem Tony sold a dog for a certain amount. After spending \$5.00 of the money he got for the dog, he had \$12.50 left. How many dollars did Tony get for the dog? \$17.50

Explanation Let x = number of dollars received for the dog

If Tony got x dollars for the dog and spent \$5.00 of this money, he had $(x - 5)$ dollars left. But you know he had \$12.50 left, so $(x - 5)$ dollars and \$12.50 both represent the same amount. You can now write this equation:

$$x - \$5.00 = \$12.50$$

Adding \$5.00 to both sides, you get

$$x = \$17.50$$

So Tony got \$17.50 for the dog.

Check: Does $\$17.50 - \$5.00 = \$12.50$?

3. In solving the above problem, you first found two different phrases, $(x - 5)$ dollars and \$12.50, which represent the amount Tony had left. You next formed an equation by stating the equality of these phrases. You then solved this equation.

4. Follow these steps to solve a problem with an equation:

(1) Let x, or any letter, represent the unknown number. Stress.

(2) Study the problem and find two phrases that represent the same quantity.

(3) Express the equality of these phrases as an equation.

(4) Solve the equation and check your work.

5. When Ann gets 15¢ more, she will have enough money to buy a book that costs 87¢. How much has Ann now? 72¢

Let x = number of cents Ann has now; $x + 15 = 87$; $x = 72$

▶ To form the equation, first find two phrases that represent the amount of money Ann must have to buy the book.

Go over the explanation in ex. 2 step by step. In ex. 3 be sure the difference between phrase and equation is clear to the students. In discussing the procedure in ex. 4, emphasize the importance of step (2).

Solve each problem by the equation method: See the Guide for the equations.
Emphasize that students must check all work.

1. When the pupils in the eighth grade collect \$2.75 more, they will have the \$15 they plan to spend on their picnic at Woodland Park. How much money do they have now? \$12.25

2. After 1477 people had bought tickets for the school play 273 tickets were left that were not sold. How many tickets were there at first? 1750

3. On the day of the blizzard only 35 pupils came to school. This was 140 fewer than the number of pupils enrolled in the school. How many pupils were enrolled in the school? 175

4. Ellen has a large box and a small box. She is putting chocolates into each box. The small box will hold $\frac{1}{4}$ as many chocolates as the large box. If the small box holds 18 chocolates, how many chocolates does the large box hold? 72

▶ Let x = number of chocolates that the large box holds. Then $\frac{1}{4}x$ = number of chocolates the small box holds.

5. One fifth of the boys who entered the field day races won prizes. If 9 boys won prizes, how many boys entered? 45

6. June is saving money to buy a bicycle that costs \$43.50. If she has \$9.50 already, how much more must she save? \$34.00

7. Ray earns \$3.50 a week. How many weeks will it take him to earn \$28.00? 8

▶ Let x = the number of weeks; then \$3.50 x = the amount he will earn in x weeks. What else represents the amount he will earn in x weeks? \$28.00

8. Mr. Niles drives his car an average of 35 mi. an hour. How many hours will it take him to drive 210 mi.? 6

9. When Louise saves 3 times as much as she has saved this week she will have enough to buy a sweater that costs \$6.75. How much did Louise save this week? \$2.25

10. Jim walked $6\frac{1}{2}$ mi. in 2 hr. Find, in miles per hour, Jim's average rate of walking. $3\frac{1}{4}$

Do some of these exercises with the class, using the procedure given in ex. 4 on page 306. Let different students explain the first two steps and then have the class complete the solutions.

Teach the students the solution of equations requiring the use of more than one principle (pages 310–311).

Using More than One Principle

1. In the equation $4x + x = 30$, you find that x appears twice; so you begin by adding the x's. When x is written alone, it means $1x$, so $4x + x = 5x$. Therefore, $5x = 30$. What must you do to both sides of $5x = 30$ to find the value of x? Divide by 5

$4x + x = 30$
$5x = 30$
$x = 6$
Check: $24 + 6 = 30$

Have the students explain the meaning of these equations in words (a number added to 3 times the number is 16).

Solve each equation and check the work:

2. $3x + x = 16$ 4
3. $6n + n = 56$ 8
4. $5r + 3r = 24$ 3
5. $7x + x = 40$ 5
6. $7x - 2x = 5$ 1
7. $3x - 2x = 9$ 9
8. $4n - 2n = 8$ 4
9. $3a - a = 2$ 1
10. $6x - x = 15$ 3
11. $8a - 2a = 24$ 4
12. $2x + 2x = 20$ 5
13. $5n - n = 10$ $2\frac{1}{2}$

14. **Problem** To solve the equation $4x - 3 = 17$, it is necessary to use more than one principle as shown below:

Explanation

Given:	$4x - 3 = 17$
Add 3 to both sides:	$4x = 20$
Divide both sides by 4:	$x = 5$
Check:	$20 - 3 = 17$

When using more than one principle to solve an equation, do the addition or subtraction before the multiplication or division.

15. Explain each step in the solution below: Emphasize checking.

Given: $\frac{1}{2}x + 3 = 5$
$\frac{1}{2}x = 2$
$x = 4$

Solve these equations and check the work:

16. $5x + x = 42$ 7
17. $8a - 4a = 12$ 3
18. $6n - n = 25$ 5
19. $3x + 8 = 20$ 4
20. $10x + 8 = 48$ 4
21. $\frac{1}{2}x + 5 = 11$ 12
22. $\frac{1}{4}x + 6 = 13$ 28
23. $x + 3x = 40$ 10
24. $\frac{1}{3}n = 5$ 15
25. $\frac{1}{5}a = 7$ 35
26. $\frac{3}{4}x = 6$ 8
27. $\frac{2}{3}x = 6$ 9

If the students ask, explain that multiplication or division can be done first, but the work would usually be more difficult in these equations.

First review the steps used in solving problems by equations (page 306). Notice that only the answers are given here; see the Guide for the equations and solutions.

Can You Solve These Problems?

Solve each problem by the equation method:

1. Jean said, "I am thinking of a certain number. If I multiply the number by 7 and subtract 5, I will get 37." What is the number? 6

2. Dick said, "5 times my number plus the number itself gives 18. What is my number?" 3

3. Bob picked 25 quarts of strawberries this week. That was 1 quart more than twice as many as he picked last week. How many quarts did he pick last week? 12

4. Fred said, "If you multiply my age by 3, add 30, and then subtract my age, you will get 56." How old was Fred? 13 yr.

5. In June, Henry put \$7 in the bank. If that was \$1 more than three times as much as he put in the bank in May, how much did he put in the bank in May? \$2

6. There are 11 girls in Betty's class. This is 1 more than half the number of boys in the class. How many boys are there in the class? 20

7. Frank has \$50 saved. This is \$5 more than three times as much as he had saved a year ago. What had he saved then? \$15

8. Andy now pays 5¢ less than twice as much as he used to pay for a ticket to the movies. If a ticket costs him 55¢ now, what was the price he used to pay? 30¢

9. Mr. Brooks sold his house for \$15,800, which was \$200 less than twice as much as the house had cost him. What had the house cost him? \$8000

10. Jim and Fred were bicycling to Newton. After they had gone 12 miles, Jim said that they had gone $\frac{3}{5}$ of the way. How many miles was the whole trip? 20

11. Charles works on commission. This week he earned \$15, which is 3 times as much as he earned last week. How much did he earn last week? \$5

Correct the students' work carefully and analyze errors. Have volunteers explain the solutions at the board so that others can find and correct their mistakes. Help the students to see why they made errors. Further work in writing algebraic phrases may be needed.

Teach the students how to find two unknown numbers by using an equation. Emphasize that we must describe how each of the unknown numbers will be represented before we can write the equation.

Finding Two Unknown Numbers

1. Problem The Cubs have played 24 games this year. If they have won three times as many games as they have lost, tell how many games they have won and how many games they have lost this year. Won 18, lost 6

Explanation Here you must find two quantities, the number of games lost and the number won. The number won is three times as great as the number lost.

Let x = number of games lost

Then $3x$ = number of games won

The equation is: $x + 3x = 24$

Add the x's: $4x = 24$

Divide by 4: $x = 6$

Since $x = 6$, $3x = 18$. So 6 games were lost and 18 were won.

Check: $6 + 18 = 24$

Emphasize after further demonstration.

When two numbers are to be found in a problem, let x represent one of them and express the other in terms of x.

2. On Friday the boys played baseball for x hours. On Saturday they played twice as long. How can you express the number of hours they played on Saturday? $2x$ the total number of hours they played on the two days? $3x$

3. If the boys played baseball for 6 hr. in all (see ex. 2), form an equation (1) and find how long they played each day. (2)
(1) See Guide; (2) Friday, 2 hr.; Saturday, 4 hr.

4. Alice paid 80¢ for candy and some apples. If the candy cost 30¢ more than the apples, find the cost of each. Apples, 25¢; candy, 55¢

▶ Let x = cost of apples, then $x + 30$ = cost of candy.

5. In playing darts, Jack and Walter together scored 500 points. If Jack scored 50 more points than Walter, how many points did each boy score? Walter, 225; Jack, 275

6. Mary, Joe, and Bill were picking blackberries. Together they picked 12 quarts. Mary picked twice as many quarts as Bill, and Joe picked three times as many quarts as Bill. How many quarts did each one pick? Bill, 2; Mary, 4; Joe, 6

▶ Let x = the number of quarts Bill picked.

In ex. 1 point out that one number is represented by x, and the other is expressed "in terms of" x. Also emphasize that the value of x is not the complete answer.

Present a review of computation and problem solving.

Review

Find the answers:

1. \$8000 − \$396.73 \$7603.27
2. $\frac{3}{4}$ of 1% of \$800 \$6
3. 321% of \$5000 \$16,050
4. $7\frac{3}{4} + 9\frac{2}{3} + 4\frac{5}{6} + 3\frac{1}{2} + 10\frac{2}{3}$ $36\frac{5}{12}$
5. 7.81 + 6.53 + 7.25 + 8.87 + 1.75 32.21
6. 9204 + 748 + 8099 + 3615 21,666

Find the answers correct to the nearest cent:

7. 4% of \$89.28 \$3.57
8. $\frac{1}{2}$% of \$2736 \$13.68
9. \$2845.32 ÷ 4826 \$.59
10. 7429 × \$2.315 \$17,198.14

Find what per cent the first number is of the second. In ex. 13 and 14, find the answer to the nearest tenth of 1%:

11. 37; 50 74%
12. 216; 288 75%
13. 29; 52 55.8%
14. 304; 1214 25.0%

Solve these problems:

15. A real estate agent received \$607.50 by selling a house for \$13,500. What was the per cent of his commission? 4.5% How much did the former owner receive? \$12,892.50
16. Find the net price of garden furniture marked \$80 and bought at a discount of 25%, with an extra 2% for cash. \$58.80
17. Draw an isosceles triangle having two equal sides of 2 in. and the angle between them 90°. On the drawing, mark the size of the other two angles. 45° and 45°
18. Write the formula for the volume of a cylinder. $V = \pi r^2 h$ Find the volume of a water tank 24 ft. high and $10\frac{1}{2}$ ft. in diameter. 2079 cu. ft.
19. If you buy a 4% \$1000 bond at 83, find, to the nearest tenth of 1%, the annual rate of income on your investment. 4.8%
20. Mr. Lee can buy a boat for \$185 cash, or for \$20 down and \$14 a month for 13 mo. By the formula on page 140, find, to the nearest whole per cent, the yearly rate of interest that he pays if he buys the boat on the installment plan. 18%

Have the students review the formula for finding the rate of interest charged in installment buying (page 140). Through explanations of their work, determine the causes of errors and the need for reteaching.

Show the meaning of a ratio, how to write one, and how to change a ratio to another of equal value (pages 314-315).

Comparing Numbers

1. One morning at camp, Don caught 6 fish and Paul caught 2 fish. If you want to compare the number of Don's fish with Paul's, you may say that Don caught 4 more fish than Paul. Then you are comparing the numbers by **subtraction** by finding the **difference** between the two numbers.

2. You may also say that Don caught 3 times as many fish as Paul. Then you are comparing the numbers by **division** by showing **how many times** one number is contained in the other. Thus, when you divide 6 by 2, you see that Don caught 3 times as many fish as Paul.

3. In the same way, you can compare the number of Paul's fish with Don's by dividing 2 by 6 to find **what part** 2 is of 6. Paul caught $\frac{2}{6}$, or $\frac{1}{3}$, as many fish as Don.

4. When two numbers are compared by division, the result is called their **ratio.** Thus the ratio of the number of Don's fish to Paul's is 3, and the ratio of the number of Paul's fish to Don's is $\frac{1}{3}$. A ratio may be a fraction, such as $\frac{3}{4}$ or $\frac{4}{3}$, or a whole number, such as 2, or a mixed number, such as $1\frac{1}{2}$.

5. How many times as large as 4 is 8? 2 What is the ratio of 8 to 4? 2 What part of 8 is 4? $\frac{1}{2}$ What is the ratio of 4 to 8? $\frac{1}{2}$

6. What is the ratio of 20 to 4? 5 the ratio of 4 to 20? $\frac{1}{5}$

7. Jane has \$3 and Sam has \$9. What is the ratio of Jane's money to Sam's money? $\frac{1}{3}$ of Sam's money to Jane's money? 3

8. Dan weighs 100 lb. and Ted weighs 75 lb. What is the ratio of Dan's weight to Ted's weight? of Ted's weight to $1\frac{1}{3}$ Dan's weight? $\frac{3}{4}$ Change each answer to lowest terms.

Find the ratio, in simplest form, of the first number to the second.

9. 6, 7 $\frac{6}{7}$	**13.** 15, 19 $\frac{15}{19}$	**17.** 10, 120 $\frac{1}{12}$	**21.** 150, 25 6
10. 3, 8 $\frac{3}{8}$	**14.** 30, 10 3	**18.** 15, 180 $\frac{1}{12}$	**22.** 100, 40 $2\frac{1}{2}$
11. 4, 4 1	**15.** 48, 24 2	**19.** 20, 300 $\frac{1}{15}$	**23.** 350, 75 $4\frac{2}{3}$
12. 9, 6 $1\frac{1}{2}$	**16.** 90, 40 $2\frac{1}{4}$	**20.** 45, 135 $\frac{1}{3}$	**24.** 200, 150 $1\frac{1}{3}$

Emphasize that comparisons are made by division in finding a ratio. In ex. 4 be sure the students understand that a ratio may be a fraction, a whole number, or a mixed number. Let different students explain their answers for ex. 9-24.

Emphasize the ways ratio may be written and the fact that any common fraction may be considered to be a ratio.

How to Write Ratios

1. When you compare two quantities, such as 4 in. and 3 in., you may say that they have the ratio of 4 in. to 3 in. or of 4 to 3. The ratio of 4 to 3 may be written as $\frac{4}{3}$, $4 \div 3$, or 4:3. The symbol : indicates division and is read **divided by** or **to.** 4 and 3 are called the **terms** of the ratio.

2. You may write the ratio of 2 to 8 as $\frac{2}{8}$. But $\frac{2}{8} = \frac{1}{4}$; so the ratio of 2 to 8 is equal to the ratio of 1 to 4. If you multiply both terms of $\frac{2}{8}$ by 3, you find that $\frac{2}{8} = \frac{6}{24}$. So the ratio of 2 to 8 is equal to the ratio of 6 to 24.

Emphasize after understanding is assured.

You can multiply or divide both terms of a ratio by the same number without changing its value.

3. Write in three ways the ratio of 5 to 8; $\frac{5}{8}$, $5 \div 8$, 5 : 8 of 9 to 6; $\frac{3}{2}$, $3 \div 2$, 3 : 2 of 1 to 7; $\frac{1}{7}$, $1 \div 7$, 1 : 7 of 600 to 200. $\frac{3}{1}$, $3 \div 1$, 3 : 1

4. Write as fractions three ratios each equal to $\frac{9}{12}$; to $\frac{50}{300}$. Answers will vary.

5. Which of these ratios equals $\frac{2}{3}$: $\frac{8}{12}$, $\frac{16}{24}$, $\frac{4}{8}$, $\frac{12}{21}$, $\frac{6}{9}$?

6. If you want to compare 3 ft. with 27 in., you should first change 3 ft. to 36 in. Then the ratio of 36 in. to 27 in. is $\frac{4}{3}$. What is the ratio of 2 ft. to 15 in.? $\frac{8}{5}$ of 27 in. to 6 ft.? $\frac{3}{8}$

If the ratio of two quantities is to be found, the quantities should first be expressed in the same unit of measure. Emphasize.

7. What is the ratio of a nickel to \$1? $\frac{1}{20}$ of 50¢ to \$1? $\frac{1}{2}$

Express these ratios as fractions in their simplest forms:

8. $5 \div 10$ $\frac{1}{2}$
9. $36 \div 24$ $\frac{3}{2}$
10. $28 \div 21$ $\frac{4}{3}$
11. $19 \div 57$ $\frac{1}{3}$
12. $\$9 \div \$9$ 1
13. $\$7 \div \$21$ $\frac{1}{3}$
14. 5 oz. : 9 oz. $\frac{5}{9}$
15. 9 in. : 1 yd. $\frac{1}{4}$
16. 1 hr. : 2 hr. $\frac{1}{2}$
17. 6 oz. : 1 lb. $\frac{3}{8}$
18. 1 qt. : 1 pt. 2
19. 1 hr. : 6 min. 10
20. 50 to 20 $\frac{5}{2}$
21. 24 to 40 $\frac{3}{5}$
22. 80 to 32 $\frac{5}{2}$
23. 55 to 11 5
24. 64 to 80 $\frac{4}{5}$
25. 50 to 50 1

Refer to the work on baseball standings (pages 54-55) and explain that a standing can be the ratio of the number of games won to the number played.

Teach the meaning of proportion, ways of writing a proportion, and an important principle relating to all proportions.

Proportion

1. Last week Sally earned \$10 and saved \$2. This week she earned \$15 and saved \$3. Write the ratio of Sally's savings to her earnings each week. How do the ratios compare?

2. The ratios $\frac{2}{10}$ and $\frac{3}{15}$ are equal, because each can be changed to $\frac{1}{5}$. The sentence $\frac{2}{10} = \frac{3}{15}$, which states that two ratios are equal, is called a **proportion.** The numbers 2, 10, 3, and 15 are called the **terms** of the proportion.

3. The proportion $\frac{2}{10} = \frac{3}{15}$ may be written as $2 : 10 = 3 : 15$. This proportion is read "2 is to 10 as 3 is to 15."

4. By studying the proportion below, you can discover an important principle that is true for any proportion. Multiply the numerator of each fraction by the denominator of the other fraction, as shown by the dotted lines. You find that the two products are equal, because $2 \times 15 = 3 \times 10$. These two products are called the **cross-products** of the proportion. Be sure the students understand why they are called "cross products."

$$\frac{2}{10} = \frac{3}{15}$$

The cross-products of a proportion are equal. Emphasize.

5. You can tell whether or not the sentence $\frac{2}{5} = \frac{3}{8}$ is a true proportion by seeing if the cross-products are equal. Here the cross-products are 2×8 and 3×5. Since 16 does not equal 15, the sentence above is not true.

Find the cross-products and tell which are true proportions:

6. $\frac{3}{4} = \frac{15}{20}$ Yes $\qquad \frac{7}{8} = \frac{28}{34}$ No $\qquad \frac{4}{3} = \frac{12}{8}$ No $\qquad \frac{2}{16} = \frac{3}{24}$ Yes

7. Last month Ben earned \$20 and saved \$5. This month he earned \$24 and saved \$6. Find the ratio of his savings to his earnings for each month. Are the ratios equal? Yes Is $\frac{5}{20} = \frac{6}{24}$ a true proportion? Yes

8. In Tom's class 24 out of 32 pupils can swim. Tom says this is the same as saying that 3 out of every 4 pupils can swim. Write this as a proportion. 24: 32 = 3: 4 Is Tom right? Yes Betty says this is the same as saying that 7 out of every 8 pupils can swim. Is Betty right? No

Emphasize the meaning of a proportion and illustrate further. Point out that a proportion is an equation in which the two sides are ratios.

Show how to find the unknown term in a proportion.

Finding Unknown Numbers

1. Sometimes one of the terms of a proportion is unknown. You can find the value of the unknown term by using the rule of cross-products. For example, study this proportion:

$$\frac{x}{40} = \frac{9}{15}$$

The cross-products are $15\,x$ and 9×40. Since the cross-products are equal, you have the equation:

$$15\,x = 360$$

To solve for x, divide both sides of the equation by 15. Then

$$x = 24$$

Check by substituting 24 for x in the original proportion.

Find the value of x in each proportion and check:

2. $\frac{x}{15} = \frac{10}{30}$ 5 **5.** $\frac{x}{7} = \frac{12}{21}$ 4 **8.** $\frac{x}{4} = \frac{18}{9}$ 8 **11.** $\frac{x}{48} = \frac{25}{30}$ 40

3. $\frac{x}{3} = \frac{12}{18}$ 2 **6.** $\frac{30}{x} = \frac{12}{10}$ 25 **9.** $\frac{7}{x} = \frac{28}{16}$ 4 **12.** $\frac{2}{3} = \frac{x}{12}$ 8

4. $\frac{x}{3} = \frac{14}{21}$ 2 **7.** $\frac{x}{5} = \frac{2}{10}$ 1 **10.** $\frac{x}{45} = \frac{10}{15}$ 30 **13.** $\frac{x}{70} = \frac{14}{56}$ $17\frac{1}{2}$

14. When a proportion is written in the form $x : 64 = 5 : 40$, you may rewrite it in the fractional form $\frac{x}{64} = \frac{5}{40}$ and solve for x. What is the value of x in this proportion? 8

Write in fractional form and solve for x. Check the work:

15. $x : 11 = 5 : 22$ $2\frac{1}{2}$ **18.** $4 : x = 8 : 6$ 3 **21.** $15 : 60 = x : 20$ 5

16. $x : 15 = 12 : 15$ 12 **19.** $9 : x = 12 : 8$ 6 **22.** $13 : 39 = x : 6$ 2

17. $x : 4 = 16 : 32$ 2 **20.** $90 : x = 50 : 10$ 18 **23.** $30 : 80 = x : 4$ $1\frac{1}{2}$

Slower students may need help with ex. 24.

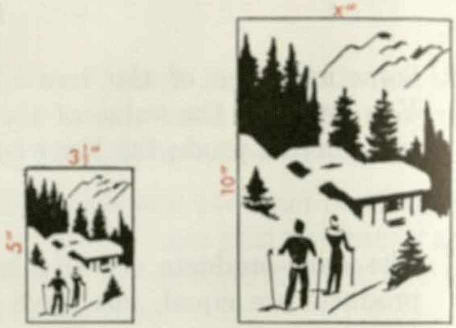
24. Last year Carl pitched 18 games and won 2 out of every 3 games that he pitched. The number of games won was to the number of games pitched as 2 is to 3. Express this as a proportion and find how many games Carl won. 12 Let x represent the number of games won.

The procedure in ex. 1 should be carefully explained to the students. Do a few more examples and let different students explain the solutions. Then have the students complete ex. 2-13. Be sure they understand how to rewrite proportion as in ex. 14 before assigning ex. 15-24.

Show how to use proportion to solve problems about enlarging pictures.

Enlarging Pictures

1. Problem Sam has a snapshot 5 in. long and $3\frac{1}{2}$ in. wide which he wants to have enlarged. If the length of the enlarged snapshot is to be 10 in., what will the width be?



Explanation When the picture is enlarged, the length and width of the enlarged picture will have the same ratio as the length and width of the small picture.* You can express this as a proportion thus: Stress.

Let x = width in inches of large picture

Form the proportion $\frac{x}{10} = \frac{3\frac{1}{2}}{5}$

Then $5x = 35$

Solve for x: $x = 7$

Be sure the students understand that if one ratio is expressed as "width to length," the other must be expressed this way also.

The enlarged picture will be 7 in. wide. Check the result.

See the Guide for the equations for ex. 2-10.

2. Polly had a picture 5 in. long and 4 in. wide enlarged so that the width was 10 in. How many inches long was the enlarged picture? $12\frac{1}{2}$

3. When pictures are printed in the newspaper, they are usually reduced in size. A picture that was 12 in. long and 10 in. wide was reduced so that its width was only 5 in. What was its length? 6 in. One that measured 12 in. by 8 in. was reduced to a width of 2 in. What was its length? 3 in.

4. A painting in the museum is 18 in. wide and 28 in. long. The length of a small copy of it is 7 in. What is the width of the small picture? $4\frac{1}{2}$ in.

5. Nancy's camera takes pictures that are $2\frac{1}{4}$ in. wide and $3\frac{1}{4}$ in. long. She wants to have one of these pictures enlarged so that it will be 9 in. wide. How many inches long will the enlarged picture be? 13 Make drawings showing the size of Nancy's pictures before and after enlarging.

* If possible, bring to class some pictures and their enlargements to illustrate this. Discuss the solution of ex. 1 carefully with the class. Let different students explain each step. Have the class discuss the solutions for ex. 2-3. Then have the students complete ex. 4-10.

6. John enlarges pictures. If he makes the length of a picture twice as long, what happens to its width? It is also doubled.

7. Mr. Allen has a new camera that uses small film, so he has all prints enlarged when they are made. He had some prints made that were 3 in. wide and $4\frac{1}{4}$ in. long, but he thought they were too small. If he had the next pictures enlarged so that they were $4\frac{1}{2}$ in. wide, how long were these larger prints? $6\frac{3}{8}$ in.

8. Mr. Allen had a print that was 3 in. wide and $4\frac{1}{4}$ in. long enlarged to make a picture large enough to frame. The width of this large picture was 9 in. How long was it? $12\frac{3}{4}$ in.

9. Dick saved money and bought a camera that uses 35 mm. film and takes pictures 24 mm. wide and 36 mm. long. Draw a rectangle this size by using the ruler pictured on page 32. You see that this is a very small picture. Dick always has the prints made much larger. If he has the width made 72 mm., how many millimeters long is that print? 108 About how many inches wide and long is it? $2\frac{13}{16}$" by $4\frac{1}{4}$"

10. Betty's camera takes pictures that are $2\frac{1}{4}$ in. by $3\frac{1}{4}$ in. If one of these pictures is enlarged to make a picture $6\frac{3}{4}$ in. wide, how long is the enlargement? $9\frac{3}{4}$ in.

When the students complete the work, the solutions should be put on the board and discussed thoroughly. Urge the students to ask questions if they do not understand any part of this work. Let other students answer the questions where possible.

Teach the students how to solve several types of problems by using proportions.

Solving Problems by Proportion

1. Mr. Reed's car used 5 gal. of gasoline on a trip of 75 mi. At this rate, how many gallons of gasoline would the car use on a trip of 180 mi.? 12

▶ You know that the longer the trip, the more gasoline the car uses. You can say that the amount of gasoline used **is proportional to** the length of the trip. In this problem, the ratio of the distances traveled is 75 : 180. If x represents the number of gallons of gasoline used on the longer trip, then $5 : x$ is the ratio of the amounts of gasoline used. The two ratios are equal, so you can form the proportion $5 : x = 75 : 180$. Solve this proportion for x. How many gallons of gasoline does the car use on a trip of 180 mi.?

See the Guide for the proportions for ex. 2–8.

2. If Mr. Reed's car used 4 gal. of gasoline on a trip of 60 mi., how many miles could he travel on 6 gal.? 90

▶ Let x = number of miles he can travel on 6 gal. of gasoline.

3. You also know that the distance you can drive in a car is proportional to the number of hours that you drive. If Miss Lewis drives 100 mi. in 3 hr., how many miles can she drive in $7\frac{1}{2}$ hr. at the same rate? 250

4. Mr. Jackson borrowed some money. The interest on this money for 90 da. was \$3. How much interest would he have paid if he had kept the money for 120 da.? \$4

5. A recipe for a certain cake calls for 6 eggs and 2 cups of sugar. If you wish to make a smaller cake and use only 4 eggs, how many cups of sugar should you use? $1\frac{1}{3}$

6. George can do 6 long examples in column addition in 4 min. At this rate, how many examples of this kind can George do in 6 min.? 9

7. On a road map, a distance of 60 mi. is represented by a line $2\frac{1}{2}$ in. long. On the same map, what distance is represented by a line 4 in. long? 96 mi.

8. Martha can buy 2 special drawing pencils for 25¢. At this price, how much will she have to pay for 6 special drawing pencils? 75¢ for 8 special drawing pencils? 100¢, or \$1.00

In discussing ex. 1, be sure the students understand why the ratio of gasoline used is $5:x$, not $x:5$. Have a volunteer solve the proportion. Do ex. 2 also with the class.

Discuss inequalities, and teach how to find the solutions of inequalities.

1. You have learned to compare two numbers by subtraction and by division. It is often desirable to compare two numbers by indicating a difference without actually performing the subtraction to find this difference. The mathematical symbol $<$ is used to express "less than." You know that 4 is less than 12; so you can write this as $4 < 12$. The symbol $>$ is used to express "greater than." You know that 20 is greater than 7; so you can write this as $20 > 7$. The sentences $4 < 12$ and $20 > 7$ are called **inequalities.**

Make a true sentence with each of the following pairs of number phrases and the symbol $<$, $>$, or $=$.

2. $3 \overset{<}{,} 6$ $\quad 5 \overset{>}{,} 2$ $\quad 17 \overset{<}{,} 35$ $\quad 46 \overset{=}{,} 46$ $\quad 15628 \overset{<}{,} 16528$

3. $2 \overset{>}{,} 1$ $\quad 0 \overset{<}{,} 1$ $\quad 101, 59$ $\quad .5, \frac{1}{2}$ $\quad 1002 \overset{<}{,} 1020$

4. $\frac{5}{6} \overset{<}{,} 1$ $\quad 2 \overset{>}{,} \frac{9}{5}$ $\quad 39 \overset{>}{,} 13 + 21$ $\quad 25 - 9 \overset{<}{,} 6 + 17$

5. $\frac{1}{4} \overset{>}{,} \frac{3}{16}$ $\quad 2^3 \overset{<}{,} 3^2$ $\quad 7 \overset{=}{,} 0 + 7$ $\quad 3^2 + 1 \overset{>}{,} 2^2 + 2^2$

6. $0 + 3 \overset{=}{,} 3 + 0$ $\quad 1 + 2 + 3 \overset{<}{,} 4^2 - 3^2$ $\quad 1 + 3 + 5 \overset{>}{,} 2^3$

7. MDCLV $\overset{<}{,}$ MCM $\quad 10_8 \overset{>}{,} 10_4$ $\quad 21_5 \overset{>}{,} 100_2$ $\quad$ 𓍢𓍢𓍢 $\overset{>}{,}$ 𓍢𓍢𓎆𓎆𓎆𓏺𓏺𓏺𓏺

8. Problem What whole number (or numbers) plus 4 is less than 10?

Explanation A letter, such as x, can be used in an inequality to represent an unknown number or unknown numbers. So you can write the inequality

$$x + 4 < 10$$

where x represents the unknown number or numbers. Is this sentence true when $x = 1$? $x = 2$? $x = 6$? $x = 7$? $x = 0$? A value of x that makes the inequality a true sentence is called a **solution** of the inequality. When you have found all the solutions, you have solved the inequality. You can see that the whole numbers that are solutions of this inequality are 0, 1, 2, 3, 4, and 5.

Find three whole numbers that are solutions of each inequality. See Guide.

9. $x + 1 < 20$ $\quad x + 2 > 5$ $\quad x + 12 < 16$ $\quad x - 7 > 5$

10. $x + \frac{1}{2} > 10$ $\quad x - 4.5 < 6$ $\quad x - 9 > 0$ $\quad 2x + 3 < 9$

Point out the convenience of the inequality symbols < and > for writing mathematical sentences. Emphasize the meaning of a solution of an inequality. In ex. 9, note that any number greater than 3 is a solution of $x + 2 > 5$.

Teach how to use inequalities to solve problems (pages 320-321).

Using Inequalities

1. **Problem** Bill had 7 model airplanes. After buying some more model airplanes, he still had fewer than 12 airplanes. How many model airplanes could Bill have bought?

   Explanation Let x represent the number of model airplanes Bill could have bought. Then you can write the inequality

   $$7 + x < 12$$

   What whole numbers are solutions of this inequality? Are all of these solutions answers for this problem? Why will you not include the solution $x = 0$ as an answer? You see that Bill could have bought 1, 2, 3, or 4 airplanes.

Write an inequality for each of these problems. Then use this inequality to solve the problem.

2. Dan started his stamp collection with some stamps that his father gave him. After Dan had added 32 more stamps to his collection, he had fewer than 50 stamps. How many stamps could Dan's father have given him to start his collection? From 1 to 17 $x + 32 < 50$

3. Jean had some tickets for the school play to sell. After she had sold 8 tickets, she had fewer than 20 tickets left to sell. $x - 8 < 20$ How many tickets could Jean have had to sell? From 0 to 27

4. After John had delivered 30 of the newspapers on his paper route, he still had more than 15 papers to deliver. How many customers could John have on his paper route?

   ▶ If you let x represent the number of customers on John's paper route, you get the inequality $x - 30 > 15$. All whole numbers greater than 45 are solutions of this inequality. But not all of these solutions are reasonable answers for this problem. For example, $x = 500$ is a solution, but it is unlikely that John would have 500 customers. To give a definite answer to the question in this problem you would need more information. Suppose that no newsboy in John's city has more than 60 customers. Then what solutions of the inequality are reasonable answers for this problem? From 46 to 60

Emphasize the need for writing a mathematical sentence to express the relationship given in a problem. Have the students use words first and then mathematical symbols.

5. Mary and Judy together have fewer than 35 phonograph
$x + 21 < 35$ records. If Mary has 21 records, how many records can Judy have? From 0 to 13

6. To earn money for the class, Jim sold 9 magazine subscriptions and Henry sold 13 subscriptions. Ted sold more subscriptions than Jim and Henry together. How many subscriptions could Ted have sold? $x > 22$ From 23 to 30

▶ Suppose that no one in the class sold more than 30 subscriptions.

7. Mrs. Perry bought some rosebushes for her garden. After she had planted 3 of the bushes, she had fewer than 8 left
$x - 3 < 8$ to plant. How many rosebushes could Mrs. Perry have bought? From 3 to 10

8. You have found solutions of an inequality by using your knowledge of the number facts. As you continue to study mathematics, you will learn to solve inequalities by methods similar to those you use to solve equations. In ex. 8 on page 319, you solved the inequality

$$x + 4 < 10$$

If you subtract 4 from both sides, you get

$$x + 4 - 4 < 10 - 4$$

or $$x < 6$$

What values of x that are whole numbers make this sentence true? Are these values the same as the solutions you found for the inequality $x + 4 < 10$?

When solving an inequality, you may first subtract the same number from each side; the solutions of the resulting inequality will be the same as the solutions of the given inequality.

Find the whole numbers that are solutions of each inequality. First change each inequality to another inequality that has only x on one side. See Guide.

9. $x + 3 < 7$ $\quad$ $x + 5 > 19$ $\quad$ $x + 12 < 67$ $\quad$ $x + 9 > 24$

Point out that only whole numbers are sensible answers to the problems in ex. 1–7, but that fractions as well as whole numbers are solutions of the inequalities used to solve these problems.

Chapter Review

Write the following phrases in symbols:

1. n increased by 9; $n + 9$ x decreased by 5 $x - 5$
2. x increased by 3 times its value $x + 3x$
3. 5 times Ann's age less 6 yr. $5a - 6$
4. The sum of 4 times a number and 5 times the number $4n + 5n$
5. The sum of twice a number and 8 $2n + 8$
6. 4 yr. less than 3 times Bill's age $3a - 4$
7. One third of a number; $\frac{1}{3}n$, or $\frac{n}{3}$ one half of a number $\frac{1}{2}n$, or $\frac{n}{2}$
8. One fourth of a number increased by 6 $\frac{1}{4}n + 6$, or $\frac{n}{4} + 6$
9. The product of a and b; ab the product of 3 and c $3c$

Solve these equations and check the work:

10. $x + 9 = 13$ 4	**15.** $6x = 24$ 4	**20.** $17 = 2x + 5$ 6
11. $a - 6 = 11$ 17	**16.** $\frac{1}{2}a = 11$ 22	**21.** $21 = 6a - 9$ 5
12. $b + 8 = 20$ 12	**17.** $21 = 9x$ $2\frac{1}{3}$	**22.** $33 = 4c + 1$ 8
13. $n + 6 = 14$ 8	**18.** $16 = \frac{1}{4}x$ 64	**23.** $50 = 3n + 8$ 14
14. $d - 15 = 13$ 28	**19.** $5x = 75$ 15	**24.** $28 = 5a - 7$ 7

Solve these problems by using equations:

25. Grandfather said, "I am so old that if you multiply my age by 3 and add 1, you get 250." How old is Grandfather? 83 yr.
26. Jack says if he subtracts 8 yr. from twice his age, he will get Tom's age. If Tom is 16 yr. old, how old is Jack? 12 yr.
27. Ann has $\frac{1}{4}$ of the number of books Lucy has. Ann has 23 books in all. How many books has Lucy? 92
28. Peter said, "If you multiply the number I am thinking of by 4 and subtract 7, you will get 45." What is the number Peter is thinking of? 13

After checking the papers and noting the kinds of errors, return the papers to the students so that they can find and correct any mistakes. Try to determine the causes of errors and reteach as needed.

1. If you add 8 times a number to twice the number, you will get 120. Find the number. 12

2. Today Mr. Grant received \$150, which was 5% interest for a year on money he had loaned Mr. Ford. Find the amount of money Mr. Grant had loaned Mr. Ford. \$3000

3. Find, to the nearest tenth of 1%, the annual rate of income that a stock yields when it is bought at 49 and pays an annual dividend of \$2.25. 4.6%

4. Mr. Hunt was earning \$450 a month; then his salary was reduced 10%. Later, his reduced salary was increased 10%. Find his annual salary after the increase. \$5346.00

5. Patty's grandfather said: "If you double the number representing my age and then subtract 20 from the result, you will get 100." Find the age of Patty's grandfather. 60 yr.

6. At a tax rate of \$3.90 per \$100, find Mr. Fisher's tax bill on property assessed at \$12,500. \$487.50

7. Mr. Palmer paid \$560 for some lumber after receiving a discount of 20%. What was the list price of the lumber? \$700

8. The annual premium on a 20-payment life insurance policy bought at age 30 is \$26.15 per \$1000. How much will Mr. Chase pay in premiums for 5 yr. if the face of his policy is \$10,000? \$1307.50

9. A water tank shaped like a cylinder has a diameter of $3\frac{1}{2}$ ft. When filled to a depth of 8 ft., how many gallons of water does it hold? Use $\pi = 3\frac{1}{7}$. $7\frac{1}{2}$ gal. = 1 cu. ft. $577\frac{1}{2}$

10. Tim paid \$4 less for a tennis racket than he paid for a tennis net. Together the two articles cost \$20.50. What was the cost of each one? Net, \$12.25; racket, \$8.25

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Have the students explain their solutions so that you can determine the causes of errors and the need for reteaching or review. Group students who need remedial work.

Present a diagnostic test of skills and concepts taught in Chapter 9, with practice-page references.

How Much Have You Learned?

If you miss more than one example in a row, turn to the Practice Page for that row.

Solve these equations and check the work:

				Practice Pages
1.	$x + 12 = 93$ 81	$x + 15 = 60$ 45		301
2.	$15 + d = 88$ 73	$120 + x = 200$ 80		301
3.	$c - 11 = 10$ 21	$r - 20 = 5$ 25		300
4.	$a - 100 = 12$ 112	$b - 38 = 25$ 63		300
5.	$\frac{1}{3}x = 4$ 12	$\frac{1}{2}x = 11$ 22	$\frac{1}{4}a = 24$ 96	302
6.	$\frac{b}{2} = 10$ 20	$12 = \frac{n}{7}$ 84	$\frac{r}{5} = 20$ 100	302
7.	$6x = 18$ 3	$9a = 45$ 5	$7c = 21$ 3	303
8.	$10 = 8d$ $1\frac{1}{4}$	$6x = 4$ $\frac{2}{3}$	$15 = 10r$ $1\frac{1}{2}$	303
9.	$4n + 6n = 30$ 3	$42 = 5d + 9d$ 3		308
10.	$7c - 4c = 27$ 9	$56 = 9a - 2a$ 8		308
11.	$8x + 5 = 61$ 7	$53 = 6x + 11$ 7		308
12.	$5x - 11 = 49$ 12	$68 = 8c - 20$ 11		308
13.	$2x - 24 = 14$ 19	$10 = 3x - 17$ 9		308

Solve these problems by equations:

14. Five times a number increased by 13 gives 58. Find the number. 9 — 307

15. Phil can ride his bicycle at the rate of 8 mi. per hour. How many hours will it take him to ride 20 mi. at that rate? $2\frac{1}{2}$ — 307

16. This year our school wants to raise \$200 for the Athletic Association. This is \$4 more than twice the amount raised last year. How many dollars were raised last year? \$98 — 309

Notice that the test covers the use of the four principles of equations, and the use of equations to solve problems. The letter in an equation is found on the right side in some examples.

See the Guide for specific aims of Chapter 10.

Chapter 10

How Long Is Jim's Step?

Present an introduction to the work on similar triangles and proportion.

1. You know that a boy with long legs takes longer steps than a boy with short legs. The length of a boy's step is proportional to the length of his legs. Jim's legs are $2\frac{1}{2}$ ft. long, but when he walks on stilts, it is the same as making his legs 5 ft. long. The ratio of these lengths is $\frac{2\frac{1}{2}}{5}$.

2. When Jim walks on the ground without stilts his step is 2 ft. long. When he walks on stilts you may call his step x ft. long. The ratio of the lengths of these steps is $\frac{2}{x}$. This ratio is equal to the ratio in ex. 1, so you can make this proportion:

$$\frac{2\frac{1}{2}}{5} = \frac{2}{x}$$

The cross-products are $\quad 2\frac{1}{2}\,x = 10$

Solve for x: $\quad x = 4$

Jim's step is 4 ft. long when he walks on stilts.

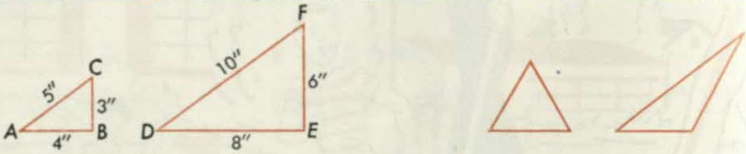
3. The giant in the parade was a man on stilts. His legs were 3 ft. long and his normal step was $2\frac{1}{2}$ ft. long. His stilts made each leg 9 ft. longer. How long were his legs then? 12 ft. How long a step did he take on stilts? 10 ft.

Make diagrams to illustrate ex. 1-3. Point out that the boy's legs and length of step, when viewed from the side, form approximately an isosceles triangle. Be sure the students understand how proportion is formed.

Teach the meaning of similar triangles and show that their corresponding sides are proportional.

Similar Triangles

1. The first two triangles below are called similar triangles because they have the same shape but not the same size. The last two triangles are not similar; can you tell why?
Reproduce the similar triangles on the board to aid in discussion.



2. In the similar triangles shown above, the bases AB and DE are called **corresponding sides** because they have **like positions** in their triangles. BC and EF are also corresponding sides. Name another pair of corresponding sides.
Draw other similar triangles on the board. Have students pick out corresponding sides.

3. In these similar triangles, the corresponding sides AB and DE have the ratio $\frac{4}{8}$, or $\frac{1}{2}$. Likewise, the corresponding sides BC and EF have the ratio $\frac{3}{6}$, or $\frac{1}{2}$. Also, the corresponding sides AC and DF have the ratio $\frac{5}{10}$, or $\frac{1}{2}$. You see each ratio equals $\frac{1}{2}$, so the three ratios are equal. Stress.

In similar triangles, the ratio of any pair of corresponding sides equals the ratio of any other pair.

4. In ex. 3 you have three equal ratios. Any two of these equal ratios will make a proportion as shown below:

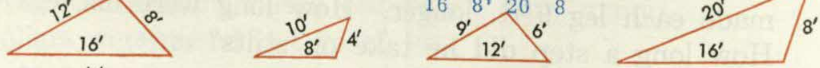
$$\frac{AB}{DE} = \frac{BC}{EF} \qquad \frac{AB}{DE} = \frac{AC}{DF} \qquad \frac{BC}{EF} = \frac{AC}{DF}$$

By substituting the values, these proportions become

$$\frac{4}{8} = \frac{3}{6} \qquad \frac{4}{8} = \frac{5}{10} \qquad \frac{3}{6} = \frac{5}{10}$$

In each proportion show that the cross-products are equal.

5. Which triangle below is similar to M? S Write the three proportions for these two triangles as shown in ex. 4.(1) Which triangle below is similar to N? T Write the three proportions for these two triangles.(2) (1) $\frac{16}{12}=\frac{12}{9}$, $\frac{16}{12}=\frac{8}{6}$, $\frac{12}{9}=\frac{8}{6}$; (2) $\frac{8}{16}=\frac{10}{20}$, $\frac{8}{16}=\frac{4}{8}$, $\frac{10}{20}=\frac{4}{8}$



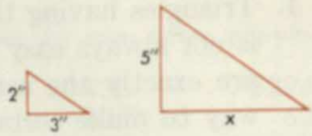
M N S T

Be sure the meaning of corresponding sides is understood. Do ex. 3 slowly and make clear that only ratios of pairs of corresponding sides are equal. Give the students more work as in ex. 5 before proceeding further.

Show how to find the length of an unknown side in one of two similar triangles.

How to Find an Unknown Side

1. The triangles at the right are similar but the length of side x in the second triangle is unknown. You can find the length of side x by forming a proportion from two equal ratios as follows: The vertical sides marked 2″ and 5″ are corresponding sides and their ratio is $\frac{2}{5}$; the bases marked 3″ and x are also corresponding sides and their ratio is $\frac{3}{x}$.* These ratios are equal, so you can form this proportion:



* Be sure the students understand why the ratio is not expressed as $\frac{x}{3}$.

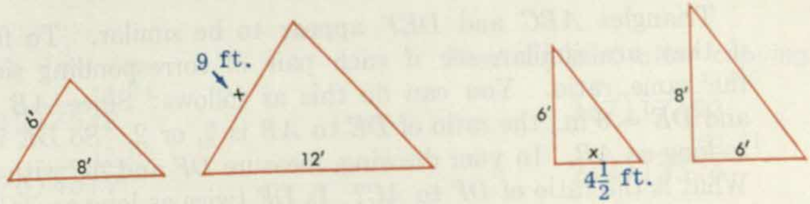
$$\frac{2}{5} = \frac{3}{x}$$

Solve for x: $\qquad x = 7\frac{1}{2}$

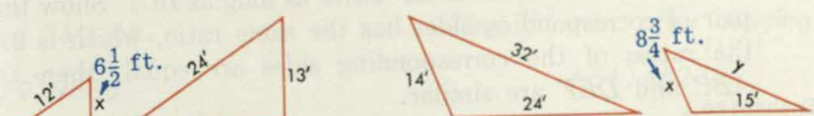
So the unknown side is $7\frac{1}{2}$ in. long.

In ex. 2 and 3 find the length of the unknown side x in each pair of similar triangles: See Guide for the proportions for ex. 2-5.

2.

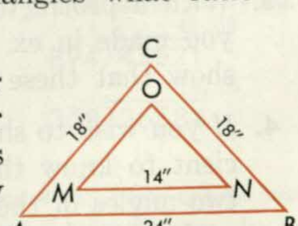


3.



4. In the second pair of triangles in ex. 3 also find the length of the unknown side y. 20 ft. In these triangles what ratio is equal to the ratio $\frac{y}{32}$? $\frac{15}{24}$

5. In the figure at the right the inner triangle MNO is similar to the outer triangle ABC. Find the lengths of sides $10\frac{1}{2}$ in. MO and NO of the inner triangle by forming two different proportions.



Reproduce on the board the triangles in ex. 1. Have a volunteer point out the corresponding sides. Emphasize that the ratio is formed from the corresponding sides and that the arrangement of the proportion is important. In ex. 5 ask why both sides had to be equal.

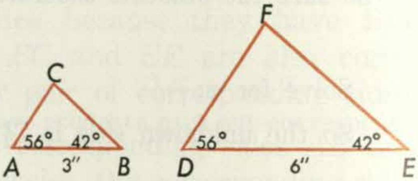
Teach that two triangles must be similar if two angles of one triangle are equal to two angles of the other triangle.

The Angles of Similar Triangles

1. Triangles having the same shape are similar triangles, but it is not always easy to tell whether the shapes of the triangles are exactly the same merely by looking at them. An easy way to make certain that two triangles are similar is to see whether two angles in one triangle are equal to two angles in the other. If they are, then the triangles are similar.

2. The following experiment shows that two triangles are similar if two angles of one equal two angles of the other.

▶ With your ruler draw *AB* 3 in. long. With your protractor draw angle *A* = 56° and angle *B* = 42°. Extend the upper sides of each angle to meet at *C*, thus forming triangle *ABC*. In the same way, draw triangle *DEF*, making *DE* = 6 in., angle *D* = 56°, and angle *E* = 42°.



Triangles *ABC* and *DEF* appear to be similar. To find out if they are similar, see if each pair of corresponding sides has the same ratio. You can do this as follows: Since *AB* = 3 in. and *DE* = 6 in., the ratio of *DE* to *AB* is $\frac{6}{3}$, or 2. So *DE* is twice as long as *AB*. In your drawing, measure *DF* and *AC* with a ruler. What is the ratio of *DF* to *AC*? Is *DF* twice as long as *AC*? Also measure *EF* and *BC*. Is *EF* twice as long as *BC*? Show that each pair of corresponding sides has the same ratio, which is 2. Since the ratios of the corresponding sides are equal, then triangles *ABC* and *DEF* are similar.

Emphasize.

If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.

3. With a protractor measure angles *C* and *F* on the drawing you made in ex. 2. How do they compare? Equal How can you show that these angles are equal without measuring them? See Guide.

4. If you wish to show that two triangles are similar, is it sufficient to know that two angles of one triangle are equal to two angles of the other? Yes Or is it better to know that three angles of one are equal to three angles of the other? Unnecessary

After the students read ex. 1, ask them why it is important to have an easy way of telling whether two triangles are similar. Have them make the construction in ex. 2. Be sure to discuss ex. 3–4 thoroughly and use the fact that the sum of the angles in any triangle is 180°.

Present the last sets of improvement tests in multiplication and division.

Improving by Practice

Multiplication Test 6a. Time: $4\frac{1}{2}$ min. after copying.

1. 85714 × 369	76082 × 258	79683 × 714	59436 × 905	73296 × 630
31,628,466	19,629,156	56,893,662	53,789,580	46,176,480

(5)

Multiplication Test 6b. Time: $4\frac{1}{2}$ min. after copying.

2. 71945 × 708	62475 × 249	91258 × 517	10374 × 584	48569 × 310
50,937,060	15,556,275	47,180,386	6,058,416	15,056,390

(5)

Multiplication Test 6c. Time: $4\frac{1}{2}$ min. after copying.

3. 49158 × 126	67405 × 385	49163 × 602	27386 × 947	73852 × 260
6,193,908	25,950,925	29,596,126	25,934,542	19,201,520

(5)

Division Test 6a. Time: $5\frac{1}{2}$ min. after copying.

4. $512\overline{)7548}$ 14.74 $\qquad$ $367\overline{)17493}$ 47.66

5. $376\overline{)4519}$ 12.02 $\qquad$ $214\overline{)13526}$ 63.21

(4)

Division Test 6b. Time: $5\frac{1}{2}$ min. after copying.

6. $879\overline{)9836}$ 11.19 $\qquad$ $651\overline{)46239}$ 71.03

7. $765\overline{)8594}$ 11.23 $\qquad$ $233\overline{)13592}$ 58.33

(4)

Division Test 6c. Time: $5\frac{1}{2}$ min. after copying.

8. $642\overline{)8537}$ 13.30 $\qquad$ $894\overline{)43521}$ 48.68

9. $466\overline{)5463}$ 11.72 $\qquad$ $358\overline{)26584}$ 74.26

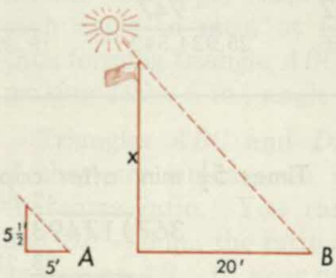
(4)

To the Pupil. In ex. 4–9, find quotients to the nearest hundredth. These are the last sets of Improvement Tests in multiplication and division you will have this year. Try to get a score of 10 on each test.

The students should complete the graphs begun on page 48 and compare their scores with previous ones to note further needs. Confer with those who had errors to determine the causes. Assign remedial work after difficulties are cleared up.

Teach the students how to measure heights indirectly, using the shadow method (pages 330-332).

Measuring Heights Indirectly



1. Problem Mary is $5\frac{1}{2}$ ft. tall if you measure to the top of her hood. Her shadow on the ground is 5 ft. long. At the same time of day, a flagpole near the club house of the Skiing Club casts a shadow 20 ft. long. How many feet tall is the flagpole? 22

Explanation If you connect the top of the pole and the end of its shadow by a dotted line, you have a triangle like the large one shown above. If you draw a line from the top of Mary's hood to the end of her shadow, you form a triangle like the small one above. These triangles are similar because 2 angles of one triangle equal 2 angles of the other. Show that each triangle has a right angle; also show that angle A = angle B, since the sun's rays are parallel.

Let x = height, in feet, of the flagpole

Since the triangles are similar, you can write this proportion:

$$\frac{x}{5\frac{1}{2}} = \frac{20}{5}$$

Be sure the students understand why the ratio used is $\frac{20}{5}$ and not $\frac{5}{20}$.

Solve: $x = 22$

The flagpole is 22 ft. high. Check the result. Emphasize the importance of checking.

When you find the height of the flagpole without measuring it, as was done above, you are **measuring indirectly.**

Discuss the explanation of ex. 1 carefully with the class. Be sure they understand why angle A = angle B. Lead the students to suggest the ratios involved and to set up the proportion.

Have the students draw the diagrams for all these examples.

2. When a 5-foot fence post casts a shadow of 7 ft., a flagpole casts a shadow of 49 ft. How high is the flagpole? 35 ft. Make a drawing that shows the two similar triangles that you use. (See ex. 1.)

3. Judy, who is 4 ft. tall, casts a shadow 3 ft. long when a tree casts a shadow 40 ft. long. How high is the tree? $53\frac{1}{3}$ ft.

4. Bob, who is 5 ft. tall, casts a shadow 2 ft. long at the same time that a smokestack of a factory casts a shadow 50 ft. long. Find the height of the smokestack. 125 ft.

5. A ship's mast casts a shadow of 15 ft. on the deck. At the same time, a sailor 6 ft. tall casts a shadow of 3 ft. How high is the mast? 30 ft.

6. Bill, who is 4 ft. tall, casts a shadow of 6 ft. when a store casts a shadow of 48 ft. How high is the store? 32 ft.

7. When George, who is 5 ft. 8 in. tall, casts a shadow 7 ft. long, a pine tree casts a shadow 42 ft. long. How high is the pine tree? 34 ft.

8. Tom says that the height of a flagpole is proportional to the length of its shadow. Is he right? Yes

Do ex. 2–4 with the class. Let different students draw on the board the diagrams for these problems. Have volunteers point out corresponding sides and set up proportions. Discuss thoroughly. Then have the class complete the solutions.

Using the Shadow Method

1. On the school grounds, find the height of a tree or flagpole by comparing the length of its shadow with the length of the shadow of one of your classmates. Do this around noon, measuring both shadows at that time. Then do the work around 3 P.M., measuring both shadows again at that time. Will you get the same answer each time for the height of the tree or flagpole? Yes

2. How do the lengths of the two shadows at noon compare with the lengths of the two shadows at 3 P.M.? Does this affect your answers in any way? No
Shadows at noon are shorter.

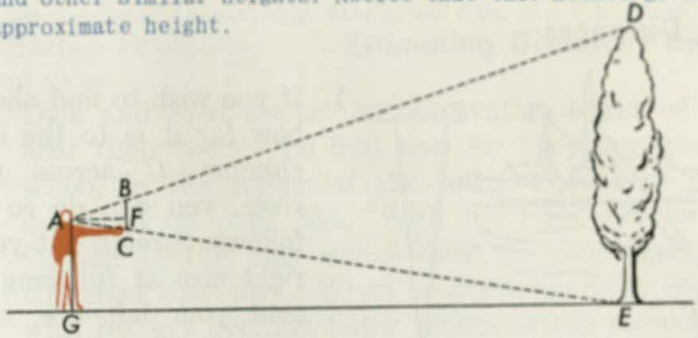
3. In ex. 1, draw to scale the similar triangles that you got when you made the measurements at noon. Do the same, using the measurements obtained at 3 P.M. Will either pair of similar triangles give you the right answer? Yes

Find the height of each object by the shadow method. Make the comparison with an upright pole whose height and shadow are given in each case:

	Shadow of Pole	Shadow of Object	Height of Pole	
4. Fir tree	2 ft.	12 ft.	7 ft.	42 ft.
5. Flagpole	4 ft.	32 ft.	5 ft.	40 ft.
6. Smokestack	$1\frac{1}{2}$ ft.	13 ft.	6 ft.	52 ft.
7. Telegraph pole	2 ft.	14 ft.	5 ft.	35 ft.
8. Lighthouse	1 ft.	17 ft.	3 ft.	51 ft.
9. Brick house	3 ft.	24 ft.	4 ft.	32 ft.
10. Maple tree	2 ft.	$6\frac{1}{4}$ ft.	8 ft.	25 ft.
11. Church steeple	$1\frac{1}{2}$ ft.	30 ft.	5 ft.	100 ft.
12. Diving tower	6 ft.	20 ft.	3 ft.	10 ft.
13. Brick chimney	4 ft.	30 ft.	6 ft.	45 ft.

If possible, have the class do ex. 1 under your supervision. In ex. 2 be sure the students understand why the answers are not affected. Have students draw the diagrams for each problem (ex. 4–13).

Teach the students another method of measuring the heights of trees and other similar heights. Notice that this method gives only the approximate height.



Another Way to Find Heights

See the Guide for a discussion of the theorem on which the method is based.

1. The approximate height of a tree may be found as follows: Emphasize. Hold a ruler **vertically** at arm's length so that the top B of the ruler is in line with the top D of the tree; next move your thumb C along the ruler until it is in line with the bottom E of the tree. Then read the distance BC on the ruler; suppose it is 9 in. ($\frac{3}{4}$ ft.). Measure the horizontal distance AF from your eye to the ruler; suppose it is 2 ft. Pace the distance GE between you and the tree; suppose it is 48 ft. The ratio of AF to GE is equal to the ratio of BC to DE. So you can form this proposition: $AF:GE = BC:DE$.

▶ Let x = height of tree in feet

Substitute your measurements in the proportion which gives

$$\frac{2}{48} = \frac{\frac{3}{4}}{x}$$

Solve for x: $x = 18$

This shows that the tree is 18 ft. high.

Have different students explain their answers for ex. 2–9.

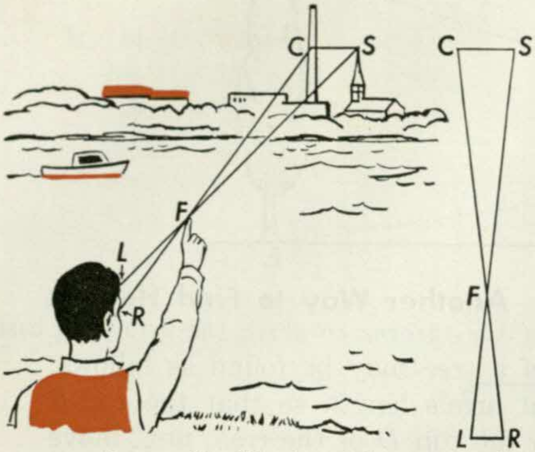
By the above method, find height DE of a tree, using these measurements:

	AF	GE	BC			AF	GE	BC	
2.	$2'$	$60'$	$\frac{5}{6}'$	25 ft.	**6.**	$2'$	$90'$	$\frac{11}{12}'$	$41\frac{1}{4}$ ft.
3.	$1\frac{3}{4}'$	$70'$	$\frac{3}{4}'$	30 ft.	**7.**	$1\frac{5}{6}'$	$165'$	$\frac{7}{12}'$	$52\frac{1}{2}$ ft.
4.	$2\frac{1}{4}'$	$81'$	$\frac{5}{6}'$	30 ft.	**8.**	$1\frac{11}{12}'$	$46'$	$\frac{3}{4}'$	18 ft.
5.	$2\frac{1}{6}'$	$52'$	$1'$	24 ft.	**9.**	$2'$	$88'$	$\frac{3}{4}'$	33 ft.

Read and discuss ex. 1 with the class. Emphasize that BC should be expressed in feet. Demonstrate this method in class, and then measure each student's horizontal distance AF. Give some practice in using this method, if possible.

Teach the students a method of estimating distances straight ahead of them (pages 334-335).

Estimating Distances



1. If you wish to find about how far it is to the tall chimney C across the river, you can do so as follows: Stretch out your right arm at full length, shut your left eye, and point to the chimney C so that the line of sight from your right eye R to the chimney will pass over the tip of your finger F. Next, without moving your arm or finger, open your left eye and shut your right eye; then sight with the left eye L across the tip of your finger as before. You now find that your finger is no longer pointing at the chimney C; instead it is pointing at the steeple S at the right of the chimney. Estimate the distance CS from the chimney to the steeple and suppose you call it about 60 ft. Multiply 60 ft. by 10, which gives 600 ft., the approximate distance from you to the chimney.

▶ This method makes use of two triangles, FLR and FSC, which are shown at the right of the picture. Since these triangles are similar, the ratio of RF to RL in triangle FLR equals the ratio of FC to CS in triangle FSC. It has been found that the average distance, RF, from one's right eye to the end of one's outstretched right arm is about 10 times the distance, RL, between the pupils of the eyes; therefore RF is 10 times as long as RL. Since triangles FLR and FSC are similar, FC is also 10 times as long as CS. Since you estimated CS to be 60 ft. long, then FC is about 600 ft. long.

If there is no second object S to help you to estimate the distance CS, then estimate the distance from the chimney to the spot at the right of the chimney toward which your finger points last. Stress.

The above method uses the fact that it is much easier to estimate a **short** distance, like CS, which is nearly perpendicular to the line of sight, than a **long** distance, like FC, which is straight ahead.

Emphasize that the right arm must be extended at full length and held without moving until the sighting is completed. Also, CS must be estimated carefully.

Before doing these problems, give the students some practice in using these methods to estimate distances from the classroom to distant buildings or objects, and so on.

Estimating Distances and Heights

1. Dick and Fred are in the observation tower on South Hill near their town. To find how far they are from the high school in the center of the town, Dick uses the method described on page 334. With his right eye he sights the flagpole of the high school over the forefinger of his outstretched right arm. When he closes his right eye and sights with his left eye, his finger points to the steeple of the old church, which is 3 blocks from the high school. If each block is 350 ft. long, about how many miles are the boys from the high school? 2 1 mi. = 5280 ft.

2. One day at the seashore, Sam sighted a large steamship on the horizon. Stretching out his arm, he sighted the ship over the tip of his forefinger. As he closed his right eye and opened his left eye, his finger marked off on the horizon a distance equal to about twice the length of the ship. If the ship was 800 ft. long, about how far away was it? 3 mi.

3. Suppose you are trying to find the height of a tree by using the method that is described on page 333. Instead of holding the ruler in a vertical position, suppose you are careless and hold it in a slanting position so that the top of the ruler slants toward the tree as you sight the top and bottom of the tree. Will the height of the tree as you find it be too large or too small? Try it and see.
Too large; see Guide.

4. When Roy wants to find the height of a tree by the shadow method, he pushes a stick about $3\frac{1}{2}$ ft. long vertically into the ground so that it stands up 3 ft. above the ground. Then he waits until the time of day when the shadow of the stick is just 3 ft. long, which is the same as the height of the stick. At that time he measures the length of the shadow of the tree, which he finds to be 47 ft. long. Without making a proportion, tell the height of the tree. Explain.
47 ft.

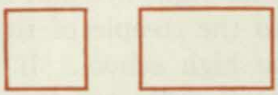
5. Find the height of a tree by the shadow method. Then find the height of the same tree by the method described on page 333. Which method do you prefer and why?

Do ex. 1-4 with the class. Remind the students that the estimate must be multiplied by 10. Encourage them to do ex. 5 and report their findings to the class. They will need practice in using this method out of doors.

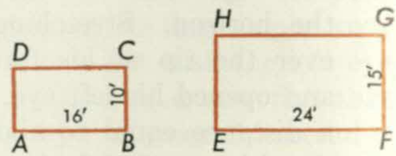
Define similar rectangles and show that scale drawings of floor plans are an application of similar rectangles (pages 336–337).

Similar Rectangles

1. If two rectangles have the same shape but differ in size, they are called **similar rectangles.** You have learned that two triangles are similar if two angles of one equal two angles of the other. Two rectangles, however, are not necessarily Stress. similar just because the angles of one equal the angles of the other. For example, the rectangles at the left are not similar though the four angles of one equal the four angles of the other. Do the four angles of one rectangle equal the four angles of any other rectangle? Explain your answer.



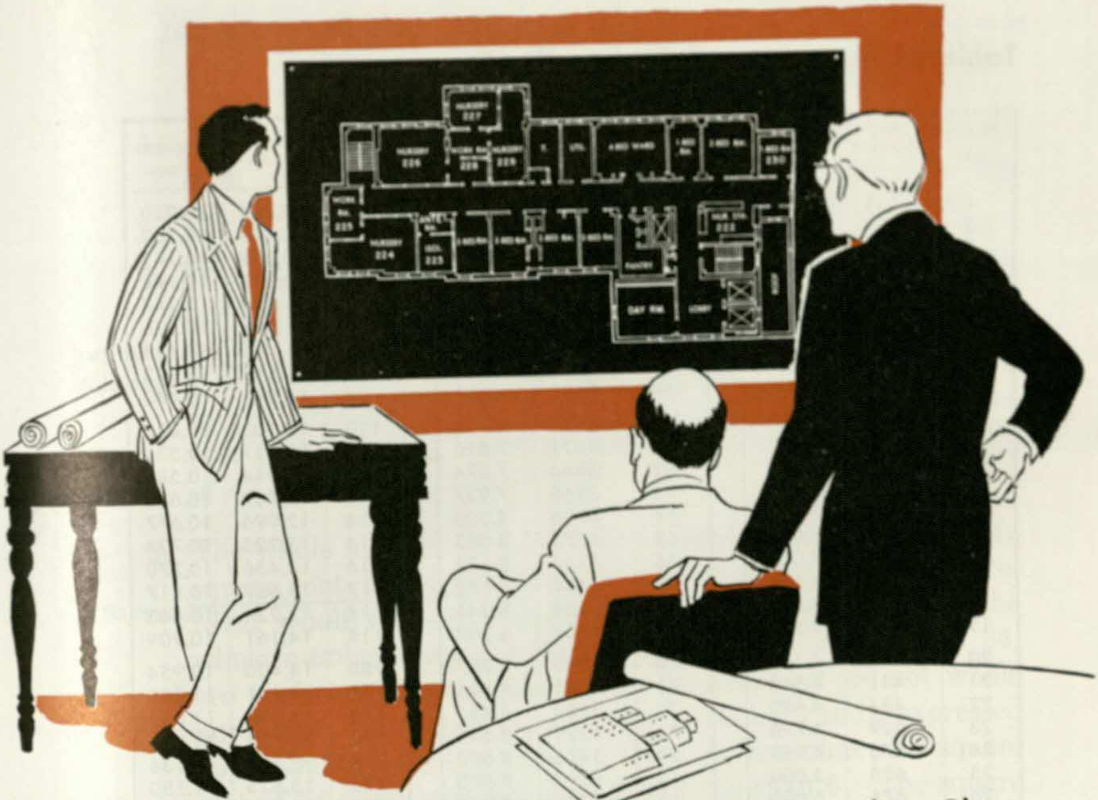
2. In order for two rectangles to be similar it is necessary that their corresponding sides have the same ratio. For example, in the rectangles at the right, the ratio of AB to EF is $\frac{16}{24}$, or $\frac{2}{3}$; likewise, the ratio of BC to FG is $\frac{10}{15}$, or $\frac{2}{3}$; the other corresponding sides have this same ratio. Since all these ratios are equal, the rectangles are similar. Have students explain why.



3. Is a rectangle that is 12 ft. wide and 15 ft. long similar to one that is 8 ft. wide and 10 ft. long?(1) Is it similar to a rectangle that is 9 ft. wide and 12 ft. long?(2) Explain your answer in each case. (1) Yes, because $\frac{12}{15} = \frac{8}{10}$; (2) no, because $\frac{12}{15}$ does not equal $\frac{9}{12}$.

4. Make a scale drawing of a rectangular garden 30 ft. long and 20 ft. wide, letting 1 in. = 10 ft. What is the length of your drawing in inches? What is the length of the garden in feet? in inches? Show that the ratio of the length of your drawing to the length of the garden is $\frac{3}{360}$, or $\frac{1}{120}$. In the same way, find the ratio of the width of your drawing to the width of the garden. Are these ratios equal? Do the angles of your drawing equal the corresponding angles of the garden? Is the rectangle you drew similar to the actual garden? Why? Is any scale drawing of a rectangular garden always similar to the actual garden? Discuss thoroughly.

In discussing ex. 2, be sure the students understand why the rectangles are similar. Again stress the fact that rectangles are similar if the ratios of corresponding sides are equal.



Studying Floor Plans

1. The men in the picture are looking at the floor plan of a building. All floor plans are scale drawings. Since the rooms of a building are usually rectangles, these floor plans contain scale drawings of many rectangles.

2. Make a scale drawing of a room 20 ft. long and 15 ft. wide, letting 1 in. = 5 ft. How many inches long will your drawing be? 4 How many inches long is the actual room? 240 in. What is the ratio of the length of your drawing to the length of the room? $\frac{1}{60}$ What is the ratio of the width of your drawing to the width of the room? $\frac{1}{60}$ Are these ratios equal? Yes Are the angles equal? Yes Is the rectangle you drew similar to the actual room? Yes Explain why.

Floor plans are usually made up of rectangles each of which is similar to the room it represents. Emphasize.

Show how to use a table of squares and square roots (pages 338-339).

Table of Squares and Square Roots

Number	Square	Square Root	Number	Square	Square Root	Number	Square	Square Root
1	1	1.000	51	2601	7.141	101	10,201	10.050
2	4	1.414	52	2704	7.211	102	10,404	10.100
3	9	1.732	53	2809	7.280	103	10,609	10.149
4	16	2.000	54	2916	7.348	104	10,816	10.198
5	25	2.236	55	3025	7.416	105	11,025	10.247
6	36	2.449	56	3136	7.483	106	11,236	10.296
7	49	2.646	57	3249	7.550	107	11,449	10.344
8	64	2.828	58	3364	7.616	108	11,664	10.392
9	81	3.000	59	3481	7.681	109	11,881	10.440
10	100	3.162	**60**	3600	7.746	**110**	12,100	10.488
11	121	3.317	61	3721	7.810	111	12,321	10.536
12	144	3.464	62	3844	7.874	112	12,544	10.583
13	169	3.606	63	3969	7.937	113	12,769	10.630
14	196	3.742	64	4096	8.000	114	12,996	10.677
15	225	3.873	65	4225	8.062	115	13,225	10.724
16	256	4.000	66	4356	8.124	116	13,456	10.770
17	289	4.123	67	4489	8.185	117	13,689	10.817
18	324	4.243	68	4624	8.246	118	13,924	10.863
19	361	4.359	69	4761	8.307	119	14,161	10.909
20	400	4.472	**70**	4900	8.367	**120**	14,400	10.954
21	441	4.583	71	5041	8.426	121	14,641	11.000
22	484	4.690	72	5184	8.485	122	14,884	11.045
23	529	4.796	73	5329	8.544	123	15,129	11.091
24	576	4.899	74	5476	8.602	124	15,376	11.136
25	625	5.000	75	5625	8.660	125	15,625	11.180
26	676	5.099	76	5776	8.718	126	15,876	11.225
27	729	5.196	77	5929	8.775	127	16,129	11.269
28	784	5.292	78	6084	8.832	128	16,384	11.314
29	841	5.385	79	6241	8.888	129	16,641	11.358
30	900	5.477	**80**	6400	8.944	**130**	16,900	11.402
31	961	5.568	81	6561	9.000	131	17,161	11.446
32	1024	5.657	82	6724	9.055	132	17,424	11.489
33	1089	5.745	83	6889	9.110	133	17,689	11.533
34	1156	5.831	84	7056	9.165	134	17,956	11.576
35	1225	5.916	85	7225	9.220	135	18,225	11.619
36	1296	6.000	86	7396	9.274	136	18,496	11.662
37	1369	6.083	87	7569	9.327	137	18,769	11.705
38	1444	6.164	88	7744	9.381	138	19,044	11.747
39	1521	6.245	89	7921	9.434	139	19,321	11.790
40	1600	6.325	**90**	8100	9.487	**140**	19,600	11.832
41	1681	6.403	91	8281	9.539	141	19,881	11.874
42	1764	6.481	92	8464	9.592	142	20,164	11.916
43	1849	6.557	93	8649	9.644	143	20,449	11.958
44	1936	6.633	94	8836	9.695	144	20,736	12.000
45	2025	6.708	95	9025	9.747	145	21,025	12.042
46	2116	6.782	96	9216	9.798	146	21,316	12.083
47	2209	6.856	97	9409	9.849	147	21,609	12.124
48	2304	6.928	98	9604	9.899	148	21,904	12.166
49	2401	7.000	99	9801	9.950	149	22,201	12.207
50	2500	7.071	**100**	10,000	10.000	**150**	22,500	12.247

First discuss ex. 1-3 on page 339 before explaining the use of the table. Point out that the square roots (except exact ones) given in this table are correct to the nearest thousandth.

Point out that most whole numbers have square roots that can be expressed only approximately.

Squares and Square Roots

1. If you multiply 4 by 4, the product 16 is called the **square** of 4. You know that 4×4 can be written 4^2 and that this can be read "4 squared." $4^2 = 16$. Find 8^2; 64 7^2; 49 12^2. 144

2. It is often necessary to start with a number and reverse the above process. This means that you must express a number as the product of **two equal factors.** For example, $49 = 7 \times 7$; and 7 is called the **square root** of 49. Express 36 as the product of two equal factors. What is the square root of 36? 6 What is the square root of 81? 9 of 25? 5 of 100? 10 of 4? 2

Emphasize meaning.

3. The whole number 25 is called a **perfect square** because its square root, 5, is a whole number; 36 is also a perfect square. Most whole numbers are not perfect squares. None of the whole numbers between 25 and 36, such as 26, 28, and 34, are perfect squares because their square roots are not whole numbers. The square root of such a number can be expressed approximately as a decimal. You can see that the square root of 34 must be larger than 5 but less than 6. The square root of 34 is equal to 5.8 to the nearest tenth, 5.83 to the nearest hundredth, and 5.831 to the nearest thousandth.

Stress.

4. The squares and square roots of numbers can be found in tables. A short table is given on page 338. If you look in the left-hand column of the table for 34, the first number to the right of 34 is 1156, which is the **square** of 34; the next number to the right is 5.831, which is the **square root** of 34.

Ex. 5–6 may be done orally with the class. Give further practice if needed.

Using the table on page 338, find the square (1) and also the square root (2) of each number:

5. 15 (1) 225 (2) 3.873 43 1849 6.557 81 6561 9.000 91 8281 9.539 121 14,641 11.000 112 12,544 10.583 108 11,664 10.392

6. 35 1225 5.916 50 2500 7.071 72 5184 8.485 65 4225 8.062 149 22,201 12.207 135 18,225 11.619 150 22,500 12.247

7. In the table, find the square root of 52 and then square your answer. Is the product close to 52? Repeat, using the square root of 39; of 17; of 10; of 122; of 147.

Discuss ex. 1–3 with the class. Be sure the students understand the meaning of square, square root, and perfect square. Have them follow the directions in ex. 4 and check to see that all locate the square and the square root of 34.

Teach the students how to find, to the nearest tenth, the square roots of numbers that are not in the table (pages 340–341). See the Guide for a discussion of the modern treatment of square root.

Using the Table Backward

1. The table on page 338 does not give the square roots of numbers over 150 but the square roots of larger numbers can be found in the table **provided** the numbers are perfect squares. In such cases the table is used backward. For example, to find the square root of 1296, look for 1296 in one of the columns headed "Square." Then, in the column to the left of 1296, you find 36, which is the square root of 1296. In the table find the square root of 4761; 69 of 3844; 62 of 1936; 44 of 7921; 89 of 9216; 96 of 13,225; 115 of 10,609. 103

2. The table on page 338 may also be used to find the square roots of larger numbers that are **not** in the column headed "Square" but lie between two numbers in that column. Such numbers are **not** perfect squares, so their square roots have to be found approximately. Ex. 3 is an example. Stress.

3. **Problem** Find the square root of 263 to the nearest tenth. 16.2

Explanation In the second column of the table, headed "Square," you do not find 263 but you find that 263 lies between 256 (the square of 16) and 289 (the square of 17). Thus the square root of 263 lies between 16 and 17.

You must now decide whether 263 is closer to 256 or to 289. Since the difference between 263 and 256 is 7, while the difference between 263 and 289 is 26, you see that 263 is closer to 256 than to 289. Thus the square root of 263 is closer to 16 than it is to 17. Next actually square some numbers close to 16, such as 16.1, 16.2, 16.3, and write the results as shown at the right. Examining 259.21, 262.44, and 265.69 you see that 262.44 is closest to 263. So 16.2 is the square root of 263 to the nearest tenth.

$$16.1^2 = 259.21$$
$$16.2^2 = 262.44$$
$$16.3^2 = 265.69$$

Do part of ex. 4 with class to assure understanding.

4. Find, to the nearest tenth, the square root of 234; 15.3 408; 20.2 300; 17.3 202; 14.2 177; 13.3 266; 16.3 152; 12.3 172; 13.1 498; 22.3 541; 23.3 8875. 94.2

5. The symbol for square root is $\sqrt{\ }$. $\sqrt{16}$ means that the square root of 16 is to be found; therefore $\sqrt{16} = 4$.

Have pupils do ex. 6 independently.

6. Find to the nearest tenth: $\sqrt{587}$, 24.2 $\sqrt{338}$, 18.4 $\sqrt{369}$. 19.2

The procedure in ex. 3 should be done step by step with the class. Emphasize that you are expressing 263 as the product of two equal factors, approximately.

Give further instruction and practice in finding square roots to the nearest tenth.

In the exercises below, use the table on page 338:

1. **Problem** Find, to the nearest tenth, the square root of 476. 21.8

Explanation In the table of squares on page 338, you find that 476 lies between 441 (the square of 21) and 484 (the square of 22); but it is much nearer to 484 than to 441. So the square root of 476 is closer to 22 than to 21. Square 21.7, 21.8, and 21.9 to find which square is closest to 476. Show that 21.8 is the square root of 476 to the nearest tenth.

$21.7^2 = 470.89$
$21.8^2 = 475.24$
$21.9^2 = 479.61$

2. Find to the nearest tenth: $\sqrt{167}$, $\sqrt{191}$, $\sqrt{220}$, $\sqrt{253}$. 12.9, 13.8, 14.8, 15.9

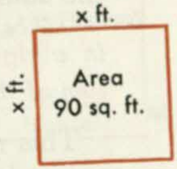
3. **Problem** Find, to the nearest tenth, the square root of 240.

Explanation Since 240 lies about halfway between 225 and 256, its square root is about halfway between 15 and 16. Try 15.4, 15.5, and 15.6, squaring each as shown at the right. You find that 15.5 is the approximate square root of 240. Why?

$15.4^2 = 237.16$
$15.5^2 = 240.25$
$15.6^2 = 243.36$

4. Find to the nearest tenth: $\sqrt{343}$, $\sqrt{385}$, $\sqrt{417}$, $\sqrt{463}$, $\sqrt{185}$, $\sqrt{1400}$, $\sqrt{2011}$, $\sqrt{1332}$, $\sqrt{18{,}205}$. 18.5, 19.6, 20.4, 21.5, 13.6, 37.4, 44.8, 36.5, 134.9

5. If you want the area of a square garden to be 90 sq. ft., you can find the length of one side of the garden by taking the square root of 90. Find the length x of one side of the garden, to the nearest tenth of a foot. 9.5



▶ Get the square root directly from the table and round off the result to the nearest tenth.

6. Find, to the nearest whole number, the square root of 220; 15 of 330; 18 of 5940; 77 of 7050; 84 of 18,470; 136 of 22,210. 149

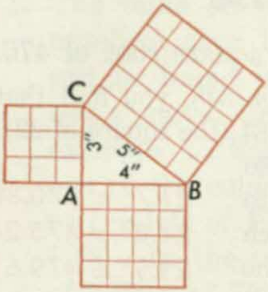
▶ In the column of squares in the table, 220 lies between 196 (the square of 14) and 225 (the square of 15). It is seen that 220 is closer to 225 than to 196, so the square root of 220 is closer to 15 than to 14. The square root of 220, to the nearest whole number, is 15.

Do ex. 1 with the class. Have different students do and explain ex. 2 at the board. Then discuss ex. 3. Be sure all the students understand this work before assigning ex. 4. Part of ex. 6 should be done with the class. Then have the students complete ex. 6.

Show, in a right triangle, how to find the length of the hypotenuse if the other two sides are given (pages 342-343). Emphasize the meaning

The Pythagorean Formula

of hypotenuse and legs and point out that the hypotenuse is always the largest side of the triangle.



1. In a right triangle there is only one right angle. The side opposite the right angle is called the **hypotenuse;** the other two sides are called the **legs.** Stress.

2. Draw a right triangle ABC whose legs, CA and AB, measure 3 in. and 4 in. Then measure the hypotenuse, BC. It should be exactly 5 in. long.

Next construct three squares having CA, AB, and BC as sides. Divide these squares into square inches, as shown in the drawing above.

Stress. The square on leg CA contains 9 sq. in. The square on leg AB contains 16 sq. in. The square on the hypotenuse BC contains 25 sq. in., which is the sum of 9 sq. in. and 16 sq. in. The area of the square on BC equals the sum of the areas of the squares on the legs. Stress. Stress.

3. Draw a right triangle whose legs are 6 in. and 8 in. Measure the hypotenuse. Is it 10 in. long? Then find the area of the square on each side of the triangle. Now show that $6^2 + 8^2 = 10^2$. Does the square of the hypotenuse equal the sum of the squares of the other two sides? Yes

Emphasize.

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

This rule is called the **Pythagorean formula;** it was discovered by **Pythagoras,** a Greek philosopher, about 540 B.C.

4. The legs of a right triangle are 5 in. and 12 in. Find the length of the hypotenuse, using the Pythagorean formula. 13 in.

▶ $5^2 + 12^2 = 25 + 144 = 169$. Thus the square of the hypotenuse is 169 sq. in. The length of the hypotenuse is the square root of 169. Find $\sqrt{169}$ in the table on page 338.

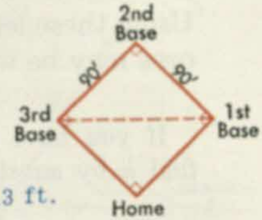
5. Find the hypotenuse of a right triangle whose legs are 9 in. and 12 in.; 15 in. of one whose legs are 7 in. and 24 in.; 25 in. of one whose legs are 12 in. and 35 in. 37 in.

Have the students make constructions in ex. 2-3. Emphasize that "the square of the hypotenuse" means "the area of the square on the hypotenuse." Be sure the meaning of "the sum of the squares of the other two sides" is also understood.

Using the Pythagorean Formula

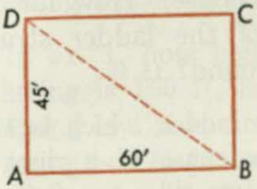
Stress.

Compute each answer to the nearest tenth. Use the table on page 338 to find squares and square roots:

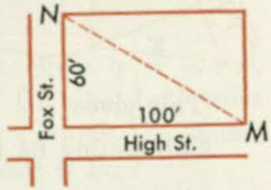


1. A baseball diamond is a square measuring 90 ft. on each side. How far does a player throw a ball when he throws it from first base to third base? 127.3 ft. from second base to home plate? 127.3 ft.

2. Mrs. Brown's back yard is a rectangle 30 ft. long and 20 ft. wide. Draw a diagram of the yard and tell how long a clothesline must be to stretch diagonally from one corner of the yard to the opposite corner. 36.1 ft. After finding the length, add 4 ft. to it to allow for tying the line at each end. 40.1 ft.



3. A rectangular swimming pool is 60 ft. long and 45 ft. wide. If Jane swam from one corner B to the opposite corner D, how many feet did she swim? 75 If she swam from A to C, would the distance be the same as BD or longer? Same



4. There is an empty corner lot near Jack's house. The lot is 100 ft. long and 60 ft. wide with a diagonal path NM across it. When Jack goes to school each day, he cuts corners by taking the path instead of walking on the sidewalks on Fox Street and High Street. How many feet does he save by taking the path instead of the sidewalks? 43.4

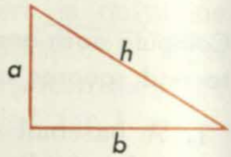
5. To drive a car from Wells to Lee, you must drive 12 mi. north to Dover and then 16 mi. east to Lee. How many miles could you save if there were a road leading direct from Wells to Lee? 8 Using a scale of $\frac{1}{4}$ in. = 1 mi., draw a diagram to scale showing the roads, including the direct road from Wells to Lee. Then measure the length of the direct road and see if it agrees with what you computed.

Do ex. 1 with the students and discuss the solution thoroughly. Then have them complete ex. 2–5. Have them draw the diagram for ex. 5. Check the solutions carefully and give more practice in finding square roots to nearest tenth, if needed.

Show, in a right triangle, how to find the length of one leg of a triangle if the hypotenuse and the other leg are given.

Problems about Ladders

1. In this right triangle a, b, and h represent the lengths of the legs and the hypotenuse. Using these letters, the formula of Pythagoras may be written:



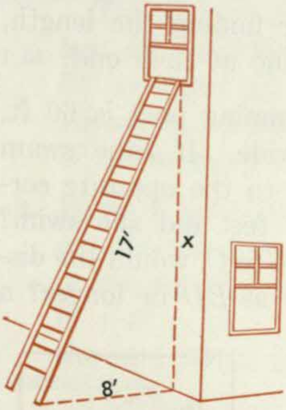
$$h^2 = a^2 + b^2$$

If you have a triangle in which $a = 9$ and $b = 12$, you can find h by substituting these values in the formula, thus

$$h^2 = 81 + 144$$
$$h^2 = 225$$

Taking the square root of 225, you get

$$h = 15$$



2. Problem The ladder at the left is 17 ft. long and its foot is 8 ft. from the side of the house. How far is it from the top of the ladder straight down to the ground? 15 ft.

Explanation The ladder, which is 17 ft. long, is the hypotenuse of a right triangle; the other two sides are 8 ft. and x ft. Substituting these values in the formula, $h^2 = a^2 + b^2$, you get

$$17^2 = 8^2 + x^2$$

This gives
$$289 = 64 + x^2$$

Subtracting 64 from each side, you get

$$225 = x^2$$

Taking the square root of 225, you get

$$15 = x$$

The top of the ladder is 15 ft. from the ground.

Have the students draw the triangles for ex. 3–4.

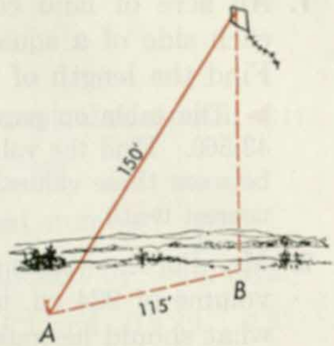
3. A ladder is 23 ft. long and its foot is 7 ft. from the side of a house. How far is it, to the nearest tenth of a foot, from the top of the ladder to the ground? 21.9

4. The foot of a 25-foot ladder is 12 ft. from the side of a barn. How far is it from the top of the ladder to the ground? 21.9 ft.

Do ex. 1–2 with the class. Point out that the formula is an equation and any one of the letters can be considered as an unknown quantity which can be found if the other two quantities are known. Have volunteers explain the work in ex. 3–4.

Show other uses of the Pythagorean formula and of square root (pages 345–346).

How High Is the Kite?



1. If you are flying a kite, can you tell about how high it is above the ground? Here is one way to do it. First, put a mark on your kite string 150 ft. from the place where it is attached to the kite. Then fly your kite. When you have let out 150 ft. of string, fasten it to a stake A in the ground. Then pace or measure the distance on the ground from point A to point B, which is directly under the kite. Suppose $AB = 115$ ft.; you can now compute the height of the kite above B by using the formula of Pythagoras. 96.3 ft. The kite string is the hypotenuse of the triangle.

2. In ex. 1, how high above the ground will the kite be if the string is 150 ft. long and the distance AB is 90 ft.? 120 ft. if the string is 125 ft. long and the distance AB is 100 ft.? 75 ft. if the string is 175 ft. long and AB is 140 ft.? 105 ft.

3. Problem A circle has an area of 616 sq. ft. Find the length of the radius of the circle. 14 ft. Use $\pi = 3\frac{1}{7}$.

Explanation The formula for the area of a circle is $A = \pi r^2$. Substituting 616 for A, you get $616 = \pi r^2$. Dividing both sides of the equation by $3\frac{1}{7}$, you get $196 = r^2$; so r equals the square root of 196, which is 14. The radius of the circle is 14.

4. How long is the radius of a circle if its area is 154 sq. in.? 7 in. 1386 sq. in.? 21 in. 2464 sq. in.? 28 in. 3850 sq. in.? 35 in. 5544 sq. in.? 42 in.

Find the third side of these triangles if the hypotenuse and one leg have the following lengths:

5. 37 ft., 35 ft. 12 ft.	**8.** 82 in., 80 in. 18 in.	**11.** 29 ft., 21 ft. 20 ft.
6. 50 in., 48 in. 14 in.	**9.** 65 ft., 63 ft. 16 ft.	**12.** 40 in., 32 in. 24 in.
7. 61 ft., 11 ft. 60 ft.	**10.** 74 in., 70 in. 24 in.	**13.** 73 ft., 55 ft. 48 ft.

Do ex. 1 with the class. Point out and explain that the height found will be approximate. Have the students do ex. 2. In ex. 3 have them note that the formula for the area is an equation, and the principles learned in Chapter 9 may be used to solve for r^2.

Problems

1. An acre of land contains 43,560 sq. ft. How long would each side of a square field be if the field contained 1 acre? Find the length of each side to the nearest tenth of a foot. 208.7

   ▶ The table on page 338 does not contain a square as large as 43,560. Find the value of 208^2 and 209^2 and show that 43,560 lies between these values. Then find the square root of 43,560 to the nearest tenth.

2. Mr. Porter wishes to make a cylindrical container with a volume of 924 cu. in. If he makes its radius equal to 7 in., what should he make the height? 6 in.

   ▶ The formula for the volume of a cylinder is $V = \pi r^2 h$. If you substitute in this formula 924 for V, 7 for r, and $3\frac{1}{7}$ for π, you get $924 = 154\,h$. Solve for h.

3. A cylindrical tank is to have a volume of 1650 cu. ft. If its length is to be 21 ft., what should be its radius? 5 ft.

   ▶ If you substitute in the formula used in ex. 2, 1650 for V, 21 for h, and $3\frac{1}{7}$ for π, you get $1650 = 66\,r^2$. Solve this equation for r^2, and then find r.

4. If you are asked to draw a triangle whose sides are 14, 48, and 50, you can tell before drawing it whether it is a right triangle. If the sentence $50^2 = 14^2 + 48^2$ is true, then the triangle is a right triangle. If $50^2 < 14^2 + 48^2$ or if $50^2 > 14^2 + 48^2$, then the triangle is not a right triangle. Is $50^2 = 14^2 + 48^2$ a true sentence? Yes Is the triangle a right triangle? Yes Draw it to scale.

5. A triangle has sides of 19, 15, and 8. Test these sides as described in ex. 4 and see if they make a right triangle. No Also draw the triangle. Do the same for a triangle whose sides are 36, 35, and 12. No

Have the students write a true sentence for each triangle.

In each exercise below, the three numbers are the sides of a triangle. Tell which triangles are right triangles:

6. 100, 90, 56 No	**8.** 34, 30, 16 Yes	**10.** 41, 40, 9 Yes
7. 29, 21, 20 Yes	**9.** 50, 48, 20 No	**11.** 82, 64, 48 No

With reference to ex. 4, you might point out that if the square of the longest side of any triangle is greater than the sum of the squares of the other two sides, the angle opposite the longest side is an obtuse angle. If it is less, the angle is an acute one.

Present mixed practice on computation and problems.

Mixed Practice

1. What per cent of 132 is 19.8? 15%
2. 477 is 75% of what number? 636
3. Round off 3.1416 correct to the nearest thousandth. 3.142
4. Find the product of $4\frac{1}{8}$, $5\frac{1}{3}$, and 15. 330
5. 978 is $\frac{1}{3}$ of what number? 2934 $\frac{2}{3}$ of what number? 1467
6. Find the quotient when 152.88 is divided by .049. 3120
7. Subtract 175.29 from the sum of 4.72 and 219.39. 48.82
8. Find the prime factors of 308; 2 x 2 x 7 x 11 of 675. 3 x 3 x 3 x 5 x 5
9. Find the interest on $720 at 6% for 30 da.; $3.60 for 24 da. $2.88
10. Find the average of 15, $4\frac{1}{8}$, $23\frac{1}{2}$, $6\frac{1}{4}$, $11\frac{5}{8}$, $9\frac{1}{2}$, 17. $12\frac{3}{7}$
11. What is the sum of 1.92, 3.17, 4.64, 7.32, 18.08? 35.13
12. Change 195% to a decimal; 1.95 change .074 to a per cent. 7.4%
13. What is 1200% of $36? $432 What is $\frac{1}{12}$ of 1% of $36? $.03
14. Tell which of these fractions equal $\frac{3}{4}$: $\underline{\frac{6}{8}}$, $\frac{8}{12}$, $\underline{\frac{9}{12}}$, $\underline{\frac{16}{20}}$, $\frac{20}{35}$.
15. Bill Gates earned $4100 last year and saved $205. What per cent of his income did he save? 5%
16. One acute angle in a right triangle is 41°. Find the number of degrees in the other acute angle. 49°
17. Mr. Cook bought $600 worth of travelers checks. Find the total cost if the service charge was 1% of their value. $606
18. Mr. Long bought a house for $17,500. He paid $3500 down and will pay the balance in equal monthly installments during 20 yr. Find his monthly payment if he pays 5% interest on the unpaid balance. Use the table on page 208. $92.40
19. Last year the Wills Steel Company paid out dividends on its common stock amounting to $18,750. If there were 7500 shares of common stock, find the amount of the dividend on each share. $2.50

Have different students explain their answers at the board. Encourage class questions and discussion of this work. Plan further review or reteaching as needed.

Pages 348-351 provide a comprehensive review of problem solving covering topics presented in Grade 8. Conduct a class discussion of solutions to help clarify the students' thinking.

Problem Review

1. Mr. Chapman's taxes on property assessed at \$8500 were \$246.50. At this rate, find Mr. Miller's taxes on property assessed at \$12,000. \$348 Solve by using proportion.

2. The cooking classes made $18\frac{3}{4}$ lb. of candy for the school fair. Of this, $6\frac{1}{4}$ lb. was chocolate fudge, 5 lb. was taffy, and the rest was maple fudge. The chocolate fudge sold at \$1 a pound, the taffy at \$.85 a pound, and the maple fudge at \$1.25 a pound. How much money was taken in from the sale of this candy? \$19.88

3. The weight of a cake of baking chocolate is marked in ounces and also in grams. If it weighs 8 oz., how many grams does it weigh? About 227 1 kg. = 2.2 lb.

4. Find the net price after a discount of 20%, 10%, 2% has been given on the following bill: 12 chairs at \$16.75, 6 tables at \$32.50, and 8 lamps at \$10.00. \$335.87

5. The Cub baseball team has won 38 games and lost 15 games so far this season. The Bears have won 36 games and lost 14. Which team has the higher standing? Give the standings as three-place decimals. Cubs, .717; Bears, .720; Bears have better standing.

6. A circular flower bed 14 ft. in diameter is surrounded by a cement walk $3\frac{1}{2}$ ft. wide. How many square feet are there in the area of the walk? $192\frac{1}{2}$ First draw a diagram of the flower bed and walk. Use $\pi = 3\frac{1}{7}$.

7. In ex. 6, which has the larger area, the garden or the walk? How much larger? The walk is $38\frac{1}{2}$ sq. ft. larger.

8. Mr. Mason has a shoe store. He ordered 300 pairs of shoes at a net cost of \$2115. He figures that his expenses and his profit should be 40% of their selling price. For how much should he sell a pair of shoes? \$11.75

9. A field that is shaped like a trapezoid has bases of 250 ft. and 325 ft. The height of the trapezoid is 200 ft. Find, to the nearest hundredth of an acre, the number of acres in the field. 1.32 1 acre = 43,560 sq. ft.

The problems on these pages can be used to introduce and motivate a review of topics covered during the year. Base specialized reviews on class or individual needs as shown by students' understanding of the problems.

Problem Review

Have volunteers explain their solutions at the board.

1. Recently a record total of 15.50 in. of rain fell during the hours from 6:30 A.M. to 12:30 P.M. of the same day. Find, to the nearest hundredth of an inch, the average amount of rain that fell per hour. 2.58

2. A silo casts a shadow 40 ft. long at the same time of day that a man 6 ft. tall casts a shadow 5 ft. long. Find the height of the silo. 48 ft.

3. A check on the price of a No. $2\frac{1}{2}$ can of peaches showed that 9 different stores in a city charged these prices: 37¢, 39¢, 43¢, 45¢, 42¢, 40¢, 41¢, 49¢, 49¢. Find the difference between the highest price and the lowest price charged. 12¢ Also find the average price to the nearest cent. 43¢

4. Each dollar paid to the American railroads one year was used as follows: (185°) 51.5% for wages, (119°) 33% for materials and (31°) fuel, 8.5% for taxes, 6.5% (23°) for interest and rents, and 0.5% (2°) for the owners. Draw a circle graph to show these facts. See Guide.

5. The rates for electric current in Springfield are $1.00 for the first 15 kw-hr, 4¢ per kw-hr for the next 50 kw-hr, and $2\frac{1}{2}$¢ per kw-hr for all over 65 kw-hr. Find the amount of an electric bill for the interval between a reading of 6574 and one of 6818 on the electric meter. $7.48

6. When Judy was born, her father deposited $1500 in a savings account for her. This bank pays 3% interest, compounded semiannually. Using the table on page 212, find how much Judy will have in her account when she is 15 yr. old, if she makes no deposits or withdrawals. $2344.62 Find the total amount of compound interest earned in the 15 yr. $844.62

Individual conferences with students who had errors will still be helpful at this time in strengthening problem-solving skills. Help them to analyze and discover the causes of errors.

Problem Review

1. Mr. Oliver bought a new house for \$16,000. He took out a fire insurance policy for 80% of the value of the house and paid \$.24 per \$100 for the premium on it. How much was the premium on this insurance policy? \$30.72

2. Mr. Adams borrowed \$4000 from a savings and loan association to be repaid in 20 yr. at \$28.68 per month. How much interest did Mr. Adams pay in all? \$2883.20 What was the average amount of interest he paid per year? \$144.16

3. Find the exact number of days between Dec. 15 and May 14 of the following year. 150 (or 151 in leap year)

4. An empty corner lot is rectangular in shape and is 180 ft. by 240 ft. People have made a diagonal path across the lot for a short cut. How many feet shorter is it to cross the lot diagonally than to go around the corner? 120 If Jim's step is $2\frac{1}{2}$ ft. long, how many steps does he save by using the short cut instead of the sidewalk? 48

5. Mr. Walters wants to buy a house valued at \$18,000. If the savings bank will lend him $66\frac{2}{3}\%$ of the value of the house, how much will he have to pay in cash? \$6000

6. Mr. King plans to borrow \$9000 from a bank at $5\frac{1}{2}\%$ to build a house. If he repays the loan over a 15-year period, his monthly payment on the loan will be \$73.62. How much interest will Mr. King pay for the loan? \$4251.60

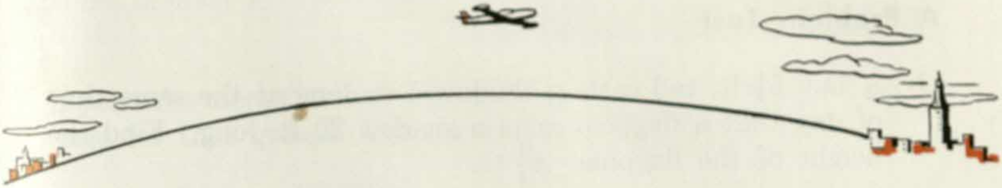
Solve by the equation method: See the Guide for the equations for ex. 7-9.

7. A tennis racket and a box of tennis balls together cost \$11.81. The racket costs \$7.31 more than the box of balls. Find the cost of the racket \$9.56 and also the cost of the balls. \$2.25

8. After saving up enough money for 3 wk. at camp, Andy received \$10 from his father to add to his camp money. Now he has \$95.50 in all. Find the cost of a week at camp. \$28.50

9. If 6 times a number is increased by 5, the total is 47. Find the number. 7

If errors are computational, provide more practice in this area after clearing up difficulties. See pages 348-349 for other suggestions.



Problem Review

1. A recent record flight of a jet airplane from the Pacific coast to New York took approximately 3 hr. 23 min. If the plane arrived in New York at 12:27 P.M. Eastern Daylight Saving Time, when did it leave the Pacific coast, Pacific Daylight Saving Time? 6:04 A.M.

2. If the distance traveled by the plane in ex. 1 was 2460 mi., what was its speed to the nearest mile per hour? 728 First express 3 hr. 23 min. as a decimal to the nearest hundredth.

3. Bill saved \$30. Without using the table, find the amount to which this money will grow in 4 yr. at 2% compounded semiannually? \$32.45 In computing the interest, disregard the cents in each new principal.

4. A broker sold 200 shares of Bell Oil stock for Mr. Peters at $49\frac{1}{4}$. At the rates given on page 190, what was the brokerage? \$87.26 Find the net amount Mr. Peters received. \$9762.74

5. Mr. Rice bought 100 shares of Ace Railroad stock at 50, paying a brokerage fee of \$40.00. Several years later, he sold the stock at $60\frac{3}{4}$. At the rates given on page 190, what was the brokerage? \$45.08 What was Mr. Rice's net profit? \$989.92

6. Mr. Page has preferred stock that he bought at 150. The annual dividend per share that he receives is \$6.00. Omitting brokerage, find the rate of income on this stock. 4%

See the Guide for a discussion of the scale drawing for ex. 7.

7. To find the distance across a river, two boys made three measurements so that they could draw a triangle *ABC* to scale and thus find the side representing the distance across the river. They found *AB* was 325 ft., angle *A* was 90°, and angle *B* was 40°. By making the scale drawing, find the distance across the river, which is *AC* in the triangle. 275 ft.

See the suggestions on pages 348–350.

Present the last problem test.

A Problem Test

1. A boy $5\frac{1}{4}$ ft. tall casts a shadow 4 ft. long at the same time of day that a flagpole casts a shadow 20 ft. long. Find the height of the flagpole. $26\frac{1}{4}$ ft.

2. In a right triangle the sides of the right angle are 8 ft. and 6 ft. Find the length of the hypotenuse of the triangle. 10 ft.

3. In a month the price of eggs per dozen increased from 49¢ to 67¢. Find, to the nearest tenth of 1%, the per cent of increase in the price. 36.7%

4. Mr. Case borrowed $900 from the bank on May 18 to be repaid on July 20. Find the bank discount at 4% $6.30 and the proceeds. $893.70

5. After earning the same amount of money each week for 7 wk., Joe spent $42.50 for a bicycle. He had $6.50 left. Using an equation, find how much Joe earned per week. $7.00

6. Without using a compound interest table find the amount to which $10,000 will grow in 2 yr. at 4% if the interest is compounded semiannually. $10,824.32

7. A rectangular swimming pool is 80 ft. long and 48 ft. wide. The average depth of the pool is 6 ft. When the pool is filled to within $\frac{1}{2}$ ft. of the top, how many gallons of water are in the pool? 158,400 $7\frac{1}{2}$ gal. of water occupy 1 cu. ft.

8. A 4% $1000 bond was bought at 92. Find, to the nearest tenth of 1%, the rate of income on this investment. 4.3%

9. A city where the assessed valuation of property is $253,750,000 needed to raise $8,323,000 to run the city for the next year. Find the tax rate per $100. $3.28

10. Find the annual premium on a life insurance policy for $12,000 at the rate of $23.51 per $1000. $282.12

SCORE	0–5 You need help	6–7 Fair	8–9 Good	10 Excellent

Record a summary of the results of these problem tests on the progress cards. Discussion of the problems will still be helpful at this time to reinforce the students' skills in problem solving.

Provide comprehensive diagnostic tests (pages 353-354) of skills taught in Grade 8, with practice-page references.

How Much Have You Learned?

If you miss more than one example in a group, turn to the Practice Page for that group. Use the table on page 338 for ex. 3-8.

Find x in these proportions:						Practice Pages
1. $\frac{x}{80} = \frac{6}{15}$ 32		$\frac{4\frac{1}{2}}{x} = \frac{48}{32}$ 3		$\frac{28}{12} = \frac{x}{9}$ 21		315
2. $11 : x = 52 : 13$ $2\frac{3}{4}$				$8 : 18 = 16 : x$ 36		315

Find the square of each of the following:

3. 39 1521	46 2116	52 2704	84 7056	127 16,129	339

Find the square root of each of the following:

4. 38 6.164	71 8.426	84 9.165	97 9.849	148 12.166	339
5. 841 29	5476 74	8649 93	17,161 131		340

Find, to the nearest tenth, the square root of each of the following:

6. 914 30.2	1955 44.2	4368 66.1	9293 96.4	340
7. 710 26.6	2000 44.7	3950 62.8	9381 96.9	341
8. 992 31.5	2075 45.6	4020 63.4	7330 85.6	341

Find three whole numbers that are solutions of each inequality: See Guide.

9. $x + 11 < 37$ $x + 8 > 41$ $x - 16 < 21$ 319

Column A gives the 3 sides of a triangle. Column B gives, in the same order, the 3 corresponding sides of a similar triangle. Find the unknown sides:

	A	B	
10.	6 in., 8 in., 9 in.	12 in., x, y $x = 16$ in., $y = 18$ in.	327
11.	5 in., 7 in., 8 in.	$7\frac{1}{2}$ in., x, y $x = 10\frac{1}{2}$ in., $y = 12$ in.	327
12.	9 in., 4 in., 6 in.	x, y, 21 in. $x = 31\frac{1}{2}$ in., $y = 14$ in.	327

These pages may be used as end-of-year tests. Check the papers carefully and analyze any errors. Through conferences with the students, try to clear up difficulties. Re-explain procedures as needed before assigning practice.

How Much Have You Learned?

If you miss any example, turn to the Practice Pages indicated.

Find the answers: **Practice Pages**

1. $\frac{5}{6} \times \frac{9}{10}$ $\frac{3}{4}$ $\quad 3\frac{3}{4} \times 1\frac{1}{5}$ $4\frac{1}{2}$ $\quad 11 \times 7\frac{3}{8}$ $81\frac{1}{8}$ 17
2. $\frac{5}{8} \div \frac{3}{4}$ $\frac{5}{6}$ $\quad 6\frac{3}{5} \div 2\frac{3}{4}$ $2\frac{2}{5}$ $\quad 4\frac{1}{2} \div 15$ $\frac{3}{10}$ 18, 19

Find quotients correct to the nearest tenth:

3. $3.8\overline{)42.73}$ 11.2 $\quad .45\overline{)291.2}$ 647.1 $\quad .09\overline{)32.47}$ 360.8 $\quad 1.23\overline{)6.74}$ 5.5 29, 30

Find the answers to the nearest cent:

4. 6.3% of \$255 \$16.07 $\quad$ 4.5% of \$495 \$22.28 43
5. $\frac{3}{4}$% of \$175 \$1.31 $\quad$ $\frac{1}{2}$ of 1% of \$835 \$4.18 45
6. $152\frac{1}{4}$% of \$85 \$129.41 $\quad$ 238.7% of \$350 \$835.45 50

Solve each equation for x and check:

7. $x - 12 = 41$ 53 $\quad x - 90 = 24$ 114 300
8. $\frac{1}{2}x = 4$ 8 $\quad \frac{1}{5}x = 7$ 35 $\quad \frac{1}{3}x = 5$ 15 302
9. $2x + 5 = 27$ 11 $\quad 3x - 4 = 14$ 6 308

Find these volumes. Use $\pi = 3\frac{1}{7}$:

10. Cylinder: radius, $3\frac{1}{2}$ ft.; height, 8 ft. 308 cu. ft. 231
11. Pyramid: base, 9 sq. ft.; height, 3 ft. 9 cu. ft. 237
12. Cone: radius, 7 ft.; height, 8 ft. $410\frac{2}{3}$ cu. ft. 240
13. Sphere: radius, 35 ft. $179,666\frac{2}{3}$ cu. ft. 244

Find the per cent of increase (inc.) or decrease (dec.) that the second number shows over the first number:

14. \$1200, \$1380 15% inc. $\quad$ \$24,000, \$23,880 .5% dec. 56
15. \$1960, \$1715 $12\frac{1}{2}$% dec. $\quad$ \$56,750, \$59,020 4% inc. 56

If the students need extra summer work, refer them to the practice pages or "More Practice," pages 358–372.

Provide diagnostic tests on fundamental addition, subtraction (page 356), multiplication, and division facts (page 357).

The Addition and Multiplication Facts

Add (1) or multiply (2) orally as the teacher directs. You should be able to give all the sums or products within the time stated by the teacher:

1. (1),(2)	3 1 4, 3	8 4 12, 32	9 8 17, 72	0 8 8, 0	4 5 9, 20	5 3 8, 15	8 5 13, 40	4 0 4, 0	6 5 11, 30	3 9 12, 27
2.	4 2 6, 8	1 9 10, 9	0 4 4, 0	2 3 5, 6	3 5 8, 15	5 7 12, 35	1 2 3, 2	2 9 11, 18	7 9 16, 63	8 0 8, 0
3.	5 2 7, 10	6 1 7, 6	2 6 8, 12	8 3 11, 24	1 0 1, 0	6 3 9, 18	2 8 10, 16	0 6 6, 0	6 9 15, 54	4 7 11, 28
4.	5 8 13, 40	9 0 9, 0	7 8 15, 56	9 4 13, 36	2 7 9, 14	8 8 16, 64	9 2 11, 18	1 1 2, 1	3 0 3, 0	7 7 14, 49
5.	2 1 3, 2	3 7 10, 21	5 0 5, 0	8 2 10, 16	4 4 8, 16	1 6 7, 6	3 2 5, 6	9 3 12, 27	2 0 2, 0	1 3 4, 3
6.	7 6 13, 42	6 0 6, 0	3 4 7, 12	7 2 9, 14	4 9 13, 36	1 4 5, 4	7 1 8, 7	8 6 14, 48	0 9 9, 0	8 9 17, 72
7.	4 8 12, 32	0 3 3, 0	5 6 11, 30	6 4 10, 24	1 8 9, 8	5 9 14, 45	7 0 7, 0	6 8 14, 48	4 1 5, 4	2 2 4, 4
8.	8 1 9, 8	6 7 13, 42	0 5 5, 0	5 4 9, 20	1 7 8, 7	3 3 6, 9	2 5 7, 10	0 1 1, 0	8 7 15, 56	6 2 8, 12
9.	0 2 2, 0	5 1 6, 5	9 9 18, 81	4 3 7, 12	7 5 12, 35	0 7 7, 0	9 6 15, 54	5 5 10, 25	9 7 16, 63	7 3 10, 21
10.	9 1 10, 9	4 6 10, 24	3 8 11, 24	0 0 0, 0	2 4 6, 8	1 5 6, 5	6 6 12, 36	3 6 9, 18	7 4 11, 28	9 5 14, 45

To the Teacher. See the suggestions on page 377 for giving this diagnostic test on the fundamental addition or multiplication facts.

The tests should be given at the beginning of the year, since a mastery of facts is necessary for success in the year's work. Note the results on the students' progress cards and give each student a list of facts which he needs to study. Retest in a week or two.

The Subtraction Facts

Subtract orally. You should be able to give all the answers within the time stated by the teacher:

1.	4	9	11	15	7	14	8	10	12	4
	3	1	6	6	5	8	5	3	9	4
	1	8	5	9	2	6	3	7	3	0
2.	3	1	7	11	8	5	15	10	4	0
	1	1	3	3	8	4	8	5	1	0
	2	0	4	8	0	1	7	5	3	0
3.	9	7	12	10	16	13	15	9	13	9
	9	6	3	6	8	6	9	7	8	4
	0	1	9	4	8	7	6	2	5	5
4.	12	7	13	9	11	12	11	17	8	5
	7	0	9	2	8	6	7	8	6	1
	5	7	4	7	3	6	4	9	2	4
5.	15	10	14	6	8	10	8	12	4	7
	7	4	9	6	4	2	1	8	0	2
	8	6	5	0	4	8	7	4	4	5
6.	7	5	9	8	11	3	17	4	9	10
	4	3	0	3	5	3	9	2	3	7
	3	2	9	5	6	0	8	2	6	3
7.	13	7	18	2	13	6	12	6	14	2
	5	1	9	1	7	4	4	0	5	2
	8	6	9	1	6	2	8	6	9	0
8.	16	11	8	12	13	6	9	3	10	6
	7	4	7	5	4	1	5	2	9	3
	9	7	1	7	9	5	4	1	1	3
9.	9	11	5	7	8	10	2	9	6	1
	6	9	0	7	2	1	0	8	5	0
	3	2	5	0	6	9	2	1	1	1
10.	16	6	10	3	14	11	5	8	5	14
	9	2	8	0	7	2	5	0	2	6
	7	4	2	3	7	9	0	8	3	8

To the Teacher. See the suggestions on page 377 for giving this diagnostic test on the fundamental subtraction facts.

Divide orally. You should be able to give all the quotients within the time stated by the teacher:

1.	$\overset{2}{2\overline{)4}}$	$\overset{0}{8\overline{)0}}$	$\overset{1}{6\overline{)6}}$	$\overset{6}{4\overline{)24}}$	$\overset{2}{7\overline{)14}}$	$\overset{8}{2\overline{)16}}$
2.	$\overset{2}{1\overline{)2}}$	$\overset{2}{4\overline{)8}}$	$\overset{0}{1\overline{)0}}$	$\overset{7}{7\overline{)49}}$	$\overset{5}{3\overline{)15}}$	$\overset{7}{6\overline{)42}}$
3.	$\overset{0}{3\overline{)0}}$	$\overset{9}{1\overline{)9}}$	$\overset{8}{6\overline{)48}}$	$\overset{4}{5\overline{)20}}$	$\overset{6}{9\overline{)54}}$	$\overset{5}{4\overline{)20}}$
4.	$\overset{4}{2\overline{)8}}$	$\overset{1}{4\overline{)4}}$	$\overset{9}{7\overline{)63}}$	$\overset{6}{3\overline{)18}}$	$\overset{6}{6\overline{)36}}$	$\overset{3}{9\overline{)27}}$
5.	$\overset{3}{3\overline{)9}}$	$\overset{8}{1\overline{)8}}$	$\overset{4}{9\overline{)36}}$	$\overset{7}{4\overline{)28}}$	$\overset{3}{7\overline{)21}}$	$\overset{5}{6\overline{)30}}$
6.	$\overset{3}{1\overline{)3}}$	$\overset{3}{2\overline{)6}}$	$\overset{7}{5\overline{)35}}$	$\overset{4}{7\overline{)28}}$	$\overset{8}{9\overline{)72}}$	$\overset{8}{3\overline{)24}}$
7.	$\overset{0}{5\overline{)0}}$	$\overset{1}{5\overline{)5}}$	$\overset{2}{8\overline{)16}}$	$\overset{6}{5\overline{)30}}$	$\overset{9}{9\overline{)81}}$	$\overset{4}{3\overline{)12}}$
8.	$\overset{1}{9\overline{)9}}$	$\overset{0}{4\overline{)0}}$	$\overset{9}{2\overline{)18}}$	$\overset{6}{7\overline{)42}}$	$\overset{5}{8\overline{)40}}$	$\overset{4}{4\overline{)16}}$
9.	$\overset{5}{1\overline{)5}}$	$\overset{0}{2\overline{)0}}$	$\overset{5}{5\overline{)25}}$	$\overset{8}{8\overline{)64}}$	$\overset{5}{2\overline{)10}}$	$\overset{3}{5\overline{)15}}$
10.	$\overset{0}{6\overline{)0}}$	$\overset{1}{3\overline{)3}}$	$\overset{6}{2\overline{)12}}$	$\overset{9}{8\overline{)72}}$	$\overset{4}{6\overline{)24}}$	$\overset{4}{8\overline{)32}}$
11.	$\overset{2}{3\overline{)6}}$	$\overset{0}{7\overline{)0}}$	$\overset{6}{8\overline{)48}}$	$\overset{8}{5\overline{)40}}$	$\overset{2}{9\overline{)18}}$	$\overset{7}{2\overline{)14}}$
12.	$\overset{1}{2\overline{)2}}$	$\overset{6}{1\overline{)6}}$	$\overset{5}{7\overline{)35}}$	$\overset{7}{3\overline{)21}}$	$\overset{3}{8\overline{)24}}$	$\overset{9}{6\overline{)54}}$
13.	$\overset{7}{1\overline{)7}}$	$\overset{0}{9\overline{)0}}$	$\overset{7}{9\overline{)63}}$	$\overset{9}{3\overline{)27}}$	$\overset{7}{8\overline{)56}}$	$\overset{2}{5\overline{)10}}$
14.	$\overset{1}{7\overline{)7}}$	$\overset{1}{8\overline{)8}}$	$\overset{8}{4\overline{)32}}$	$\overset{3}{6\overline{)18}}$	$\overset{3}{4\overline{)12}}$	$\overset{8}{7\overline{)56}}$
15.	$\overset{4}{1\overline{)4}}$	$\overset{1}{1\overline{)1}}$	$\overset{9}{5\overline{)45}}$	$\overset{2}{6\overline{)12}}$	$\overset{5}{9\overline{)45}}$	$\overset{9}{4\overline{)36}}$

To the Teacher. See the suggestions on page 377 for giving this diagnostic test on the fundamental division facts.

Pages 358–372 provide practice material in fundamental operations with whole numbers, fractions, decimals, and percentage. Assign these in accordance with the individual needs of each student.

More Practice

1

1. 91 + 42 + 45 + 92 + 34 + 36 + 92 + 83 + 55 + 61 631

2. 68 + 89 + 56 + 40 + 38 + 25 + 78 + 27 + 59 + 63 543

3. 59 + 22 + 67 + 94 + 85 + 46 + 13 + 65 + 68 + 26 545

4. 11 + 94 + 89 + 33 + 65 + 13 + 58 + 10 + 75 + 45 493

5. 25 + 90 + 16 + 86 + 77 + 49 + 92 + 35 + 18 + 99 587

6. 50 + 439 + 847 + 79 + 528 + 33 + 262 + 778 3016

7. 866 + 575 + 42 + 87 + 745 + 420 + 91 + 683 3509

8. \$767.35 + \$348.34 + \$439.50 + \$202.38 + \$591.49 \$2349.06

9. \$680.20 + \$511.19 + \$842.61 + \$999.32 + \$173.83 \$3207.15

10. \$425.04 + \$513.77 + \$224.87 + \$919.69 + \$620.73 \$2704.10

11.

517	846	957	840	8325	8680	8540
482	758	680	929	6910	9762	1374
210	230	529	607	8641	7935	9998
721	619	763	751	6386	7097	2620
305	986	604	235	1448	5755	2508
573	837	237	994	2635	3240	9129
254	741	446	486	1048	8181	8757
+736	+272	+942	+855	+8566	+1339	+4633
3798	5289	5158	5697	43,959	51,989	47,559

12.

805	651	916	842	7584	8078	9604
718	994	745	965	2103	6532	2394
846	631	632	913	8554	6677	4072
473	294	358	710	4039	3466	8993
861	478	575	284	9955	6070	7655
498	803	301	922	3183	4182	2319
509	741	148	876	8626	9744	4158
215	257	265	829	5507	9948	8725
+962	+908	+159	+735	+8336	+6401	+3529
5887	5757	4099	7076	57,887	61,098	51,449

13.

\$ 681.56	\$ 21.49	\$6026.24	\$ 1.85	\$8610.56
7436.25	2678.16	8743.58	8563.52	9961.69
188.73	7860.42	9.17	78.21	5062.73
2436.34	521.69	5914.95	6962.00	9479.14
923.46	16.76	61.17	1486.13	8128.17
4243.99	6331.10	649.00	.78	.32
8202.02	3734.52	5453.68	6903.52	2.04
+1874.52	+ 241.65	+1981.56	+5495.88	+2753.22
\$25,986.87	\$21,405.79	\$28,839.35	\$29,491.89	\$43,997.87

Although these pages are correlated with ones in Chapters 1 and 2, do not confine their use to these chapters only. Use them throughout the year to overcome computational weaknesses.

2

1.	9098 −4359 4739	7563 −6268 1295	9000 −159 8841	58037 −48698 9339	489632 −129817 359,815	\$619.29 −161.86 \$457.43
2.	6183 −1614 4569	7142 −1287 5855	9202 −404 8798	84431 −51853 32,578	461239 −107396 353,843	\$730.86 −450.99 \$279.87
3.	5702 −4329 1373	8665 −2936 5729	6000 −194 5806	68434 −40977 27,457	600000 −498653 101,347	\$737.17 −190.58 \$546.59
4.	8438 −5678 2760	8139 −1574 6565	5726 −458 5268	50000 −31028 18,972	700105 −382141 317,964	\$203.61 −114.86 \$ 88.75
5.	7585 −2608 4977	7306 −4842 2464	5014 −736 4278	90007 −86399 3608	854030 −474155 379,875	\$518.74 −182.59 \$336.15

3

6.	37 × 53 1961	52 × 193 10,036	143 × 536 76,648	159 × 691 109,869	485 × 620 300,700
7.	45 × 89 4005	76 × 220 16,720	256 × 407 104,192	647 × 836 540,892	200 × 713 142,600
8.	92 × 98 9016	19 × 481 9139	319 × 752 239,888	834 × 794 662,196	390 × 395 154,050
9.	68 × 68 4624	42 × 276 11,592	207 × 951 196,857	407 × 419 170,533	604 × 506 305,624
10.	17 × 62 1054	87 × 180 15,660	284 × 728 206,752	369 × 720 265,680	189 × 231 43,659
11.	29 × 37 1073	95 × 120 11,400	568 × 639 362,952	508 × 824 418,592	464 × 500 232,000
12.	46 × 75 3450	22 × 436 9592	790 × 438 346,020	241 × 506 121,946	357 × 375 133,875
13.	83 × 90 7470	65 × 502 32,630	258 × 563 145,254	783 × 783 613,089	504 × 405 204,120

4

14.	68 × 1359 92,412	936 × 5006 4,685,616	204 × 4285 874,140	1746 × 7035 12,283,110
15.	53 × 9038 479,014	153 × 8276 1,266,228	951 × 7034 6,689,334	6005 × 9758 58,596,790
16.	47 × 9257 435,079	817 × 5840 4,771,280	675 × 4962 3,349,350	5062 × 3809 19,281,158
17.	61 × 9510 580, 110	935 × 1045 977,075	390 × 2937 1,145,430	8963 × 5012 44,922,556
18.	27 × 5836 157,572	902 × 4193 3,782,086	428 × 1973 844,444	5840 × 6148 35,904,320
19.	90 × 3904 351,360	870 × 7317 6,365,790	749 × 8436 6,318,564	3784 × 2004 7,583,136
20.	85 × 9037 768,145	136 × 2731 371,416	804 × 6009 4,831,236	1852 × 7920 14,667,840
21.	26 × 8347 217,022	962 × 6720 6,464,640	342 × 7495 2,563,290	2501 × 9728 24,329,728

More Practice

5

1.	7941 ÷ 3 2647	2705 ÷ 7 386 R3	20,304 ÷ 4 5076	14,816 ÷ 5 2963 R1
2.	4599 ÷ 2 2299 R1	5672 ÷ 8 709	30,444 ÷ 6 5074	25,530 ÷ 4 6382 R2
3.	9245 ÷ 9 1027 R2	1780 ÷ 4 445	56,277 ÷ 9 6253	42,525 ÷ 7 6075
4.	8574 ÷ 6 1429	3270 ÷ 5 654	58,872 ÷ 8 7359	12,327 ÷ 6 2054 R3

6

5.	3228 ÷ 43 $75\frac{3}{43}$	26,188 ÷ 52 $503\frac{8}{13}$	27,480 ÷ 327 $84\frac{4}{109}$	109,277 ÷ 233 469
6.	9647 ÷ 41 $235\frac{12}{41}$	15,532 ÷ 65 $238\frac{62}{65}$	32,725 ÷ 711 $46\frac{19}{711}$	382,844 ÷ 528 $725\frac{1}{12}$
7.	5272 ÷ 81 $65\frac{7}{81}$	49,882 ÷ 54 $923\frac{20}{27}$	31,828 ÷ 436 73	164,115 ÷ 315 521
8.	2088 ÷ 72 29	51,984 ÷ 83 $626\frac{26}{83}$	13,275 ÷ 225 59	344,029 ÷ 441 $780\frac{1}{9}$
9.	8750 ÷ 25 350	29,647 ÷ 41 $723\frac{4}{41}$	16,542 ÷ 459 $36\frac{2}{51}$	202,939 ÷ 209 971
10.	1980 ÷ 22 90	12,005 ÷ 64 $187\frac{37}{64}$	59,706 ÷ 642 93	157,248 ÷ 512 $307\frac{1}{8}$
11.	2738 ÷ 74 37	41,376 ÷ 32 1293	27,090 ÷ 315 86	225,515 ÷ 125 $1804\frac{3}{25}$
12.	4977 ÷ 53 $93\frac{48}{53}$	21,831 ÷ 25 $873\frac{6}{25}$	17,018 ÷ 254 67	248,896 ÷ 256 $972\frac{1}{4}$
13.	1989 ÷ 55 $36\frac{9}{55}$	13,885 ÷ 53 $261\frac{52}{53}$	39,420 ÷ 730 54	324,786 ÷ 627 518
14.	2781 ÷ 44 $63\frac{9}{44}$	29,199 ÷ 74 $394\frac{43}{74}$	30,056 ÷ 442 68	477,456 ÷ 406 1176

7

15.	4417 ÷ 88 50 R17	34,046 ÷ 37 920 R6	34,937 ÷ 483 72 R161	182,325 ÷ 187 975
16.	2976 ÷ 48 62	34,993 ÷ 66 530 R13	13,224 ÷ 196 67 R92	158,404 ÷ 398 398
17.	1547 ÷ 57 27 R8	48,751 ÷ 87 560 R31	33,822 ÷ 663 51 R9	312,858 ÷ 382 819
18.	1001 ÷ 36 27 R29	16,214 ÷ 19 853 R7	16,576 ÷ 592 28	176,886 ÷ 279 634
19.	6079 ÷ 28 217 R3	26,470 ÷ 87 304 R22	14,168 ÷ 184 77	141,812 ÷ 293 484
20.	2148 ÷ 37 58 R2	34,500 ÷ 46 750	71,960 ÷ 196 367 R28	110,976 ÷ 867 128
21.	1955 ÷ 57 34 R17	65,988 ÷ 78 846	60,465 ÷ 285 212 R45	274,360 ÷ 760 361
22.	3230 ÷ 38 85	31,543 ÷ 69 457 R10	82,464 ÷ 576 143 R96	287,082 ÷ 399 719 R201
23.	3940 ÷ 19 207 R7	78,716 ÷ 98 803 R22	23,298 ÷ 274 85 R8	175,720 ÷ 382 460
24.	3229 ÷ 46 70 R9	74,080 ÷ 37 2002 R6	57,390 ÷ 188 305 R50	455,940 ÷ 868 525 R240
25.	7955 ÷ 38 209 R13	64,500 ÷ 27 2388 R24	79,644 ÷ 272 292 R220	257,105 ÷ 685 375 R230

8

1.	23,875 ÷ 28 852 R19	51,758 ÷ 76 681 R2	10,925 ÷ 125 87 R50	241,900 ÷ 375 645 R25
2.	35,400 ÷ 64 553 R8	34,091 ÷ 71 480 R11	12,575 ÷ 255 49 R80	171,573 ÷ 159 1079 R12
3.	15,138 ÷ 87 174	35,211 ÷ 33 1067	35,535 ÷ 515 69	186,204 ÷ 789 236
4.	13,144 ÷ 53 248	25,077 ÷ 39 643	31,312 ÷ 192 163 R16	216,098 ÷ 647 334
5.	12,288 ÷ 32 384	12,084 ÷ 76 159	39,732 ÷ 129 308	136,968 ÷ 439 312
6.	28,750 ÷ 46 625	28,455 ÷ 35 813	24,645 ÷ 257 95 R230	102,297 ÷ 129 793

9

Change to whole numbers or mixed numbers:

7. $\frac{11}{2}$ $5\frac{1}{2}$ $\frac{16}{8}$ 2 $\frac{17}{3}$ $5\frac{2}{3}$ $\frac{25}{25}$ 1 $\frac{18}{8}$ $2\frac{1}{4}$ $\frac{26}{4}$ $6\frac{1}{2}$ $\frac{15}{3}$ 5 $\frac{20}{6}$ $3\frac{1}{3}$ $\frac{35}{10}$ $3\frac{1}{2}$ $\frac{42}{16}$ $2\frac{5}{8}$

8. $\frac{14}{3}$ $4\frac{2}{3}$ $\frac{12}{4}$ 3 $\frac{21}{5}$ $4\frac{1}{5}$ $\frac{18}{18}$ 1 $\frac{32}{8}$ 4 $\frac{22}{4}$ $5\frac{1}{2}$ $\frac{14}{2}$ 7 $\frac{42}{8}$ $5\frac{1}{4}$ $\frac{37}{12}$ $3\frac{1}{12}$ $\frac{40}{18}$ $2\frac{2}{9}$

Change to improper fractions:

9. $3\frac{1}{3}$ $\frac{10}{3}$ $6\frac{1}{2}$ $\frac{13}{2}$ $5\frac{3}{8}$ $\frac{43}{8}$ $2\frac{1}{4}$ $\frac{9}{4}$ $4\frac{2}{5}$ $\frac{22}{5}$ $3\frac{5}{6}$ $\frac{23}{6}$ $7\frac{3}{4}$ $\frac{31}{4}$ $6\frac{2}{3}$ $\frac{20}{3}$ $4\frac{1}{12}$ $\frac{49}{12}$

10. $6\frac{1}{5}$ $\frac{31}{5}$ $8\frac{1}{6}$ $\frac{49}{6}$ $2\frac{3}{4}$ $\frac{11}{4}$ $7\frac{1}{8}$ $\frac{57}{8}$ $6\frac{5}{6}$ $\frac{41}{6}$ $5\frac{3}{5}$ $\frac{28}{5}$ $3\frac{7}{8}$ $\frac{31}{8}$ $1\frac{4}{5}$ $\frac{9}{5}$ $2\frac{5}{16}$ $\frac{37}{16}$

10

11.	$8\frac{1}{6}+1\frac{1}{2}+3\frac{2}{3}=13\frac{1}{3}$	$3\frac{5}{6}+1\frac{5}{6}+2\frac{5}{6}=8\frac{1}{2}$	$3\frac{1}{2}+2\frac{5}{6}+7\frac{1}{3}=13\frac{2}{3}$	$1\frac{1}{4}+1\frac{1}{3}+8\frac{1}{4}=10\frac{5}{6}$	$4\frac{1}{2}+2\frac{11}{12}+6\frac{2}{3}=14\frac{1}{12}$	$9\frac{5}{8}+1\frac{1}{4}+3\frac{7}{16}=14\frac{5}{16}$	$4\frac{1}{4}+7\frac{1}{3}+1\frac{7}{12}=13\frac{1}{6}$	$5\frac{4}{5}+7\frac{1}{2}+9\frac{7}{10}=23$
12.	$5\frac{7}{8}+2\frac{1}{2}+1\frac{3}{4}=10\frac{1}{8}$	$6\frac{1}{8}+3\frac{7}{8}+1\frac{5}{8}=11\frac{5}{8}$	$3\frac{1}{6}+1\frac{1}{6}+4\frac{1}{6}=8\frac{1}{2}$	$7\frac{1}{2}+1\frac{2}{3}+1\frac{1}{6}=10\frac{1}{3}$	$2\frac{3}{10}+2\frac{2}{5}+6\frac{1}{2}=11\frac{1}{5}$	$5\frac{1}{2}+9\frac{5}{16}+1\frac{3}{4}=16\frac{9}{16}$	$1\frac{3}{4}+7\frac{7}{12}+5\frac{3}{4}=15\frac{1}{12}$	$2\frac{2}{5}+2\frac{7}{10}+2\frac{1}{2}=7\frac{3}{5}$
13.	$6\frac{1}{4}-4\frac{1}{4}=2$	$7\frac{1}{8}-5\frac{5}{8}=1\frac{1}{2}$	$6\frac{1}{2}-3\frac{1}{3}=3\frac{1}{6}$	$2\frac{1}{8}-1\frac{1}{4}=\frac{7}{8}$	$9\frac{5}{12}-4\frac{1}{2}=4\frac{11}{12}$	$4\frac{1}{4}-3\frac{7}{16}=\frac{13}{16}$	$4\frac{7}{12}-1\frac{3}{4}=2\frac{5}{6}$	$9\frac{3}{10}-2\frac{9}{10}=6\frac{2}{5}$
14.	$9\frac{1}{6}-3\frac{1}{2}=5\frac{2}{3}$	$5\frac{1}{4}-2\frac{2}{3}=2\frac{7}{12}$	$7\frac{5}{8}-5\frac{3}{4}=1\frac{7}{8}$	$7\frac{1}{2}-4\frac{4}{5}=2\frac{7}{10}$	$5\frac{3}{8}-2\frac{11}{16}=2\frac{11}{16}$	$8\frac{1}{10}-2\frac{3}{5}=5\frac{1}{2}$	$6\frac{5}{12}-2\frac{11}{12}=3\frac{1}{2}$	$7\frac{5}{12}-2\frac{5}{6}=4\frac{7}{12}$
15.	$7\frac{3}{8}-5\frac{3}{8}=2$	$8-4\frac{2}{3}=3\frac{1}{3}$	$9\frac{2}{5}-4\frac{4}{5}=4\frac{3}{5}$	$7\frac{3}{8}-2\frac{1}{4}=5\frac{1}{8}$	$8\frac{1}{2}-3\frac{15}{16}=4\frac{9}{16}$	$7\frac{1}{12}-4\frac{5}{6}=2\frac{1}{4}$	$9\frac{3}{16}-8\frac{15}{16}=\frac{1}{4}$	$6\frac{1}{10}-3\frac{3}{5}=2\frac{1}{2}$

More Practice

11

1.	$2\frac{3}{4}+1\frac{1}{6}=3\frac{11}{12}$	$8\frac{1}{8}+7\frac{1}{6}=15\frac{7}{24}$	$5\frac{1}{4}+2\frac{5}{6}=8\frac{1}{12}$	$6\frac{1}{8}+6\frac{1}{6}=12\frac{7}{24}$	$1\frac{5}{6}+4\frac{1}{4}=6\frac{1}{12}$	$7\frac{3}{8}+3\frac{5}{6}=11\frac{5}{24}$	$7\frac{7}{12}+3\frac{1}{8}=10\frac{17}{24}$	$8\frac{3}{4}+4\frac{3}{10}=13\frac{1}{20}$
2.	$7\frac{5}{6}+2\frac{1}{8}=9\frac{23}{24}$	$1\frac{1}{6}+4\frac{5}{8}=5\frac{19}{24}$	$3\frac{1}{4}+3\frac{1}{6}=6\frac{5}{12}$	$2\frac{3}{4}+6\frac{5}{6}=9\frac{7}{12}$	$3\frac{3}{8}+5\frac{1}{6}=8\frac{13}{24}$	$9\frac{3}{4}+4\frac{5}{6}=14\frac{7}{12}$	$2\frac{1}{4}+8\frac{7}{10}=10\frac{19}{20}$	$6\frac{3}{8}+5\frac{1}{12}=11\frac{11}{24}$
3.	$1\frac{1}{2}+1\frac{2}{3}+1\frac{3}{8}=4\frac{13}{24}$	$9\frac{5}{8}+1\frac{1}{3}+4\frac{1}{6}=15\frac{1}{8}$	$4\frac{2}{3}+5\frac{3}{8}+2\frac{1}{6}=12\frac{5}{24}$	$8\frac{1}{6}+2\frac{1}{8}+3\frac{1}{4}=13\frac{13}{24}$	$1\frac{3}{8}+5\frac{5}{6}+4\frac{1}{4}=11\frac{11}{24}$	$2\frac{1}{6}+2\frac{1}{3}+2\frac{1}{4}=6\frac{3}{4}$	$2\frac{1}{6}+4\frac{7}{8}+9\frac{1}{12}=16\frac{1}{8}$	$3\frac{9}{10}+4\frac{4}{5}+1\frac{1}{4}=9\frac{19}{20}$
4.	$3\frac{2}{3}+6\frac{5}{6}+8\frac{3}{4}=19\frac{1}{4}$	$6\frac{2}{3}+1\frac{1}{4}+2\frac{1}{8}=10\frac{1}{24}$	$7\frac{2}{3}+5\frac{1}{4}+2\frac{5}{6}=15\frac{3}{4}$	$1\frac{1}{3}+3\frac{3}{4}+1\frac{5}{6}=6\frac{11}{12}$	$2\frac{1}{6}+1\frac{1}{3}+1\frac{1}{8}=4\frac{5}{8}$	$1\frac{1}{2}+6\frac{2}{5}+8\frac{1}{6}=16\frac{1}{15}$	$6\frac{3}{10}+8\frac{2}{5}+2\frac{1}{4}=16\frac{19}{20}$	$2\frac{1}{5}+6\frac{3}{4}+2\frac{7}{10}=11\frac{13}{20}$
5.	$9\frac{5}{6}+1\frac{1}{2}+3\frac{2}{3}+6\frac{1}{4}=21\frac{1}{4}$	$2\frac{3}{8}+9\frac{1}{2}+8\frac{1}{6}+6\frac{2}{3}=26\frac{17}{24}$	$2\frac{3}{4}+6\frac{1}{6}+8\frac{3}{4}+4\frac{1}{2}=22\frac{1}{6}$	$2\frac{1}{2}+3\frac{2}{3}+2\frac{1}{4}+2\frac{5}{8}=11\frac{1}{24}$	$3\frac{1}{3}+6\frac{3}{4}+1\frac{3}{8}+4\frac{1}{6}=15\frac{5}{8}$	$3\frac{5}{6}+3\frac{2}{3}+5\frac{3}{4}+2\frac{1}{2}=15\frac{3}{4}$	$2\frac{3}{10}+4\frac{1}{4}+6\frac{1}{2}+5\frac{3}{5}=18\frac{13}{20}$	$6\frac{1}{2}+6\frac{2}{3}+3\frac{5}{12}+7\frac{3}{4}=24\frac{1}{3}$
6.	$6\frac{3}{4}-5\frac{5}{6}=\frac{11}{12}$	$8\frac{5}{6}-4\frac{3}{4}=4\frac{1}{12}$	$7\frac{3}{8}-6\frac{1}{6}=1\frac{5}{24}$	$4\frac{3}{4}-2\frac{1}{6}=2\frac{7}{12}$	$5\frac{1}{4}-1\frac{5}{6}=3\frac{5}{12}$	$9\frac{1}{8}-6\frac{5}{6}=2\frac{7}{24}$	$8\frac{3}{10}-4\frac{5}{6}=3\frac{7}{15}$	$9\frac{1}{4}-5\frac{7}{10}=3\frac{11}{20}$
7.	$9\frac{1}{8}-8\frac{1}{6}=\frac{23}{24}$	$5\frac{7}{8}-2\frac{5}{6}=3\frac{1}{24}$	$6\frac{1}{4}-2\frac{5}{6}=3\frac{5}{12}$	$9\frac{5}{8}-1\frac{1}{6}=8\frac{11}{24}$	$3\frac{3}{8}-2\frac{5}{6}=\frac{13}{24}$	$7\frac{3}{8}-5\frac{1}{6}=2\frac{5}{24}$	$4\frac{1}{8}-3\frac{3}{10}=\frac{33}{40}$	$5\frac{3}{8}-1\frac{7}{12}=3\frac{19}{24}$
8.	$7\frac{1}{6}-1\frac{3}{4}=5\frac{5}{12}$	$2\frac{3}{8}-1\frac{5}{6}=\frac{13}{24}$	$8\frac{5}{8}-4\frac{1}{6}=4\frac{11}{24}$	$9\frac{3}{4}-3\frac{1}{6}=6\frac{7}{12}$	$3\frac{5}{6}-1\frac{1}{8}=2\frac{17}{24}$	$6\frac{7}{8}-4\frac{1}{6}=2\frac{17}{24}$	$7\frac{1}{4}-5\frac{3}{10}=1\frac{19}{20}$	$8\frac{3}{10}-3\frac{3}{4}=4\frac{11}{20}$
9.	$7\frac{3}{4}-2\frac{1}{6}=5\frac{7}{12}$	$8\frac{5}{6}-4\frac{1}{4}=4\frac{7}{12}$	$4\frac{1}{8}-1\frac{5}{6}=2\frac{7}{24}$	$9\frac{5}{6}-3\frac{3}{8}=6\frac{11}{24}$	$3\frac{1}{6}-1\frac{3}{4}=1\frac{5}{12}$	$7\frac{1}{8}-4\frac{1}{6}=2\frac{23}{24}$	$6\frac{1}{12}-1\frac{1}{8}=4\frac{23}{24}$	$5\frac{3}{4}-4\frac{3}{10}=1\frac{9}{20}$
10.	$5\frac{1}{8}-3\frac{5}{6}=1\frac{7}{24}$	$9\frac{1}{6}-5\frac{1}{4}=3\frac{11}{12}$	$7\frac{3}{4}-3\frac{5}{6}=3\frac{11}{12}$	$4\frac{7}{8}-2\frac{5}{6}=2\frac{1}{24}$	$2\frac{1}{4}-1\frac{5}{6}=\frac{5}{12}$	$5\frac{1}{6}-4\frac{1}{4}=\frac{11}{12}$	$8\frac{7}{10}-5\frac{1}{4}=3\frac{9}{20}$	$6\frac{11}{12}-3\frac{5}{8}=3\frac{7}{24}$

12

1.	$\frac{3}{8} \times 5$ $1\frac{7}{8}$	$\frac{1}{2} \times 2\frac{3}{5}$ $1\frac{3}{10}$	$13 \times 5\frac{1}{4}$ $68\frac{1}{4}$	$1\frac{1}{3} \times 5\frac{1}{4}$ 7	$3\frac{2}{3} \times 6$ 22
2.	$\frac{4}{5} \times \frac{7}{8}$ $\frac{7}{10}$	$\frac{2}{3} \times 5\frac{1}{4}$ $3\frac{1}{2}$	$12 \times 2\frac{3}{8}$ $28\frac{1}{2}$	$7\frac{1}{2} \times 1\frac{1}{2}$ $11\frac{1}{4}$	$6\frac{1}{4} \times 8$ 50
3.	$\frac{2}{3} \times \frac{4}{5}$ $\frac{8}{15}$	$\frac{5}{6} \times 1\frac{1}{5}$ 1	$14 \times 1\frac{1}{2}$ 21	$6\frac{2}{3} \times 2\frac{1}{4}$ 15	$8\frac{2}{5} \times 6$ $50\frac{2}{5}$
4.	$\frac{3}{4} \times \frac{2}{5}$ $\frac{3}{10}$	$\frac{4}{5} \times 3\frac{1}{3}$ $2\frac{2}{3}$	$10 \times 3\frac{7}{8}$ $38\frac{3}{4}$	$1\frac{2}{5} \times 3\frac{1}{7}$ $4\frac{2}{5}$	$4\frac{1}{2} \times 9$ $40\frac{1}{2}$
5.	$\frac{5}{6} \times 8$ $6\frac{2}{3}$	$\frac{7}{8} \times 5\frac{3}{5}$ $4\frac{9}{10}$	$30 \times 5\frac{1}{4}$ $157\frac{1}{2}$	$3\frac{3}{4} \times 1\frac{1}{2}$ $5\frac{5}{8}$	$2\frac{1}{6} \times 4$ $8\frac{2}{3}$
6.	$\frac{5}{6} \times \frac{9}{16}$ $\frac{15}{32}$	$\frac{5}{6} \times 2\frac{1}{2}$ $2\frac{1}{12}$	$33 \times 7\frac{2}{3}$ 253	$1\frac{1}{5} \times 2\frac{1}{4}$ $2\frac{7}{10}$	$2\frac{5}{8} \times 6$ $15\frac{3}{4}$
7.	$\frac{4}{5} \times \frac{5}{12}$ $\frac{1}{3}$	$\frac{3}{5} \times 1\frac{1}{8}$ $\frac{27}{40}$	$36 \times 4\frac{1}{4}$ 153	$5\frac{3}{5} \times 1\frac{3}{4}$ $9\frac{4}{5}$	$4\frac{1}{4} \times 8$ 34
8.	$\frac{1}{4} \times \frac{9}{10}$ $\frac{9}{40}$	$\frac{1}{2} \times 2\frac{2}{3}$ $1\frac{1}{3}$	$72 \times 5\frac{3}{8}$ 387	$3\frac{3}{4} \times 2\frac{4}{5}$ $10\frac{1}{2}$	$6\frac{1}{2} \times 6$ 39
9.	$\frac{5}{6} \times \frac{9}{10}$ $\frac{3}{4}$	$\frac{3}{4} \times 1\frac{1}{3}$ 1	$54 \times 1\frac{5}{6}$ 99	$1\frac{1}{5} \times 2\frac{1}{8}$ $2\frac{11}{20}$	$8\frac{3}{4} \times \frac{2}{5}$ $3\frac{1}{2}$
10.	$\frac{1}{2} \times 15$ $7\frac{1}{2}$	$\frac{7}{8} \times 3\frac{1}{7}$ $2\frac{3}{4}$	$18 \times 1\frac{2}{3}$ 30	$2\frac{1}{6} \times 1\frac{4}{5}$ $3\frac{9}{10}$	$7\frac{1}{5} \times \frac{5}{6}$ 6
11.	$\frac{2}{3} \times 16$ $10\frac{2}{3}$	$\frac{1}{4} \times 2\frac{4}{5}$ $\frac{7}{10}$	$16 \times 9\frac{3}{4}$ 156	$2\frac{3}{4} \times 2\frac{2}{3}$ $7\frac{1}{3}$	$6\frac{1}{4} \times \frac{2}{5}$ $2\frac{1}{2}$
12.	$\frac{3}{4} \times 20$ 15	$\frac{3}{5} \times 7\frac{1}{3}$ $4\frac{2}{5}$	$20 \times 4\frac{1}{6}$ $83\frac{1}{3}$	$2\frac{2}{5} \times 1\frac{3}{8}$ $3\frac{3}{10}$	$6\frac{2}{3} \times \frac{1}{5}$ $1\frac{1}{3}$
13.	$\frac{2}{3} \times \frac{15}{16}$ $\frac{5}{8}$	$\frac{1}{3} \times 4\frac{4}{5}$ $1\frac{3}{5}$	$14 \times 2\frac{3}{4}$ $38\frac{1}{2}$	$3\frac{1}{3} \times 4\frac{1}{2}$ 15	$1\frac{1}{4} \times \frac{3}{5}$ $\frac{3}{4}$
14.	$8 \times \frac{3}{16}$ $1\frac{1}{2}$	$\frac{5}{8} \times 2\frac{1}{5}$ $1\frac{3}{8}$	$15 \times 3\frac{1}{5}$ 48	$1\frac{7}{8} \times 1\frac{1}{5}$ $2\frac{1}{4}$	$1\frac{1}{2} \times \frac{2}{3}$ 1
15.	$9 \times \frac{7}{12}$ $5\frac{1}{4}$	$\frac{5}{6} \times 1\frac{1}{8}$ $\frac{15}{16}$	$20 \times 1\frac{5}{8}$ $32\frac{1}{2}$	$1\frac{1}{6} \times 1\frac{1}{2}$ $1\frac{3}{4}$	$2\frac{2}{5} \times \frac{5}{6}$ 2
16.	$5 \times \frac{3}{10}$ $1\frac{1}{2}$	$\frac{1}{6} \times 8\frac{2}{5}$ $1\frac{2}{5}$	$36 \times 2\frac{7}{8}$ $103\frac{1}{2}$	$4\frac{1}{2} \times 2\frac{2}{3}$ 12	$2\frac{1}{4} \times \frac{3}{4}$ $1\frac{11}{16}$
17.	$8 \times \frac{5}{12}$ $3\frac{1}{3}$	$\frac{2}{3} \times 4\frac{1}{2}$ 3	$27 \times 8\frac{2}{3}$ 234	$1\frac{1}{8} \times 3\frac{1}{3}$ $3\frac{3}{4}$	$1\frac{1}{4} \times \frac{9}{10}$ $1\frac{1}{8}$
18.	$4 \times \frac{9}{10}$ $3\frac{3}{5}$	$\frac{3}{8} \times 5\frac{1}{3}$ 2	$15 \times 6\frac{2}{3}$ 100	$3\frac{1}{7} \times 5\frac{1}{4}$ $16\frac{1}{2}$	$7\frac{1}{2} \times \frac{1}{12}$ $\frac{5}{8}$
19.	$\frac{4}{5} \times \frac{5}{16}$ $\frac{1}{4}$	$\frac{3}{5} \times 3\frac{1}{3}$ 2	$21 \times 3\frac{1}{3}$ 70	$3\frac{1}{2} \times 2\frac{1}{2}$ $8\frac{3}{4}$	$1\frac{7}{8} \times \frac{3}{10}$ $\frac{9}{16}$
20.	$\frac{3}{16} \times 18$ $3\frac{3}{8}$	$\frac{3}{10} \times 1\frac{2}{3}$ $\frac{1}{2}$	$48 \times 2\frac{3}{16}$ 105	$3\frac{3}{4} \times 2\frac{2}{3}$ 10	$3\frac{1}{3} \times \frac{3}{10}$ 1
21.	$\frac{7}{10} \times 16$ $11\frac{1}{5}$	$\frac{5}{16} \times 1\frac{1}{2}$ $\frac{15}{32}$	$15 \times 2\frac{1}{10}$ $31\frac{1}{2}$	$2\frac{1}{4} \times 3\frac{1}{3}$ $7\frac{1}{2}$	$2\frac{2}{3} \times \frac{3}{16}$ $\frac{1}{2}$
22.	$\frac{9}{16} \times 24$ $13\frac{1}{2}$	$\frac{7}{10} \times 1\frac{1}{4}$ $\frac{7}{8}$	$10 \times 2\frac{1}{12}$ $20\frac{5}{6}$	$1\frac{1}{2} \times 2\frac{1}{2}$ $3\frac{3}{4}$	$2\frac{1}{2} \times \frac{3}{10}$ $\frac{3}{4}$
23.	$\frac{7}{12} \times 18$ $10\frac{1}{2}$	$\frac{3}{10} \times 1\frac{3}{5}$ $\frac{12}{25}$	$32 \times 1\frac{3}{16}$ 38	$7\frac{1}{2} \times 1\frac{1}{3}$ 10	$1\frac{1}{5} \times \frac{5}{12}$ $\frac{1}{2}$
24.	$\frac{15}{16} \times 20$ $18\frac{3}{4}$	$\frac{5}{12} \times 3\frac{1}{5}$ $1\frac{1}{3}$	$25 \times 3\frac{3}{10}$ $82\frac{1}{2}$	$1\frac{3}{5} \times 1\frac{1}{2}$ $2\frac{2}{5}$	$1\frac{1}{3} \times \frac{9}{16}$ $\frac{3}{4}$
25.	$\frac{11}{12} \times 24$ 22	$\frac{3}{10} \times 4\frac{1}{6}$ $1\frac{1}{4}$	$16 \times 2\frac{1}{16}$ 33	$3\frac{1}{3} \times 1\frac{1}{4}$ $4\frac{1}{6}$	$2\frac{2}{3} \times \frac{9}{10}$ $2\frac{2}{5}$

More Practice

13

1.	$8 \div \frac{4}{5}$ 10	$4\frac{1}{5} \div \frac{3}{5}$ 7	$18 \div \frac{2}{3}$ 27	$\frac{1}{5} \div \frac{7}{10}$ $\frac{2}{7}$	$\frac{5}{8} \div \frac{1}{4}$ $2\frac{1}{2}$
2.	$6 \div \frac{1}{3}$ 18	$1\frac{1}{3} \div \frac{3}{4}$ $1\frac{7}{9}$	$25 \div \frac{5}{8}$ 40	$\frac{3}{8} \div \frac{9}{16}$ $\frac{2}{3}$	$\frac{3}{8} \div \frac{3}{4}$ $\frac{1}{2}$
3.	$7 \div \frac{2}{5}$ $17\frac{1}{2}$	$1\frac{1}{2} \div \frac{3}{8}$ 4	$21 \div \frac{3}{4}$ 28	$\frac{3}{4} \div \frac{15}{16}$ $\frac{4}{5}$	$\frac{3}{4} \div \frac{1}{6}$ $4\frac{1}{2}$
4.	$\frac{3}{8} \div \frac{3}{5}$ $\frac{5}{8}$	$7\frac{1}{2} \div \frac{4}{5}$ $9\frac{3}{8}$	$35 \div \frac{5}{6}$ 42	$\frac{3}{4} \div \frac{3}{16}$ 4	$\frac{5}{8} \div \frac{3}{4}$ $\frac{5}{6}$
5.	$\frac{5}{6} \div \frac{2}{5}$ $2\frac{1}{12}$	$1\frac{1}{8} \div \frac{3}{4}$ $1\frac{1}{2}$	$17 \div \frac{1}{3}$ 51	$\frac{1}{8} \div \frac{7}{16}$ $\frac{2}{7}$	$\frac{9}{16} \div \frac{1}{4}$ $2\frac{1}{4}$
6.	$2 \div \frac{1}{2}$ 4	$2\frac{1}{3} \div \frac{1}{6}$ 14	$28 \div \frac{7}{8}$ 32	$\frac{3}{8} \div \frac{1}{12}$ $4\frac{1}{2}$	$\frac{1}{12} \div \frac{1}{3}$ $\frac{1}{4}$
7.	$\frac{3}{5} \div \frac{3}{4}$ $\frac{4}{5}$	$5\frac{1}{3} \div \frac{2}{3}$ 8	$11 \div \frac{1}{2}$ 22	$\frac{11}{16} \div \frac{11}{12}$ $\frac{3}{4}$	$\frac{5}{16} \div \frac{3}{8}$ $\frac{5}{6}$
8.	$\frac{5}{8} \div \frac{3}{4}$ $\frac{5}{6}$	$3\frac{3}{4} \div \frac{5}{8}$ 6	$32 \div \frac{4}{5}$ 40	$\frac{9}{16} \div \frac{3}{10}$ $1\frac{7}{8}$	$\frac{1}{12} \div \frac{2}{3}$ $\frac{1}{8}$
9.	$6 \div \frac{3}{5}$ 10	$5\frac{1}{2} \div \frac{1}{4}$ 22	$15 \div \frac{1}{4}$ 60	$\frac{7}{16} \div \frac{5}{12}$ $1\frac{1}{20}$	$2\frac{1}{3} \div \frac{1}{8}$ $18\frac{2}{3}$
10.	$\frac{5}{12} \div \frac{1}{8}$ $3\frac{1}{3}$	$3\frac{3}{8} \div \frac{3}{8}$ 9	$24 \div \frac{9}{10}$ $26\frac{2}{3}$	$\frac{7}{10} \div \frac{7}{12}$ $1\frac{1}{5}$	$6\frac{1}{4} \div \frac{3}{8}$ $16\frac{2}{3}$
11.	$\frac{3}{10} \div \frac{4}{5}$ $\frac{3}{8}$	$4\frac{1}{5} \div \frac{9}{10}$ $4\frac{2}{3}$	$10 \div \frac{5}{16}$ 32	$\frac{1}{12} \div \frac{9}{10}$ $\frac{5}{54}$	$4\frac{2}{3} \div \frac{7}{8}$ $5\frac{1}{3}$
12.	$\frac{5}{16} \div \frac{1}{8}$ $2\frac{1}{2}$	$4\frac{4}{5} \div \frac{3}{10}$ 16	$14 \div \frac{7}{12}$ 24	$\frac{15}{16} \div \frac{9}{16}$ $1\frac{2}{3}$	$3\frac{1}{8} \div \frac{5}{6}$ $3\frac{3}{4}$

14

13.	$\frac{3}{4} \div 12$ $\frac{1}{16}$	$3\frac{1}{3} \div 15$ $\frac{2}{9}$	$6\frac{3}{5} \div 2\frac{2}{5}$ $2\frac{3}{4}$	$16 \div 2\frac{2}{3}$ 6	$8\frac{1}{3} \div 6\frac{1}{4}$ $1\frac{1}{3}$
14.	$\frac{5}{8} \div 10$ $\frac{1}{16}$	$4\frac{1}{2} \div 18$ $\frac{1}{4}$	$5\frac{1}{4} \div 3\frac{1}{2}$ $1\frac{1}{2}$	$14 \div 4\frac{1}{5}$ $3\frac{1}{3}$	$1\frac{4}{5} \div 2\frac{2}{5}$ $\frac{3}{4}$
15.	$\frac{1}{2} \div 2\frac{2}{3}$ $\frac{3}{16}$	$2\frac{2}{5} \div 30$ $\frac{2}{25}$	$6\frac{2}{3} \div 1\frac{7}{8}$ $3\frac{5}{9}$	$10 \div 1\frac{4}{5}$ $5\frac{5}{9}$	$1\frac{1}{2} \div 3\frac{1}{3}$ $\frac{9}{20}$
16.	$\frac{3}{4} \div 4\frac{1}{2}$ $\frac{1}{6}$	$3\frac{3}{4} \div 15$ $\frac{1}{4}$	$6\frac{1}{4} \div 3\frac{1}{3}$ $1\frac{7}{8}$	$12 \div 1\frac{3}{5}$ $7\frac{1}{2}$	$3\frac{2}{3} \div 4\frac{1}{8}$ $\frac{8}{9}$
17.	$\frac{5}{8} \div 1\frac{7}{8}$ $\frac{1}{3}$	$2\frac{5}{8} \div 14$ $\frac{3}{16}$	$3\frac{1}{3} \div 1\frac{1}{5}$ $2\frac{7}{9}$	$25 \div 2\frac{1}{2}$ 10	$2\frac{1}{4} \div 1\frac{1}{2}$ $1\frac{1}{2}$
18.	$\frac{5}{6} \div 10$ $\frac{1}{12}$	$5\frac{1}{4} \div 35$ $\frac{3}{20}$	$4\frac{1}{5} \div 3\frac{3}{4}$ $1\frac{3}{25}$	$10 \div 2\frac{1}{2}$ 4	$5\frac{5}{8} \div 3\frac{3}{4}$ $1\frac{1}{2}$
19.	$\frac{3}{4} \div 2\frac{1}{4}$ $\frac{1}{3}$	$6\frac{3}{5} \div 22$ $\frac{3}{10}$	$6\frac{2}{3} \div 2\frac{2}{5}$ $2\frac{7}{9}$	$12 \div 3\frac{1}{5}$ $3\frac{3}{4}$	$2\frac{1}{6} \div 1\frac{1}{3}$ $1\frac{5}{8}$
20.	$\frac{1}{6} \div 4\frac{1}{2}$ $\frac{1}{27}$	$8\frac{1}{3} \div 10$ $\frac{5}{6}$	$2\frac{4}{5} \div 1\frac{3}{4}$ $1\frac{3}{5}$	$22 \div 6\frac{3}{5}$ $3\frac{1}{3}$	$2\frac{4}{5} \div 1\frac{2}{5}$ 2
21.	$6 \div 3\frac{3}{8}$ $1\frac{7}{9}$	$2\frac{3}{4} \div 22$ $\frac{1}{8}$	$3\frac{1}{5} \div 2\frac{2}{3}$ $1\frac{1}{5}$	$25 \div 3\frac{1}{3}$ $7\frac{1}{2}$	$3\frac{1}{5} \div 1\frac{1}{5}$ $2\frac{2}{3}$
22.	$7 \div 1\frac{3}{4}$ 4	$9\frac{1}{3} \div 14$ $\frac{2}{3}$	$2\frac{1}{4} \div 1\frac{7}{8}$ $1\frac{1}{5}$	$21 \div 2\frac{1}{4}$ $9\frac{1}{3}$	$3\frac{3}{4} \div 4\frac{1}{2}$ $\frac{5}{6}$
23.	$6 \div 4\frac{1}{2}$ $1\frac{1}{3}$	$8\frac{1}{4} \div 11$ $\frac{3}{4}$	$4\frac{4}{5} \div 1\frac{2}{3}$ $2\frac{22}{25}$	$24 \div 5\frac{1}{3}$ $4\frac{1}{2}$	$5\frac{1}{3} \div 6\frac{2}{3}$ $\frac{4}{5}$
24.	$7 \div 5\frac{3}{5}$ $1\frac{1}{4}$	$4\frac{1}{2} \div 27$ $\frac{1}{6}$	$4\frac{3}{8} \div 2\frac{1}{2}$ $1\frac{3}{4}$	$30 \div 6\frac{1}{4}$ $4\frac{4}{5}$	$2\frac{1}{10} \div 2\frac{1}{4}$ $\frac{14}{15}$

15

1.	.42	73.9	.137	31.05	4.118	42.252
	.07	46.0	.284	60.54	9.302	5.681
	.35	30.8	.306	24.63	8.075	12.596
	.89	2.4	.145	31.27	2.164	64.401
	.50	54.3	.270	92.86	7.935	32.040
	+.23	+ .8	+.937	+89.10	+ .640	+98.647
	2.46	208.2	2.079	329.45	32.234	255.617
2.	6.5	8.21	.875	978.2	38.56	23.892
	−1.9	−5.04	−.125	−650.1	−19.74	−21.755
	4.6	3.17	.750	328.1	18.82	2.137

3. 2.5 + 7.4 + 6.5 + 3.1 + 9.8 + 7.2 + 4.6 41.1 — 7.25 − 4.03 3.22

4. .175 + 3.648 + 8.705 + .731 + 9.426 + 4.863 27.548 — 2.34 − 1.406 .934

5. .42 + .76 + .91 + .46 + .87 + .59 + .22 + .68 4.91 — .963 − .724 .239

6. 1.8 + 4.4 + 26.1 + 30.5 + 2.9 + 76.3 + 21.4 163.4 — 5.72 − 2.16 3.56

7. 2.28 + 1.32 + 7.24 + 8.05 + 2.19 + 3.82 + 1.47 26.37 — 6.29 − 3.4 2.89

8. 14.2 + 3.28 + 72.8 + 6 + 38.71 + 4.689 + 9.15 148.829 — 9.85 − 7.61 2.24

9. 2.375 + .625 + 1.5 + 2.75 + .875 + 2.25 + 7 17.375 — .675 − .431 .244

10. 6.8 + 2.875 + 3.3125 + .0625 + .375 + 7.5 20.9250 — 4.37 − 2.9 1.47

11. 5.75 + 8.35 + 11.40 + 6.38 + 7.26 + 9.47 + 1.12 49.73 — 83.1 − 27.4 55.7

16

12.	9 × 35.4 318.6	65 × 2.6 169.0	4.7 × 7.5 35.25	.63 × 6.9 4.347	.04 × .704 .02816
13.	6 × 3.81 22.86	28 × 8.7 243.6	2.3 × .41 .943	.19 × .89 .1691	7.5 × 19.6 147.00
14.	2 × .905 1.810	39 × .92 35.88	5.7 × .48 2.736	.09 × .58 .0522	8.2 × .924 7.5768
15.	7 × 83.6 585.2	74 × 6.9 510.6	.78 × .15 .1170	.07 × .09 .0063	.41 × 85.4 35.014
16.	5 × .025 .125	91 × .46 41.86	2.4 × 2.6 6.24	.08 × .73 .0584	6.1 × 28.6 174.46
17.	.5 × .007 .0035	.42 × 3.9 1.638	1.3 × .75 .975	.04 × .08 .0032	1.3 × .053 .0689
18.	.2 × .015 .0030	.12 × .16 .0192	8.9 × .37 3.293	.07 × 5.2 .364	.74 × .063 .04662
19.	.6 × 2.16 1.296	.95 × .06 .0570	.78 × .29 .2262	.09 × .79 .0711	.45 × 2.36 1.0620
20.	.9 × .058 .0522	3.5 × 8.3 29.05	2.5 × 3.7 9.25	.06 × .08 .0048	8.4 × 4.17 35.028
21.	.3 × .003 .0009	6.2 × 9.5 58.90	6.7 × .74 4.958	.08 × .35 .0280	.62 × .096 .05952
22.	.8 × .007 .0056	4.7 × 8.2 38.54	1.9 × .65 1.235	.05 × .75 .0375	.98 × .909 .89082

More Practice

17

Divide until there is no remainder. Annex zeros if necessary:

1.	8)26.16 3.27	4)3.928 .982	42)52.08 1.24	14)9.506 .679	67)159.46 2.38
2.	7)205.8 29.4	6)42.54 7.09	25)917.5 36.7	34)598.4 17.6	15)14.955 .997
3.	9)7.605 .845	8)506.4 63.3	28)8.904 .318	29)916.4 31.6	33)179.19 5.43
4.	4)391.00 97.75	16)375.0000 23.4375	35)23.590 .674	64)3596.0000 56.1875	75)5592.00 74.56
5.	8)37.20 4.65	25)344.00 13.76	45)197.10 4.38	36)3213.00 89.25	24)2001.000 83.375
6.	5)46.30 9.26	48)39.600 .825	18)1017.0 56.5	72)5148.0 71.5	56)219.800 3.925
7.	6)351.0 58.5	32)590.0000 18.4375	96)4542.0000 47.3125	28)279.300 9.975	38)131.10 3.45
8.	5)32.60 6.52	16)276.00 17.25	36)2970.0 82.5	32)23.36 .73	96)9360.0 97.5
9.	8)1.4200 .1775	25)216.00 8.64	24)193.20 8.05	42)390.6 9.3	75)61.0200 .8136
10.	4)26.4 6.6	35)133.0 3.8	97)834.2 8.6	81)607.5 7.5	44)408.100 9.275

18

Change these fractions to decimals:

11. $\frac{3}{10}$ .3 $\frac{17}{40}$ .425 $\frac{5}{16}$ .3125 $\frac{17}{20}$ .85 $\frac{23}{25}$ .92 $\frac{7}{16}$ .4375 $\frac{19}{50}$ .38 $\frac{1}{5}$ .2 $\frac{31}{32}$ .96875 $\frac{11}{16}$ .6875

12. $\frac{19}{80}$ .2375 $\frac{15}{16}$ .9375 $\frac{18}{25}$ .72 $\frac{1}{16}$ .0625 $\frac{11}{40}$ .275 $\frac{9}{32}$ .28125 $\frac{17}{50}$ .34 $\frac{5}{8}$ .625 $\frac{13}{16}$ .8125 $\frac{19}{32}$ .59375

Change to decimals correct to the nearest hundredth:

13. $\frac{16}{43}$ .37 $\frac{15}{19}$ .79 $\frac{4}{13}$ .31 $\frac{25}{82}$ .30 $\frac{3}{17}$ .18 $\frac{37}{94}$ .39 $\frac{17}{75}$ .23 $\frac{8}{9}$ .89 $\frac{23}{37}$ .62 $\frac{8}{11}$ .73

14. $\frac{18}{35}$ .51 $\frac{17}{23}$ .74 $\frac{9}{17}$ .53 $\frac{43}{62}$ .69 $\frac{8}{19}$ .42 $\frac{31}{35}$ .89 $\frac{11}{29}$ .38 $\frac{5}{6}$ .83 $\frac{3}{14}$ .21 $\frac{17}{21}$ .81

15. $\frac{19}{58}$ .33 $\frac{42}{67}$ .63 $\frac{5}{11}$ .45 $\frac{21}{34}$ .62 $\frac{5}{21}$ .24 $\frac{35}{76}$ .46 $\frac{19}{43}$ .44 $\frac{4}{7}$ .57 $\frac{13}{28}$ .46 $\frac{23}{36}$ .64

Change to decimals correct to the nearest thousandth:

16. $\frac{11}{12}$ .917 $\frac{4}{15}$ .267 $\frac{5}{12}$ .417 $\frac{19}{41}$ .463 $\frac{11}{52}$ .212 $\frac{21}{26}$ .808 $\frac{18}{29}$ .621 $\frac{1}{6}$ .167 $\frac{15}{38}$ .395 $\frac{30}{47}$ .638

17. $\frac{21}{23}$ .913 $\frac{19}{26}$ .731 $\frac{3}{11}$ .273 $\frac{37}{61}$ .607 $\frac{49}{92}$ .533 $\frac{11}{30}$ .367 $\frac{17}{19}$ .895 $\frac{5}{7}$ .714 $\frac{32}{51}$ .627 $\frac{46}{83}$ .554

18. $\frac{25}{31}$ .806 $\frac{14}{37}$ .378 $\frac{10}{13}$ .769 $\frac{14}{85}$ .165 $\frac{12}{73}$ .164 $\frac{7}{18}$ .389 $\frac{29}{41}$ .707 $\frac{7}{9}$ .778 $\frac{43}{57}$ .754 $\frac{12}{39}$ .308

19

Divide until there is no remainder. Annex zeros if necessary:

1.	$\overset{.161}{6\overline{).966}}$	$\overset{137}{1.6\overline{)219.2}}$	$\overset{7760}{.03\overline{)232.8\,0}}$	$\overset{.018}{4.6\overline{).0828}}$	$\overset{.13}{4.3\overline{).559}}$
2.	$\overset{7.5}{5\overline{)37.5}}$	$\overset{51}{.21\overline{)10.71}}$	$\overset{7.9}{.17\overline{)1.343}}$	$\overset{4.23}{.05\overline{).2115}}$	$\overset{65}{.15\overline{)9.75}}$
3.	$\overset{.96}{9\overline{)8.64}}$	$\overset{7.1}{.79\overline{)5.609}}$	$\overset{84.25}{4.8\overline{)404.4\,00}}$	$\overset{31,600}{.27\overline{)8532\,00}}$	$\overset{5}{.39\overline{)1.95}}$
4.	$\overset{.021}{7\overline{).147}}$	$\overset{186}{4.8\overline{)892.8}}$	$\overset{32}{.56\overline{)17.92}}$	$\overset{532}{.13\overline{)69.16}}$	$\overset{7}{.58\overline{)4.06}}$
5.	$\overset{230}{.6\overline{)138\,0}}$	$\overset{206}{4.4\overline{)906.4}}$	$\overset{18}{.81\overline{)14.58}}$	$\overset{96.75}{2.4\overline{)232.2\,00}}$	$\overset{6.25}{5.2\overline{)32.5\,00}}$
6.	$\overset{.07}{.8\overline{).056}}$	$\overset{3.365}{2.4\overline{)8.076\,0}}$	$\overset{35.9}{1.9\overline{)68.21}}$	$\overset{8.2}{6.2\overline{)50.84}}$	$\overset{2.36}{7.5\overline{)17.7\,00}}$
7.	$\overset{.08}{.4\overline{).032}}$	$\overset{720}{1.8\overline{)1296\,0}}$	$\overset{8.8}{6.7\overline{)58.96}}$	$\overset{7.85}{.64\overline{)5.024\,0}}$	$\overset{4.75}{9.6\overline{)45.6\,00}}$
8.	$\overset{.079}{9\overline{).711}}$	$\overset{6960}{.07\overline{)487.2\,0}}$	$\overset{32.5}{.42\overline{)13.65\,0}}$	$\overset{.021}{2.5\overline{).0525}}$	$\overset{3400}{.27\overline{)918\,00}}$
9.	$\overset{.076}{8\overline{).608}}$	$\overset{.07}{.25\overline{).0175}}$	$\overset{1830}{.28\overline{)512.4\,0}}$	$\overset{.007}{3.6\overline{).0252}}$	$\overset{2500}{.35\overline{)875\,00}}$

20

Divide. In ex. 10–13, find quotients correct to the nearest tenth; in ex. 14–18, find quotients correct to the nearest hundredth:

10.	$\overset{1.2}{.6\overline{).74}}$	$\overset{11.2}{7.1\overline{)79.8}}$	$\overset{8.2}{.38\overline{)3.11}}$	$\overset{.6}{.31\overline{).174}}$	$\overset{.3}{19.5\overline{)5.76}}$
11.	$\overset{6.4}{.7\overline{)4.5}}$	$\overset{14.9}{.19\overline{)2.83}}$	$\overset{24.9}{7.7\overline{)192}}$	$\overset{1.2}{7.7\overline{)9.17}}$	$\overset{1.7}{3.41\overline{)5.93}}$
12.	$\overset{4.8}{.9\overline{)4.3}}$	$\overset{34.1}{2.7\overline{)92.1}}$	$\overset{7.0}{6.2\overline{)43.1}}$	$\overset{.6}{.28\overline{).163}}$	$\overset{.4}{25.8\overline{)9.36}}$
13.	$\overset{2.8}{.3\overline{).83}}$	$\overset{9.5}{9.3\overline{)87.9}}$	$\overset{24.6}{.84\overline{)20.7}}$	$\overset{.6}{3.9\overline{)2.24}}$	$\overset{.7}{38.4\overline{)26.8}}$
14.	$\overset{3.83}{.6\overline{)2.3}}$	$\overset{46.07}{2.8\overline{)129}}$	$\overset{8.83}{5.3\overline{)46.8}}$	$\overset{1.65}{1.7\overline{)2.81}}$	$\overset{.21}{46.9\overline{)9.72}}$
15.	$\overset{6.33}{.3\overline{)1.9}}$	$\overset{.71}{1.2\overline{).848}}$	$\overset{4.78}{4.5\overline{)21.5}}$	$\overset{49.15}{4.7\overline{)231}}$	$\overset{.48}{17.3\overline{)8.24}}$
16.	$\overset{.87}{.6\overline{).52}}$	$\overset{24.64}{.84\overline{)20.7}}$	$\overset{8.19}{.16\overline{)1.31}}$	$\overset{11.86}{9.7\overline{)115}}$	$\overset{.18}{23.8\overline{)4.39}}$
17.	$\overset{8.22}{.9\overline{)7.4}}$	$\overset{.91}{4.2\overline{)3.82}}$	$\overset{5.04}{.95\overline{)4.79}}$	$\overset{27.90}{3.1\overline{)86.5}}$	$\overset{.01}{51.6\overline{).492}}$
18.	$\overset{6.17}{.6\overline{)3.7}}$	$\overset{2.82}{2.7\overline{)7.61}}$	$\overset{.84}{2.2\overline{)1.84}}$	$\overset{482.11}{1.9\overline{)916}}$	$\overset{.01}{42.3\overline{).586}}$

More Practice

21

Find the answers correct to the nearest cent:

1.	6% of $725 $43.50	3% of $8.95 $.27	$12\frac{1}{2}$% of $416 $52	12% of $18.37 $2.20
2.	32% of $140 $44.80	7% of $6.32 $.44	$33\frac{1}{3}$% of $207 $69	17% of $26.55 $4.51
3.	61% of $252 $153.72	2% of $9.89 $.20	$83\frac{1}{3}$% of $354 $295	75% of $21.53 $16.15
4.	50% of $753 $376.50	6% of $4.85 $.29	$62\frac{1}{2}$% of $520 $325	50% of $37.75 $18.88
5.	75% of $260 $195	5% of $7.37 $.37	$87\frac{1}{2}$% of $192 $168	25% of $31.55 $7.89
6.	60% of $135 $81	1% of $6.65 $.07	$16\frac{2}{3}$% of $438 $73	15% of $34.85 $5.23
7.	15% of $175 $26.25	4% of $2.75 $.11	$37\frac{1}{2}$% of $9.28 $3.48	$2\frac{1}{4}$% of $4688 $105.48
8.	12% of $350 $42	3% of $9.35 $.28	$16\frac{2}{3}$% of $7.56 $1.26	$5\frac{1}{3}$% of $2700 $144
9.	22% of $415 $91.30	8% of $6.50 $.52	$33\frac{1}{3}$% of $3.60 $1.20	$6\frac{3}{4}$% of $3368 $227.34
10.	25% of $588 $147	2% of $5.77 $.12	$12\frac{1}{2}$% of $6.64 $.83	$2\frac{2}{3}$% of $4239 $113.04
11.	80% of $115 $92	1% of $9.25 $.09	$66\frac{2}{3}$% of $4.68 $3.12	$1\frac{1}{2}$% of $5426 $81.39
12.	20% of $275 $55	6% of $8.38 $.50	$62\frac{1}{2}$% of $5.20 $3.25	$4\frac{1}{4}$% of $6000 $255

22

Write these per cents as decimals:

13.	18.2% .182	11.4% .114	73.4% .734	27.16% .2716	5.9% .059	2.68% .0268
14.	46.5% .465	19.1% .191	61.9% .619	83.36% .8336	8.4% .084	1.93% .0193

Write these decimals as per cents:

15.	.652 65.2%	.249 24.9%	.074 7.4%	.441 44.1%	.045 4.5%	.0336 3.36%
16.	.846 84.6%	.583 58.3%	.096 9.6%	.082 8.2%	.724 72.4%	.0714 7.14%

Find the answers correct to the nearest whole number:

17.	15.2% of 750 114	1.9% of 854 16	11.8% of 1500 177
18.	21.6% of 125 27	6.4% of 216 14	12.3% of 3275 403
19.	18.5% of 468 87	8.3% of 580 48	15.7% of 1900 298
20.	11.7% of 600 70	2.5% of 925 23	18.9% of 2400 454
21.	13.3% of 250 33	4.8% of 730 35	21.6% of 3600 778
22.	24.8% of 560 139	5.5% of 460 25	25.1% of 8150 2046

23

Estimate the answers. (1) Then find the exact answers: (2)

(1), (2) **1.**	19% of \$198 \$40, \$37.62	76% of \$1575 \$1200, \$1197	32% of \$6100 \$2000, \$1952
2.	63% of \$799 \$500, \$503.37	37% of \$2415 \$900, \$893.55	66% of \$8990 \$6000, \$5933.40
3.	12% of \$159 \$20, \$19.08	49% of \$1195 \$600, \$585.55	41% of \$4998 \$2000, \$2049.18
4.	51% of \$447 \$225, \$227.97	34% of \$2090 \$700, \$710.60	19% of \$3050 \$600, \$579.50
5.	26% of \$790 \$200, \$205.40	16% of \$1210 \$200, \$193.60	87% of \$7900 \$7000, \$6873
6.	33% of \$596 \$200, \$196.68	83% of \$1189 \$1000, \$986.87	59% of \$1520 \$900, \$896.80
7.	62% of \$810 \$500, \$502.20	13% of \$3175 \$400, \$412.75	88% of \$4005 \$3500, \$3524.40

24

8.	$\frac{1}{2}$ of 1% of \$488 \$2.44	.4% of \$2750 \$11	0.2% of \$2450 \$4.90
9.	$\frac{1}{4}$ of 1% of \$800 \$2.00	.3% of \$2240 \$6.72	0.5% of \$6400 \$32
10.	$\frac{1}{3}$ of 1% of \$690 \$2.30	.2% of \$4600 \$9.20	0.1% of \$2800 \$2.80
11.	$\frac{3}{4}$ of 1% of \$360 \$2.70	.6% of \$2550 \$15.30	0.4% of \$3000 \$12
12.	$\frac{1}{2}$ of 1% of \$750 \$3.75	$\frac{3}{5}$% of \$2000 \$12	0.6% of \$2500 \$15
13.	$\frac{2}{3}$ of 1% of \$900 \$6.00	$\frac{3}{4}$% of \$7000 \$52.50	0.3% of \$4850 \$14.55
14.	$\frac{1}{10}$ of 1% of \$350 \$.35	$\frac{7}{10}$% of \$1800 \$12.60	0.5% of \$7640 \$38.20

25

Find answers to the nearest cent:

15.	400% of \$295 \$1180	125% of \$380 \$475	135% of \$50.25 \$67.84
16.	175% of \$720 \$1260	515% of \$860 \$4429	285% of \$46.50 \$132.53
17.	120% of \$355 \$426	140% of \$175 \$245	345% of \$18.75 \$64.69
18.	112% of \$290 \$324.80	150% of \$380 \$570	250% of \$63.85 \$159.63
19.	100% of \$425 \$425	120% of \$225 \$270	310% of \$49.73 \$154.16
20.	234% of \$196 \$458.64	225% of \$760 \$1710	112.4% of \$37.76 \$42.44
21.	114% of \$288 \$328.32	110% of \$175 \$192.50	125.3% of \$55.70 \$69.79
22.	215% of \$375 \$806.25	230% of \$525 \$1207.50	150.7% of \$95.50 \$143.92
23.	145% of \$260 \$377	160% of \$329 \$526.40	175.5% of \$72.90 \$127.94

More Practice

26

Find what per cent the first number is of the second:

1.	6, 40 15%	11, 50 22%	70, 35 200%	13, 104 $12\frac{1}{2}\%$	105, 150 70%
2.	7, 25 28%	16, 30 $53\frac{1}{3}\%$	65, 52 125%	66, 400 $16\frac{1}{2}\%$	213, 284 75%
3.	9, 72 $12\frac{1}{2}\%$	17, 40 $42\frac{1}{2}\%$	57, 76 75%	28, 350 8%	375, 625 60%
4.	5, 80 $6\frac{1}{4}\%$	31, 50 62%	29, 87 $33\frac{1}{3}\%$	63, 400 $15\frac{3}{4}\%$	117, 117 100%
5.	8, 32 25%	63, 75 84%	84, 24 350%	15, 200 $7\frac{1}{2}\%$	213, 100 213%
6.	4, 80 5%	15, 40 $37\frac{1}{2}\%$	14, 80 $17\frac{1}{2}\%$	85, 250 34%	195, 240 $81\frac{1}{4}\%$

27

Find, to the nearest whole per cent, what per cent the first number is of the second:

7.	3, 11 27%	16, 29 55%	15, 109 14%	27, 19 142%	101, 69 146%
8.	5, 19 26%	21, 47 45%	26, 113 23%	39, 22 177%	215, 83 259%
9.	4, 21 19%	36, 89 40%	43, 121 36%	118, 206 57%	194, 71 273%
10.	9, 35 26%	12, 49 24%	61, 157 39%	123, 187 66%	183, 59 310%

28

Find, to the nearest whole per cent, the per cent of increase (inc.) or decrease (dec.) that the second number shows over the first number:

11.	208, 273 31% inc.	1500, 2980 99% inc.	\$3.50, \$3.61 3% inc.	\$42.50, \$53.25 25% inc.
12.	116, 111 4% dec.	7690, 9000 17% inc.	\$1.25, \$1.90 52% inc.	\$18.75, \$29.00 55% inc.
13.	153, 200 31% inc.	4815, 2500 48% dec.	\$2.15, \$1.75 19% dec.	\$32.50, \$23.84 27% dec.
14.	186, 301 62% inc.	3600, 3385 6% dec.	\$4.50, \$4.85 8% inc.	\$17.25, \$31.30 81% inc.
15.	275, 243 12% dec.	2450, 2900 18% inc.	\$6.35, \$6.00 6% dec.	\$42.30, \$35.00 17% dec.

29

What per cent of increase or decrease does the second number show over the first number?

16.	2580, 3440 $33\frac{1}{3}\%$ inc.	2500, 2200 12% dec.	4500, 4950 10% inc.	\$15.00, \$52.50 250% inc.
17.	3700, 7030 90% inc.	8900, 9879 11% inc.	4320, 3240 25% dec.	\$18.60, \$51.15 175% inc.
18.	1550, 1240 20% dec.	2600, 6500 150% inc.	2750, 2970 8% inc.	\$24.00, \$51.00 $112\frac{1}{2}\%$ inc.
19.	2400, 2274 $5\frac{1}{4}\%$ dec.	3920, 2940 25% dec.	1440, 1656 15% inc.	\$92.50, \$55.50 40% dec.
20.	1444, 2527 75% inc.	2875, 2300 20% dec.	6700, 6030 10% dec.	\$84.80, \$53.00 $37\frac{1}{2}\%$ dec.

30

Find the discount (1) and the net price (2) when the list price and the rate of discount are as shown below:

(1), (2)

1.	$645, 20% — $129, $516	$960, 12½% — $120, $840	$1700, 5% — $85, $1615	$7.50, 20% — $1.50, $6.00
2.	$360, 25% — $90, $270	$276, 33⅓% — $92, $184	$2550, 3% — $76.50, $2473.50	$9.40, 10% — $.94, $8.46
3.	$800, 15% — $120, $680	$488, 37½% — $183, $305	$1875, 6% — $112.50, $1762.50	$4.60, 15% — $.69, $3.91
4.	$680, 35% — $238, $442	$540, 16⅔% — $90, $450	$3300, 2% — $66, $3234	$8.80, 25% — $2.20, $6.60
5.	$850, 30% — $255, $595	$176, 12½% — $22, $154	$2850, 4% — $114, $2736	$6.90, 10% — $.69, $6.21

31

Find the net prices, using these list prices and discounts:

6.	$20, 5% $19	$280, 20% $224	$664, 12½% $581	$36.50, 10% $32.85
7.	$25, 8% $23	$384, 25% $288	$348, 16⅔% $290	$27.75, 20% $22.20
8.	$35, 6% $32.90	$550, 30% $385	$435, 33⅓% $290	$42.60, 25% $31.95
9.	$80, 7% $74.40	$275, 40% $165	$260, 11½% $230.10	$18.90, 10% $17.01
10.	$75, 5% $71.25	$650, 12% $572	$368, 15¼% $311.88	$25.50, 12% $22.44

32

Find the commission (1) and the net proceeds (2):

(1), (2)

11.	$500, 7% — $35, $465	$980, 10% — $98, $882	$8000, 6% — $480, $7520	$25.50, 4% — $1.02, $24.48
12.	$680, 5% — $34, $646	$775, 20% — $155, $620	$7500, 4% — $300, $7200	$17.80, 5% — $.89, $16.91
13.	$750, 3% — $22.50, $727.50	$560, 30% — $168, $392	$6800, 5% — $340, $6460	$46.50, 6% — $2.79, $43.71
14.	$920, 6% — $55.20, $864.80	$596, 25% — $149, $447	$4600, 5% — $230, $4370	$79.00, 3% — $2.37, $76.63
15.	$850, 4% — $34, $816	$425, 10% — $42.50, $382.50	$9550, 3% — $286.50, $9263.50	$66.50, 2% — $1.33, $65.17

33

The first amount represents the total sales; the second amount is the commission. Find the rate of commission:

16.	$75, $15 20%	$900, $81 9%	$500, $125 25%	$9000, $540 6%
17.	$80, $20 25%	$800, $68 8½%	$900, $135 15%	$2500, $100 4%
18.	$66, $22 33⅓%	$250, $20 8%	$585, $117 20%	$3500, $210 6%
19.	$96, $12 12½%	$600, $33 5½%	$760, $152 20%	$8880, $444 5%
20.	$60, $10 16⅔%	$400, $60 15%	$840, $126 15%	$4600, $322 7%

More Practice

34

Find the missing numbers:

1.	8 is 2% of . . . 400	78 is 12% of . . . 650	528 is 8% of . . . 6600
2.	9 is 6% of . . . 150	63 is 15% of . . . 420	216 is 5% of . . . 4320
3.	6 is 4% of . . . 150	99 is 22% of . . . 450	175 is 7% of . . . 2500
4.	9 is 3% of . . . 300	81 is 18% of . . . 450	137 is 4% of . . . 3425
5.	7 is 5% of . . . 140	54 is 24% of . . . 225	162 is 6% of . . . 2700

35

In ex. 6–10, find the profit. In ex. 11–15, find the loss:

	Selling Price	Cost	Expenses			Selling Price	Cost	Expenses	
6.	$300	$200	$50	$50	**11.**	$250	$200	$70	$20
7.	$450	$300	$120	$30	**12.**	$500	$325	$185	$10
8.	$800	$480	$200	$120	**13.**	$650	$450	$220	$20
9.	$600	$400	$140	$60	**14.**	$800	$570	$245	$15
10.	$400	$250	$125	$25	**15.**	$300	$225	$80	$5

36

Find the selling price. The per cents for expenses and profit are based on the selling price:

	Cost	Expenses	Profit			Cost	Expenses	Profit	
16.	$93.00	30%	8%	$150	**27.**	$513	34%	9%	$900
17.	$78.65	23%	12%	$121	**28.**	$832	28%	8%	$1300
18.	$22.42	28%	13%	$38	**29.**	$945	27%	10%	$1500
19.	$34.85	27%	5%	$51.25	**30.**	$319	29%	13%	$550
20.	$30.00	28%	12%	$50	**31.**	$8.80	27%	9%	$13.75
21.	$92.30	30%	5%	$142	**32.**	$4.64	29%	13%	$8
22.	$37.12	31%	11%	$64	**33.**	$7.86	30%	10%	$13.10
23.	$24.50	29%	15%	$43.75	**34.**	$9.60	35%	17%	$20
24.	$88.96	30%	6%	$139	**35.**	$8.40	28%	12%	$14
25.	$72.00	28%	8%	$112.50	**36.**	$2.97	35%	11%	$5.50
26.	$96.60	35%	9%	$172.50	**37.**	$9.23	25%	10%	$14.20

Pages 373-374 provide ready-reference pages of tables of measure.

Tables for Reference

Linear Measure

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods or 5280 feet = 1 mile (mi.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
43,560 square feet = 1 acre (A.)
160 square rods = 1 acre
640 acres = 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

Measures of Weight

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
2000 pounds = 1 ton (T.)

Measures of Time

60 seconds = 1 minute
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)

12 months = 1 year (yr.)
365 days = 1 year
366 days = 1 leap year
100 years = 1 century

Liquid Measure

2 cups = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

2 pints = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Tables for Reference

Counting Articles

12 units = 1 dozen (doz.)
12 dozen = 1 gross
20 units = 1 score

Counting Paper

24 or 25 sheets = 1 quire
20 quires = 1 ream
480 or 500 sheets = 1 ream

Angles and Arcs

60 seconds ($''$) = 1 minute ($'$)
60 minutes = 1 degree ($^\circ$)
90 degrees = 1 right angle
180 degrees = 1 straight angle

Weight per Bushel (in many states)

Apples..............	48 lb.	Corn (on cob).......	70 lb.
Barley..............	48 lb.	Oats...............	32 lb.
Beans...............	60 lb.	Peas...............	60 lb.
Buckwheat...........	48 lb.	Potatoes...........	60 lb.
Clover seed.........	60 lb.	Rye................	56 lb.
Corn (shelled)........	56 lb.	Wheat..............	60 lb.

Useful Equivalents

1 bushel = about 2150 cu. in.
1 bushel = about $1\frac{1}{4}$ cu. ft.
1 cubic foot = about 0.8 bu.
1 gallon = 231 cu. in.
1 cubic foot = about $7\frac{1}{2}$ gal.
1 barrel = $31\frac{1}{2}$ gal.
1 barrel flour = 196 lb.
1 ton hard coal = about 35 cu. ft.
1 ton soft coal = about 42 cu. ft.
1 long ton = 2240 lb.
1 cord of wood = 128 cu. ft.
1 cu. ft. water = about $62\frac{1}{2}$ lb.
1 cu. ft. ice = about $57\frac{1}{2}$ lb.
1 sea mile = 1.15 land miles

Improvement Tests. The timed tests described on page 48 are called Improvement Tests. There are 72 of these tests in this book. The purpose of these tests is to keep alive and to perfect the computational skills with whole numbers that the pupils have learned previously, and to accomplish this task in a comparatively few minutes each week, so that ample time is left in each class period for the study of new topics. The procedure in giving these tests is described below and on pages 376 and 377. For a complete list of the pages upon which Improvement Tests appear, see the Index of this book.

How to Give Improvement Tests. The Improvement Tests are arranged in sets, each set containing three tests of the same kind and of equal difficulty. See the set of three addition tests on page 49. The procedure in giving a test, such as Addition Test 1a on page 49, is as follows:

(1) Start all the pupils at the same time by saying "Go." Stop all pupils promptly at the end of 4 min., which is the time assigned for this Addition Test. Use a watch with a second hand to keep the time. For addition and subtraction tests the answers should be written along the edge of a sheet of folded paper, as described below.

(2) Read the answers. Then have each pupil count the number of exercises he has right and find his score on a scale of 10, using the Scoring Table on page 376. A partially completed exercise does not count.

(3) The pupil should keep a graphic record of his scores on these tests as described on page 377.

Never give more than one test of a set per day. For example, if Addition Test 1a, on page 49, is given today, Addition Test 1b and 1c should be given on subsequent days. In general, give 1 set (3 tests) at most per week. There are 24 sets (72 tests) in all; to distribute these tests uniformly over the school year, it is suggested that one set (3 tests) be given, on the average, about every 6 school days. After a little practice a test can be taken, scored, and recorded within 10 minutes.

The time allowed for each test is a reasonable one. If the pupil has had sufficient practice in the fundamental operations, he should have no trouble in completing the test in the assigned time. It is important that each test of a set be given exactly the same amount of time; otherwise there will be no dependable means of knowing whether the pupil is improving or not.

The graphic record of scores, which each pupil keeps, enables him quickly to see whether his skill is improving. This record also makes it possible for the pupil to compare his skill in addition with that in subtraction or multiplication, thus showing him the operation upon which he needs the most practice.

Folded Paper. When you give addition and subtraction tests like those on pages 49 and 65, the pupils should not copy the examples. Instead they

should lay a sheet of paper on the book with its edge under the top row of examples and write the answer to each example on the edge of the paper. When the answers to this row have been finished, they should be folded under and the answers to the second row should be written along the folded edge. This should be continued for each row. It saves time if the paper is folded *before* the test is taken. Each fold should be about 1 inch wide.

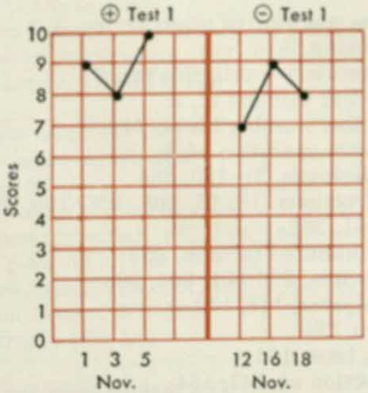
Folded paper cannot be used for tests in multiplication and division. In taking such tests the pupils should copy the examples on paper *before* the test begins. The examples should be spread over the paper so as to leave room enough to work them.

Scoring Improvement Tests. The number of minutes to be allowed for each Improvement Test is printed above the test. The pupils should work on each test only the number of minutes given for that test and should *stop work at once at the end of that time.* A comparison of the scores on the 3 tests in a set is valueless unless you allow exactly the same number of minutes for each test. Each pupil should then find his score on a scale of 10 by using the Scoring Table shown below.

SCORING TABLE FOR IMPROVEMENT TESTS

NUMBER OF EXAMPLES IN TEST	NUMBER OF EXAMPLES RIGHT															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	0	3	5	8	10											
5	0	2	4	6	8	10										
6	0	2	3	5	7	8	10									
7	0	1	3	4	6	7	9	10								
8	0	1	3	4	5	6	8	9	10							
9	0	1	2	3	4	6	7	8	9	10						
10	0	1	2	3	4	5	6	7	8	9	10					
12	0	1	2	3	3	4	5	6	7	8	8	9	10			
15	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10

To the Pupil. Suppose that you take Subtraction Test 1a on page 65. This contains 15 examples. If you get 12 examples right, you find your score as follows. Since there are 15 examples in the test, find in the above table the horizontal row beginning with 15. Then move your pencil to the right along that row until you come to the vertical column with 12 at the top of it. Your pencil will now point to the number 8. This means that your score is 8 when you have 12 examples right on a test of 15 examples. At the end of Subtraction Test 1a, the symbol 15 means that there are 15 examples in the test.



Record of Ann's Scores

Recording Scores. Each time a pupil finds his score on an Improvement Test he should record the score on a graph as shown at the right. The scores 1 to 10 are marked on the left side of the graph and the dates at the bottom of it. The first line graph, marked "⊕ Test 1" at the top, shows Ann's scores on Addition Tests 1a, 1b, and 1c. ⊕ means "addition." The first dot on this graph is on the horizontal line marked 9; this dot is also on the vertical line marked 1; this means that Ann's score was 9 on Nov. 1. The second dot shows a score of 8 on Nov. 3. The third dot shows a score of 10 on Nov. 5.

The second graph gives Ann's scores on 3 subtraction tests.

By using squared paper, each pupil can make a book in which to keep a record of his scores on all Improvement Tests.

Number Facts. In the early part of Grade 8 the pupils should be given the tests on pages 355–357 to see how well they know the fundamental number facts. A mastery of these facts is necessary for success in arithmetic.

The tests just mentioned are to be given orally to each individual pupil. As the pupil reads each fact on any given test, he gives *only the answer*. The teacher should note all the facts for which the pupil gives unsatisfactory responses. At the end of the test the teacher should give the pupil a list of these particular facts, with instructions to study them carefully. An unsatisfactory response is one that is incorrect or that is given with much hesitation, even though it is correct. The time limit for each test is 2 to 3 minutes, depending upon the ability of the pupil and the part of the school year in which the test is given.

More Practice. Pages 358–372 of this book give supplementary practice material that can be used in several ways as the teacher sees fit. References to these sets of exercises have been made on certain pages of the text where it is possible that more practice of a given type may be needed. The teacher can decide when to use this material. The supplementary exercises can also be used for the purpose of review on whole numbers, fractions, decimals, and percentage.

Index

Teacher's Guide and Key for

MASTERING MATHEMATICS

American Arithmetic
Second Edition
Grade 8

CLIFFORD B. UPTON
KENNETH G. FULLER
MILO E. WHITSON

AMERICAN BOOK COMPANY
New York Cincinnati Atlanta Dallas Millbrae

EP 10 9 8 7 6 5 4 3 2 1

Use of This Guide. This Guide, together with the Upton-Fuller-Whitson *American Arithmetic, Second Edition, Grade 8, Mastering Mathematics,* offers to the teacher a course in modern methods of teaching mathematics in Grade 8. The Guide presents the principles of teaching, while *American Arithmetic, Grade 8* illustrates these principles through the actual materials of instruction. This combination provides a more comprehensive discussion of methods in mathematics than is found in the usual methods textbook.

The Guide gives suggestions and helps on the teaching of each page of *American Arithmetic, Grade 8* and often refers to specific exercises in the text. To use the Guide successfully, you should study carefully each text exercise or explanation to which reference is made in the Guide.

Since a thorough acquaintance with a textbook helps the teacher in using that textbook effectively, certain outstanding features of this series of books are given below.

Philosophy Underlying These Books. The objectives of *American Arithmetics* are simply stated:

(1) To teach pupils to understand our number system and the four fundamental processes with whole numbers, fractions, and decimals

(2) To teach pupils properly to interpret and to solve the problems of everyday life that involve quantitative data

(3) To develop in pupils sufficient skill in computation to enable them to solve the problems of everyday life with reasonable speed and accuracy

(4) To develop in pupils an appreciation of the nature of mathematics

The attainment of these objectives comes *through insight and intelligent understanding* rather than through the blind following of rules and mechanical procedures. At appropriate grade levels concepts and methods of algebra and geometry are introduced to assist in attaining these objectives.

Meaning and Understanding. Meaning and understanding in mathematics go much farther than using objects in the primary grades to make number meaningful, or explaining the process of carrying in addition. Meaning and understanding, properly treated, must pervade all the work in mathematics throughout the elementary school and the junior high school. The constant aim of this series of texts is to teach mathematics so that the pupils understand and enjoy it. Throughout these books special attention is given to the following:

1. **Basic Concepts and Terms of Mathematics.** The meanings of fundamental concepts relating to whole numbers, fractions, and decimals, as well as the definitions of the technical terms of mathematics, are explained with unusual clarity.

2. **Comprehension of the Fundamental Processes.** This feature includes clear explanations of such topics as the addition of unlike fractions, the division of a decimal by a decimal, finding what per cent one number is of another, and so on.

3. **Meaningful Processes through Motivating Problems.** In these textbooks each new

topic or process is made meaningful through a motivating problem related to the interests of the pupils. This method shows why the new process is needed.

4. Fundamental Principles. Fundamental principles of mathematics are simply explained and applied in this series. These principles replace arbitrary rules.

5. Meaningful Problems. All problems and projects relate to the interests and experiences of children and are written in language that they can understand.

6. Our Number System. The principles underlying our number system are emphasized in each book of this series.

Number Relationships and Principles. One of the outstanding features of this series of textbooks is the use of number relationships and principles in the teaching of the basic number facts. They greatly simplify learning for the pupil and help him to attain a degree of mastery and accomplishment that is not possible by other methods. In this work, if the pupil forgets the answer to a number fact, he can get it quickly *through his own thinking* by using a simple number relationship or principle. This method makes it unnecessary for him to get the answer by counting objects, or by looking it up in a table, or by asking the teacher to give it to him. All generalizations are developed inductively and then stated algebraically. Many relationships and principles are reviewed in Chapter 1 of *American Arithmetic, Grade 8*. You will also find interesting number relationships for use with remedial work on the multiplication facts on pages 20 and 21 of *American Arithmetic, Grade 6*. The pupils will enjoy them as well as find them an aid in learning the multiplication facts they do not know.

In these texts, number relationships and fundamental principles are used effectively also in teaching the four processes with whole numbers, fractions, and decimals; in the work on percentage; in problem solving; and in the study of the formula and the equation.

Long Division. Long division is by far the most difficult topic with whole numbers. Long division with 2-figure divisors is first taught in *American Arithmetic, Grade 5*, and long division with 3-figure divisors is introduced in *American Arithmetic, Grade 6*. This work is carefully retaught in the texts for Grades 7 and 8.

In some textbooks long division is taught by a method which requires frequent correction of each quotient figure before the correct figure is found. An example of this inefficient way of doing long division is described in Method A:

Method A. In the work at the right, each quotient figure is found by dividing each partial dividend by the *first figure of the divisor*. For example, to find the first quotient figure, you divide 9 by 2, which gives 4 as a trial quotient. Since 4 is too large, you then try 3, which is correct. To get the next quotient figure, you divide 13 by 2, getting 6, which is too large. You try 5, which is also too large. Then you try 4, and that is correct. For the next quotient figure you divide 21 by 2, which gives 10; but no quotient figure can be larger than 9, so you try 9. You find that 9 is too large, so you try 8; 8 is also too large, so

$$\begin{array}{r} 347 \\ 28\overline{)9734} \\ \underline{84} \\ 133 \\ \underline{112} \\ 214 \\ \underline{196} \\ 18 \end{array}$$

you try 7, which is correct. It should be noted that the correct quotient figure was obtained each time *only after two or three trials.*

By contrast, in the *American Arithmetics* the pupils are taught to do the example as shown in Method B:

Method B. Instead of dividing each partial dividend by 2, divide in each case by 3, which is 1 *more than the first figure of the divisor.* In other words, you think of 28 as being close to 30; hence, you get each trial quotient figure by dividing by 3 instead of 2. To get the first quotient figure, divide 9 by 3; this gives 3, which is correct. To get the second quotient figure, divide 13 by 3; this gives 4, which is correct. To get the next quotient figure, divide 21 by 3; this gives 7, which is correct. By following this method, you see that the correct quotient figure is found each time *on the first trial.*

In the example shown, the divisor is 28. In the *American Arithmetics,* if the divisor were 26, 27, or 29 instead of 28, the example would be worked also by Method B, because each of these divisors is closer to 30 than to 20. In fact, for any 2-figure divisor having a second figure of 6, 7, 8, or 9, each quotient figure is estimated by dividing by 1 more than the first figure of the divisor. See page 13 of *American Arithmetic, Grade 8.* On the other hand, if the second figure of a 2-figure divisor is 1, 2, 3, 4, or 5, each quotient figure is estimated by dividing by the first figure of the divisor. See page 12 of *American Arithmetic, Grade 8.* It is thus seen that with the method of long division taught in the *American Arithmetics,* each quotient figure is found by dividing by the *first figure* of the divisor when the divisor ends in 1, 2, 3, 4, or 5, and by dividing by 1 *more than the first figure* when the divisor ends in 6, 7, 8, or 9. This method of long division is very efficient since it gives the correct quotient figure *on first trial* in 80% of all cases; in the remaining 20% of cases, with minor exceptions, the correct quotient figure is found after a single correction. In other words, the necessity of correcting each quotient figure several times does not occur when long division is taught according to the method given in the *American Arithmetics.* This method of long division is the result of years of research on the part of Professor Upton. The research was reported in detail in the *Tenth Yearbook of the National Council of Teachers of Mathematics* in an article entitled "Making Long Division Automatic."

Projects. Many interesting projects are provided in each of the books of this series. In many of these projects arithmetic is correlated with other school subjects such as history, geography, and science.

Vocabulary and Language. Problems and explanations are written in clear language, and the vocabulary has been selected with unusual care. The technical words of arithmetic are clearly defined and are reviewed in special exercises.

Maintenance Program. Full provision has been made to maintain skills in computation and problem solving as follows:

1. Review and Reteaching. This book devotes most of the first chapter to a review and reteaching of the four fundamental operations with whole numbers, fractions,

and decimals. In the second chapter the work in percentage is thoroughly retaught and extended to new topics.

2. Remedial Practice. If any pupils need supplementary practice on the four operations with whole numbers, fractions, and decimals, or on the essentials of percentage, such practice will be found on pages 358–372 of the text.

3. Diagnostic Tests. Diagnostic tests with keyed references to practice exercises are given at the end of each chapter. See pages 38, 80, 108, 144, 183, 184, 220, 252, 292, 324, 353, and 354.

4. Problem Tests. A Problem Test designed to test the pupil's ability in problem solving is given in each chapter. See pages 37, 79, 107, 143, 182, 219, 251, 291, 323, and 352.

5. Chapter Reviews. Chapter reviews covering the new work of a chapter are provided. See pages 78, 142, 181, 218, 250, 290, and 322.

6. Oral and Written Reviews. Reviews covering important topics previously taught are frequently given. See pages 66, 76, 117, 128, 138, 150, 157, 177, and so on.

7. Improvement Tests. A series of 72 improvement tests, covering the four operations with whole numbers, is distributed over the year's work. These tests provide a most efficient and interesting means of maintaining skills with whole numbers while the pupil is studying new topics. See pages 49, 65, 75, 87, 99, 113, 125, 139, and so on.

8. Mixed Practice. Computational practice on the four fundamental processes is amply provided. See pages 87, 105, 158, 170, and so on.

The Work of Grade 8. The aim of the year's work is to deepen the pupil's understanding of our number system, to increase his knowledge of mathematics, and to show him some of the important uses of mathematics in everyday life. The subject matter includes the number system, informal geometry, formulas, equations, inequalities, and applications of mathematics. You will find that *American Arithmetic, Grade 7* and *American Arithmetic, Grade 8* include many of the topics and methods of presentation that have been recommended by national groups for the improvement of the mathematics curriculum in these grades. The work of Grade 8 is outlined below:

1. The Number System. The study of the structure of the number system is continued and expanded in Grade 8. The commutative and associative principles for addition and multiplication are reviewed. The distributive principle and other previously taught principles are also reviewed. The relationship of the inverse operations subtraction and division to addition and multiplication is expressed algebraically and emphasized. Prime and composite numbers are discussed. Place value is carefully reviewed and exponents are used to express the value of a number in terms of its digits. Number systems having bases other than ten and Egyptian numerals are presented to increase the pupil's appreciation of the structure of our number system. Some computation with numbers written in bases other than ten is included.

2. Informal Geometry. This work includes geometric constructions, scale drawing, congruent and similar triangles, circles, the Pythagorean formula, areas, volumes, and indirect measurement. The terms *line*, *ray*, and *segment* are explained and used throughout the text. The formulas of mensuration are developed through classroom experimentation. The work is enlivened by challenging outdoor projects in indirect measurement. Circle graphs are used to make easier the interpretation of the numerical data of certain problems.

3. Formulas, Equations, and Inequalities. Formulas are used to express generalizations of procedures for finding areas and volumes, and for finding the interest on loans and the interest rate charged in installment buying. In the work with formulas, unknown factors of products are first found by using the relationship of division to multiplication and later by applying fundamental principles of equations. Equations and inequalities are presented as important aids to problem solving. The solution of equations is based on a few basic principles rather than upon arbitrary rules. The study of proportions and their use in problem solving is also included.

4. Applications of Mathematics. This book is outstanding in respect to its treatment of the applications of mathematics to everyday life. These applications, which may be called *consumer mathematics*, include interest, discount, commission, savings and investment, personal loans, installment purchases, buying a home, taxes, and the various kinds of insurance. Each of these topics is presented with a broad informational background so that pupils can easily understand it.

Percentage and its applications are presented with great clarity. Since percentage differs from decimals largely in the language in which its problems are expressed, many exercises are given for the purpose of acquainting the pupil with the technical expressions that are peculiar to this topic. The computational difficulties of percentage, which are closely related to those of decimals, are carefully graded. The relationship of the three types of percentage problems is shown by means of the percentage formula.

Suggestions to Teachers. Before starting the year's work, read "Suggestions to Teachers" on pages 375–377 of *American Arithmetic, Grade 8*. These suggestions contain valuable hints and give various helps for teaching specific pages of the text.

Key. All answers are given on the appropriate pages in the Guide. These answers will be a convenience for those teachers not using the annotated edition, who may prefer to transfer them to the textbook pages for ready reference. As a further convenience, many examples are worked out in detail throughout the Guide.

Chapter 1

Aims of Chapter 1. The major aims of Chapter 1 are to:

1. Review round numbers.
2. Review the commutative, associative, and distributive principles, and other number relationships and principles.
3. Review the fundamental addition, subtraction, multiplication, and division facts.
4. Review the addition, subtraction, and multiplication of whole numbers.
5. Reteach long division.
6. Reteach checking multiplication and division by casting out 9's.
7. Review the four fundamental operations with fractions.
8. Review place value and exponents.
9. Reteach writing numbers in systems having bases other than ten, and teach how to compute in such number systems.
10. Review the four fundamental operations with decimals.
11. Reteach changing fractions to decimals.
12. Introduce the metric system of measures.

Reteaching of Fundamental Operations in Grade 8. The experience of a large number of teachers has shown that in Grade 8 there are always some pupils who display weaknesses in the fundamental operations with whole numbers, fractions, and decimals. Thus, there is a need in Grade 8 for a review of these topics and for reteaching the more difficult aspects of some of these topics. *American Arithmetic, Grade 8* undertakes in the first chapter to provide such instruction. In this work, basic concepts, relationships, and generalizations are emphasized. The *commutative, associative,* and *distributive principles* are used to deepen the pupils' understanding of the structure of our number system. For the same purpose, number systems with bases other than ten are presented and the concept of place value is thoroughly reviewed. Applications to everyday life are made to show the usefulness of each topic.

The amount of time needed for Chapter 1 will depend upon the ability and previous achievement of the pupils and will vary from class to class. For average pupils, this treatment represents a brief review of fundamentals. More capable pupils can proceed rapidly to the new work on number systems with bases other than ten and on the metric system of measures, omitting any topics they have mastered. For pupils of less ability, this reteaching is supplemented by a liberal number of practice exercises, which will be found on pages 358–367 of the text.

Page 1

Aim: To present an interesting project using large numbers and previously taught operations

Workbook Reference: Upton-Uhlinger: Arithmetic Workshop, Book 8, pages 1 and 4 (published by American Book Company)

Key: **1.** 7,920,200 + 1,249,100 = 9,169,300 (motor vehicles). **2.** (1) 6,674,800 + 1,194,500 = 7,869,300 (motor vehicles); (2) 7,920,200 − 6,674,800 = 1,245,400 (passenger cars); (3) 1,249,100 − 1,194,500 = 54,600 (trucks and busses).
3. \$1822 × 6,674,800 = \$12,161,485,600. **4.** \$1968 × 1,194,500 = \$2,350,776,000.
5. (1) $\frac{2}{5}$ × 6,674,800 = 2,669,920 (4-door sedans); (2) $\frac{1}{6}$ × 6,674,800 = 1,112,466$\frac{2}{3}$, or about 1,112,467 (station wagons); (3) $\frac{1}{4}$ × 6,674,800 = 1,668,700 (hard tops and convertibles).

Page 2

Aim: To review two ways of writing large numbers

Suggestions: A short method of writing round numbers representing millions and billions is described in ex. 2 and 3. By this method the number 23,700,000,000 is written more briefly as 23.7 billion; likewise, 4,200,000 is written as 4.2 million. It is evident that the shorter forms, 23.7 billion and 4.2 million, are easier to read than the longer forms. This way of writing numbers is usually confined to round numbers that can be expressed by writing a 1-place decimal before the word *million* or *billion*, as in 6.2 million and 4.7 billion. Occasionally one finds such numbers written by using a 2-place or a 3-place decimal, as in 3.25 million and 7.328 billion, but such usage is exceptional. This method of writing large numbers is frequently used today in newspapers, magazines, and books. The work on page 2 can be used as a check on the pupil's understanding of place value.

Workbook Reference: Arithmetic Workshop, Book 8, page 2

Key: **4.** 74,300,000; \$1,200,000,000; 424,500,000; \$126,800,000,000; \$1,100,000. **5.** 2,700,000, 2.7 million; 4,700,000, 4.7 million; 16,500,000, 16.5 million; 9,100,000, 9.1 million; 8,100,000, 8.1 million; 92,600,000, 92.6 million; 7,200,000, 7.2 million; 3,400,000, 3.4 million; 9,900,000, 9.9 million. **6.** 4,800,000,000, 4.8 billion; 138,700,000,000, 138.7 billion; \$37,400,000,000, \$37.4 billion; \$12,900,000,000, \$12.9 billion.

Page 3

Aim: To review the commutative and associative principles for addition, and the fundamental addition facts

Suggestions: Relationships and general principles assist the pupil in learning the fundamental addition facts. They also give him a way of finding with ease answers that he has forgotten, or of checking answers of which he is unsure. Pupils should be encouraged to discuss different ways in which they can recall addition facts, particularly those where recall is slow or difficult. Ask pupils to identify the principles they use when they add three small numbers and check their work, as in ex. 7.

Refer to page 377 of the text for directions for giving the diagnostic test on the fundamental addition facts provided on page 355.

Key: **2.** The order of adding two numbers does not affect their sum. **3.** 7 + 8 = 7

$+ (7 + 1) = (7 + 7) + 1 = 14 + 1 = 15$. **4.** $9 + 4 = 9 + (1 + 3) = (9 + 1) + 3 = 10 + 3 = 13$. **5.** (a) n; (b) The sum of zero and any number is equal to that number. **6.** (a) The sum of three numbers is not affected by the manner in which they are grouped for addition; (b) $(6 + 9) + 2 = 15 + 2 = 17$, $6 + (9 + 2) = 6 + 11 = 17$; $(5 + 8) + 1 = 13 + 1 = 14$, $5 + (8 + 1) = 5 + 9 = 14$; $(4 + 7) + 9 = 11 + 9 = 20$, $4 + (7 + 9) = 4 + 16 = 20$; $(7 + 6) + 5 = 13 + 5 = 18$, $7 + (6 + 5) = 7 + 11 = 18$. **7.** $(3 + 6) + 2$, associative; $2 + (3 + 6)$, commutative; $2 + (6 + 3)$, commutative; $(2 + 6) + 3$, associative. **8.** (See the answers for page 355.)

Page 4

Aim: To review column addition

Suggestions: Remind the pupils to add *up* each column when finding the sum, and to add *down* each column when checking. To avoid forgetting the carry number, the pupils may write it as a small figure at the bottom of the column to the left. Such carry numbers should, however, be crossed out before an example is checked so that they will not be added twice.

At the bottom of this page you will find the first reference to the practice exercises on pages 358–372. Such references will be found on a number of pages in Chapters 1 and 2. The white number in the red circle refers you to the set of practice exercises which is related to the work of the page. These practice exercises should be assigned in accordance with each pupil's individual need. When a pupil is found especially weak in an operation, extra practice may be distributed over several weeks so that the pupil can be perfecting his skill in this operation while he is studying other topics in his mathematics class. Distributed practice is often more effective than a short period of concentrated practice.

Key: **1.** \$5.10 **2.** \$5.12. **4.** 530. **5.** 607. **6.** 470. **7.** 662. **8.** 562. **9.** 466. **10.** \$325.15. **11.** \$141.54. **12.** \$2386.32. **13.** \$2892.90.

Page 5

Aim: To review the fundamental subtraction facts and the subtraction of whole numbers

Suggestions: When reviewing the fundamental subtraction facts, stress the relationship of subtraction to addition as shown in ex. 1. Directions for giving the diagnostic test on the subtraction facts provided on page 356 will be found on page 377. At this grade level not all the pupils in a given class may be using the same method of subtraction. A pupil should continue to use the method he was originally taught if he finds it satisfactory.

In *American Arithmetics* the *decomposition* or *take-away regroup* method of subtraction is taught. However, because of the interest of teachers in the *additive* method of subtraction, this method is explained in "Suggestions to Teachers" at the end of the texts for Grades 3, 4, 5, and 6. The basic concept of additive subtraction is to find the number which should be added to the subtrahend to give the minuend. Regrouping is not required in this method, and for that reason the

additive method is often easier for pupils to learn than the take-away method. The model example below is explained by the additive method of subtraction.

Think, "8 + 4 = 12." Write 4 in ones place and add 1 ten to 7 tens, making 8 tens. Think, "8 + 2 = 10." Write 2 in tens place and add 1 hundred to 2 hundreds, making 3 hundreds. Think, "3 + 5 = 8." Write 5 in hundreds place. Check by adding 524 to 278 to see if the sum is 802.	802 Minuend 278 Subtrahend 524 Remainder

Key: **1.** 4, 5, 8, 6; 4, 5, 8, 6. **2.** (a) Zero subtracted from any number is equal to the number; (b) Any number subtracted from itself equals zero; (c) 7, 4, 0, 0. **3.** (See the answers for page 356.) **4.** 327; 8841; 5729; 65,648; 70,262; \$467.07. **5.** 295; 3008; 753; 4609; 140,724; \$188.25. **6.** \$20.78.

Pages 6–7

Aim: To review the commutative and associative principles for multiplication, the distributive principle, and the fundamental multiplication facts

Suggestions: Relationships and principles are of great assistance to the pupil in learning the fundamental multiplication facts and in maintaining his knowledge of them. Pupils should be encouraged to discuss different ways they can recall the multiplication facts, particularly those where recall is slow or difficult, and to identify the principles that they use in these different ways.

It will be interesting to show pupils that the product 3×24 can be found in several ways by first expressing 24 as the sum of two numbers and then using the distributive principle. For example:

$$\begin{aligned} 3 \times 24 &= 3 \times (20 + 4) = (3 \times 20) + (3 \times 4) = 60 + 12 = 72 \\ &= 3 \times (10 + 14) = (3 \times 10) + (3 \times 14) = 30 + 42 = 72 \\ &= 3 \times (12 + 12) = (3 \times 12) + (3 \times 12) = 36 + 36 = 72 \end{aligned}$$

Have pupils do this with other similar products.

Note that a diagnostic test on the fundamental multiplication facts is provided on page 355.

Key: **2.** (a) Yes, yes; (b) The order of multiplying two numbers does not affect the product; (c) Since reversing the two factors does not change the product, we can reverse the order of the unknown fact to that of the fact that is known. **3.** 0, 0, 0; Zero multiplied by any number equals zero; 0. **4.** 3, 7, 4, 6; One multiplied by any number equals the number; n. **5.** (See the answers for page 355.) **6.** $(2 \times 4) \times 7 = 8 \times 7 = 56$, $2 \times (4 \times 7) = 2 \times 28 = 56$; $(5 \times 6) \times 8 = 30 \times 8 = 240$, $5 \times (6 \times 8) = 5 \times 48 = 240$; $(2 \times 9) \times 8 = 18 \times 8 = 144$, $2 \times (9 \times 8) = 2 \times 72 = 144$; $(7 \times 5) \times 9 = 35 \times 9 = 315$, $7 \times (5 \times 9) = 7 \times 45 = 315$. **7.** (a) The manner of grouping three numbers for multiplication does not affect the product; (b) $(4 \times 9) \times 7 = 4 \times (9 \times 7)$; $(5 \times 1) \times 8 = 5 \times (1 \times 8)$; $(3 \times 4) \times 11 = 3 \times (4 \times 11)$; $(12 \times 3) \times 4 = 12 \times (3 \times 4)$. **8.** (a) $2 \times 35 = 70$; (b) 3, 7; 5, 11; 2, 3, 4, 6, 8, 12; 3, 5, 9, 15. **9.** $4 \times 9 = (2 \times 2) \times 9 = 2 \times (2 \times 9)$; 6×7

$= (2 \times 3) \times 7 = 2 \times (3 \times 7); 4 \times 8 = (2 \times 2) \times 8 = 2 \times (2 \times 8); 8 \times 9 = (2 \times 4) \times 9 = 2 \times (4 \times 9)$. **10.** $2 \times (4 + 8) = 2 \times 12 = 24, 2 \times (4 + 8) = (2 \times 4) + (2 \times 8) = 8 + 16 = 24$; $7 \times (2 + 3) = 7 \times 5 = 35, 7 \times (2 + 3) = (7 \times 2) + (7 \times 3) = 14 + 21 = 35$; $8 \times (7 + 9) = 8 \times 16 = 128, 8 \times (7 + 9) = (8 \times 7) + (8 \times 9) = 56 + 72 = 128$; $5 \times (6 + 8) = 5 \times 14 = 70, 5 \times (6 + 8) = (5 \times 6) + (5 \times 8) = 30 + 40 = 70$. **11.** (a) $(3 \times 6) + (3 \times 2)$; $(7 \times 3) + (7 \times 5)$; $(4 \times 8) + (4 \times 3)$; $(11 \times 7) + (11 \times 2)$; (b) $3 \times (6 + 2) = 3 \times 8 = 24, 3 \times (6 + 2) = (3 \times 6) + (3 \times 2) = 18 + 6 = 24$; $7 \times (3 + 5) = 7 \times 8 = 56, 7 \times (3 + 5) = (7 \times 3) + (7 \times 5) = 21 + 35 = 56$; $4 \times (8 + 3) = 4 \times 11 = 44, 4 \times (8 + 3) = (4 \times 8) + (4 \times 3) = 32 + 12 = 44$; $11 \times (7 + 2) = 11 \times 9 = 99, 11 \times (7 + 2) = (11 \times 7) + (11 \times 2) = 77 + 22 = 99$. **13.** $4 \times 16 = 4 \times (10 + 6) = (4 \times 10) + (4 \times 6) = 40 + 24 = 64$.

Page 8

Aim: To review multiplication of whole numbers

Suggestions: Review the meaning of the words *multiplier, multiplicand, product,* and *partial product.* When multiplying two numbers the pupil should write each partial product in such a position that its right-hand digit will come directly below the digit by which he is multiplying. For example, in ex. 1 on this page, the partial product 2898 is so written that the right-hand 8 comes directly under the 6 of the multiplier.

Key: **3.** 93,590; 51,968; 197,467; 164,610. **4.** 395,166; 129,900; 266,256; 426,755. **5.** 259,050; 573,216; 716,800; 131,250. **6.** 252,004; 80,400; 42,960; 249,165. **7.** 2 × \$.89 = \$1.78; 5 × \$.75 = \$3.75; \$1.78 + \$3.75 = \$5.53. **8.** \$121.88. **9.** (1) 125 × 75 lb. = 9375 lb.; (2) 5 T. = 10,000 lb., 10,000 lb. − 9375 lb. = 625 lb. less than 5 tons.

Page 9

Aim: To review the relationship of division to multiplication, the fundamental division facts, and division with a 1-figure divisor

Suggestions: Refer to page 377 of the text for directions for giving the diagnostic test on the fundamental division facts provided on page 357. Make sure the pupils understand the relationship of the division facts to the multiplication facts as explained in ex. 1. Since zero cannot be used as a divisor, ten of the multiplication facts do not have corresponding division facts; so there are only 90 division facts. Make sure also that the pupils understand the generalizations in ex. 2.

Many pupils in Grade 8 should be able to use, when dividing by a 1-figure number, the short form of division, in which all the steps are done mentally. You may find, however, that the short form of division is difficult for some of your pupils, if the divisor is one of the larger numbers 6, 7, 8, and 9, but that these pupils do such mental division easily when the divisor is one of the smaller numbers 2, 3, 4, and 5. In such cases you may prefer to have these pupils do the work mentally only when the divisors are 2, 3, 4, and 5.

The pupils should be familiar with the words *dividend, divisor, quotient,* and *remainder.* They should understand that the divisor and the quotient are factors of the dividend when there is no remainder.

Key: **1.** 6, 6, 7, 6; 6, 6, 7, 6. **2.** (a) Zero divided by any number other than zero is equal to zero; (b) any number divided by one is equal to that number; (c) any number, other than zero, divided by itself is equal to one; (d) 0, 0, 7, 5, 1, 1. **3.** (See the answers for page 357.) **4.** 7; 5; 21; $40\frac{1}{3}$. **5.** 120; 925; 857 R6; 982 R2; 9182. **6.** 24 R1; 740 R4; 709; 739; 3107. **7.** 43; 687 R4; 603; 1152; 8018. **8.** 132 R1; 582 R6; 552 R2; 375 R4; 11,854.

Page 10

Aim: To review casting out nines, and to develop skill in finding residues

Suggestions: After reviewing the meaning and procedure of casting out 9's, encourage pupils to discover and take advantage of short ways of finding residues. Sometimes one digit in a number can be broken into two or more addends that will make groups of nine in the sum of the digits. When finding the sum of the digits in 7884, you can think $7 + 8 + 8 + (2 + 1 + 1)$, which equals $(7 + 2) + (8 + 1) + (8 + 1)$. Thus the residue is seen to be 0 without actually finding the sum of the digits. When such short ways are used mentally, care must be exercised to avoid omitting a digit or adding a digit twice.

Key: **1.** 0, 1, 1, 0, 1, 0, 1; 2, 3, 1, 4, 0, 2, 3. **2.** 1 R1; 11 R1; 2 R2; 22 R2; 3 R3; 33 R3; 33 R4; 33 R5; 35 R5; 33 R6. **3.** 5; 5; 8; 8; 0; 0; 4; 4; 7; 7. **4.** 3; 7; 8; 5; 8; 5; 6; 7. **5.** 3; 5; 0; 5; 5; 6; 0. **6.** 3; 2; 2; 5; 3; 3; 1.

Page 11

Aim: To reteach casting out 9's as another method of checking multiplication

Suggestions: There are three methods of checking exercises in multiplication. The first and probably the most used method is to go over the work a second time to see that it is correct. The second method is to interchange the multiplicand and the multiplier and multiply again, as explained in ex. 1 on page 8. This check is an excellent one although it requires more time than the first method. The third method is to use casting out 9's as described on page 11 of the text. Casting out 9's is the quickest method of checking multiplication in cases where the multiplier contains two or more digits. In exercises having a 1-digit multiplier, the most satisfactory check is the first one described above; namely, to go over the work again. Casting out 9's, which is also called "the check of 9's," is an extremely useful and rapid check and is recommended for use, particularly in the longer exercises in multiplication. On rare occasions the check of 9's fails to detect an error in a multiplication example, as explained in ex. 2 on page 11. In this case a partial product is misplaced, and the final product, therefore, is incorrect. The check of 9's, however, did not detect this mis-

take. In fact, whenever a partial product is misplaced, the check of 9's will not disclose the error. This weakness on the part of the 9's check does not disqualify it from use, but it does show the importance of seeing that each partial product is in the correct place before applying the check of 9's.

You must remember that no method of checking is perfect. If you check multiplication by going over the work again, or by reversing the factors, you have no guarantee that the result is correct. You merely have a reason for believing that the answer is *probably* correct. In everyday life, one of the favorite methods of checking computations of all kinds is *to go over the work again;* but in such cases, if a mistake was made in the original computation, it may easily be repeated in the checking. Casting out 9's is a dependable and rapid check, not only for multiplication, but also for division. For these reasons it is strongly recommended for teaching in the seventh and eighth grades as a check for these operations.

Casting out 9's is not recommended for addition and subtraction, since it requires more time to check either of these operations by 9's than by the usual methods of checking addition and subtraction.

Key: 3. 30,498; 41,385; 330,906; 170,848. **4.** 57,036; 25,714; 342,333; 570,024.

Page 12

Aim: To reteach long division with 2-figure and 3-figure divisors when the second figure of the divisor is 1, 2, 3, 4, or 5

Suggestions: Long division is the most difficult operation with whole numbers; hence, this topic should be reviewed carefully. On page 12 the work in long division is confined to 2-figure and 3-figure divisors, in which the second figure of the divisor is 1, 2, 3, 4, or 5. With such divisors the quotient figures should always be estimated by dividing by the *first figure* of the divisor, as stated in the rule in the middle of the page. According to this rule, if the divisor were 41, 42, 43, 44, or 45, you would divide by 4 to estimate each quotient figure. Likewise, if the divisor were 418, 428, 438, 448, or 458, you would divide by 4 to estimate each quotient figure. By following this rule, along with the rule given on page 13, the necessity of correcting quotient figures will be reduced to a minimum.

In the work on page 12, emphasize the fact that the first quotient figure must always be written over the last figure of the first partial dividend. In the model example on page 12, the first figure of the quotient, which is 4, is written over the 9 of 149. In this case 149 is the first partial dividend. When the quotient figure is estimated by the rule given on page 12, the estimated quotient figure in some cases is *too large.* Show the pupils that *the quotient figure is too large if the product of the quotient figure and the divisor is greater than the partial dividend.* In such cases the quotient figure is reduced by 1. Only in very rare situations will it be necessary to correct the quotient figure a second time. It is also important to remind the pupils that when they have the correct quotient

figure, the remainder will always be *less* than the divisor. Caution the pupils always to compare the remainder with the divisor.

Key: **2.** $239\frac{5}{83}$; $534\frac{3}{44}$; $152\frac{283}{318}$; 253. **3.** $367\frac{5}{54}$; $634\frac{7}{22}$; 84; 236. **4.** $645\frac{2}{31}$; $730\frac{24}{35}$; $93\frac{3}{8}$; $221\frac{4}{91}$.

Page 13

Aim: To reteach long division with 2-figure and 3-figure divisors when the second figure of the divisor is 6, 7, 8, or 9

Suggestions: The rule on page 13 states that when the second figure of the divisor is 6, 7, 8, or 9, you should divide by 1 *more than the first figure* of the divisor to estimate each quotient figure. For example, if the divisor is 56, 57, 58, or 59, you divide by 6 to estimate each quotient figure. Likewise, if the divisor is 461, 471, 481, or 491, you divide by 5 to estimate each quotient figure. When the quotient figure is estimated by this rule, the estimated quotient figure in some cases is *too small*. Show the pupils that *the quotient figure is too small if the remainder is greater than the divisor*. In such cases the quotient figure is increased by 1.

By following the rule given on this page for its group of divisors, together with the rule in the middle of page 12 for its particular group of divisors, you will obtain the correct quotient figure *on first trial* in 80% of all cases. In the remaining 20% of the cases, in general, only one correction of the quotient figure will be necessary. Only in very rare situations will it be necessary to make two corrections of the quotient figure. The method of long division here described was first taught in this series of books in *American Arithmetic, Grade 5*. By following this method, the procedure so often observed in schoolrooms, in which the pupil tries first one quotient figure and then another until he finds the correct one, is avoided. If some of your pupils have not previously followed the rules for estimating quotient figures given on pages 12 and 13 of the text, they will need additional practice in long division which will be found on pages 360–361.

Key: **2.** 304 R22; 4006 R15; 168 R218; 235 R135. **3.** 750; 7030 R11; 115 R8; 815. **4.** 853 R7; 2001 R31; 26; 482. **5.** 2831; 2713 R32; 29; 306. **6.** 1843 R30; 4943 R12; 68 R24; 501. **7.** 842; 6791 R8; 86 R34; 745 R50.

Page 14

Aim: To reteach casting out 9's as another method of checking division

Suggestions: The check of 9's is here applied to long division. The check numbers, of course, are obtained in each case by the method described on page 10 of the text.

When the check figure of the divisor is zero and there is no remainder, as illustrated at the right, you can see that *any* number will check as the quotient for such a division example, for 0 times any check figure is 0. You might, therefore, conclude that casting out 9's is of no value in such examples.

```
     Correct
       (7)
(0)    826          9's Check
27)22302(0)             7
    216                 0
    ---                ---
     70                (0)
     54
    ---
    162
    162
    ---
```

However, if a pupil makes an error as he works such an example, he is almost certain to bring in a remainder and thus obtain different final check numbers, as shown at the right. Therefore an error in this kind of example will usually be detected by the check of casting out 9's.

The check of 9's is strongly recommended for division in Grades 7 and 8.

Incorrect

```
        (8)
(0)     863        9's Check
27)22302(0)            8
   206                 0
   ---                ---
    170                0
    162                1
    ---               ---
     82               (1)
     81
     --
      1
     (1)
```

Key: 2.

```
(2)      (7)
      (2) 934        9's Check
   29)27086(5)          7
      261               2
      ---              ---
        98             14(5)
        87
        --
       116
       116
       ---
```

3. 548 R6; 533; 781 R200. **4.** 652 R8; 492 R311; 291 R225. **5.** 609; 654 R14; 370 R375.

Pages 15–16

Aim: To review addition and subtraction of fractions

Suggestions: A basic principle of fractions is stated and explained in ex. 1. This principle shows how to change a given fraction to another fraction having the same value. When $\frac{2}{3}$ is changed to $\frac{8}{12}$, there is no change in value, but there is a change in the form of the fraction. Likewise, when $\frac{15}{20}$ is changed to $\frac{3}{4}$, there is no change in value. Because of its usefulness in a wide variety of situations, including algebra, this basic principle is very important.

In the addition and subtraction of fractions it is necessary to emphasize that fractions can be added or subtracted only if they have like denominators. If the fractions to be added or subtracted have unlike denominators, they must first be changed to fractions having the same denominators. This change is made by using the principle of fractions stated in ex. 1.

In the first subtraction example of ex. 4 on page 15, the mixed number $6\frac{1}{8}$ must be changed to $5\frac{9}{8}$. This change can be explained as follows: $6\frac{1}{8}$ means 6 *ones* and 1 *eighth*, if 1 of the *ones* is changed to 8 *eighths*, you then have 5 *ones* and 9 *eighths*. Thus, $6\frac{1}{8} = 5\frac{9}{8}$.

The model example on page 16 should be carefully studied by the pupils, since it explains an efficient method for finding the least common denominator of two or more fractions in the more difficult cases. This method is to be used in examples where the least common denominator is not easily found by inspection.

It should not be necessary for all pupils to work all the examples. However, some pupils may need careful reteaching of these two operations. Additional

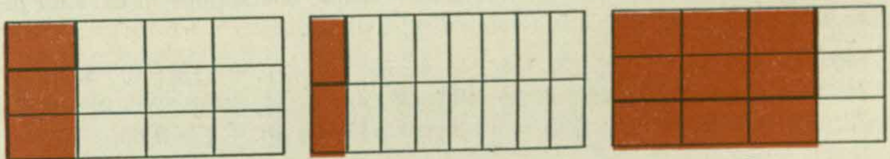
practice exercises are provided on pages 361–362. Those exercises can be used when pages 15 and 16 are first studied and from time to time during the year.

Key: Page 15 (Addition) **5.** $10\frac{3}{8}$; $17\frac{1}{4}$; $10\frac{1}{3}$; $10\frac{3}{4}$; $11\frac{13}{16}$; $11\frac{7}{12}$; $4\frac{1}{16}$. **6.** $6\frac{1}{2}$; $14\frac{5}{6}$; $12\frac{7}{8}$; $15\frac{5}{12}$; $7\frac{15}{16}$; $9\frac{1}{2}$; $7\frac{1}{5}$. **7.** 18; 18; $12\frac{1}{2}$; 20; $17\frac{7}{12}$; $14\frac{1}{16}$; $15\frac{1}{10}$. (Subtraction) **5.** $1\frac{5}{8}$; $1\frac{3}{4}$; $5\frac{1}{3}$; $1\frac{1}{2}$; $4\frac{9}{16}$; $2\frac{3}{4}$; $\frac{15}{16}$. **6.** $2\frac{5}{6}$; $1\frac{5}{6}$; $1\frac{5}{8}$; $4\frac{1}{12}$; $2\frac{9}{16}$; $5\frac{7}{10}$; $1\frac{4}{5}$.
Page 16 (Addition) **2.** $9\frac{13}{24}$; $10\frac{1}{12}$; $20\frac{7}{10}$; $13\frac{7}{24}$; $12\frac{8}{15}$; $17\frac{19}{48}$; $11\frac{17}{48}$. **3.** $8\frac{5}{12}$; $14\frac{5}{24}$; $9\frac{7}{12}$; $11\frac{19}{24}$; $12\frac{29}{40}$; $11\frac{17}{20}$; $15\frac{7}{24}$. **4.** $7\frac{19}{24}$; $5\frac{7}{12}$; $12\frac{7}{24}$; $13\frac{11}{15}$; $12\frac{7}{15}$; $7\frac{19}{20}$; $6\frac{17}{24}$. **5.** $13\frac{13}{24}$; $14\frac{11}{12}$; $2\frac{11}{24}$; $12\frac{11}{24}$; $11\frac{17}{20}$; $5\frac{13}{48}$; $13\frac{5}{24}$. (Subtraction) **3.** $6\frac{1}{12}$; $4\frac{13}{24}$; $3\frac{11}{12}$; $2\frac{23}{24}$; $4\frac{21}{40}$; $1\frac{13}{20}$; $4\frac{13}{24}$. **4.** $2\frac{13}{24}$; $2\frac{1}{12}$; $1\frac{23}{24}$; $6\frac{1}{15}$; $3\frac{13}{15}$; $5\frac{9}{20}$; $2\frac{11}{24}$. **5.** $2\frac{19}{24}$; $4\frac{7}{12}$; $\frac{19}{24}$; $3\frac{7}{24}$; $6\frac{7}{20}$; $3\frac{5}{48}$; $1\frac{1}{24}$.

Page 17

Aim: To review the multiplication of fractions

Suggestions: When the multiplier is a whole number, multiplication means repeated addition. Thus, 3×4 means $4 + 4 + 4$, and $3 \times \frac{1}{2}$ means $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$. But when the multiplier is a fraction, multiplication means finding a fractional part of the multiplicand. Thus, $\frac{1}{2} \times 8$ means $\frac{1}{2}$ *of* 8 and $\frac{1}{3} \times \frac{1}{4}$ means $\frac{1}{3}$ *of* $\frac{1}{4}$. The symbol $\times$ means *of* when the multiplier is a fraction. The diagrams below can be used to explain the multiplication of two fractions. *A* illustrates $\frac{1}{3}$ of $\frac{1}{4}$ ($\frac{1}{3} \times \frac{1}{4}$); *B* illustrates $\frac{1}{2}$ of $\frac{1}{8}$ ($\frac{1}{2} \times \frac{1}{8}$); and *C* illustrates $\frac{3}{3}$ of $\frac{3}{4}$ ($\frac{3}{3} \times \frac{3}{4}$). The explanation leads up to the rule: *To multiply a fraction by a fraction, first multiply the numerators; then multiply the denominators.*



A B C

The work of multiplying fractions can be simplified by using *cancellation*, as shown in the model examples on page 17. When you cancel, you divide the numerator of one fraction and the denominator of the other fraction by the same number. You have really divided the numerator and the denominator of the final answer by the same number and applied the basic principle of fractions stated in ex. 1 on page 15 of the text. If pupils understand the mathematical principle which justifies the process of cancellation, they are far less likely to try to use cancellation in situations where it is incorrect to do so. It should be emphasized that the kind of cancellation used in the multiplication of fractions cannot be used in the addition and subtraction of fractions. For example, in $\frac{3}{4} + \frac{2}{3}$ the 3's cannot be cancelled; likewise, 2 cannot be cancelled from the numerator 2 and the denominator 4.

When a mixed number is multiplied by a fraction or a mixed number, it is usually best first to change each mixed number to an improper fraction. When

a whole number such as 12 is multiplied by a mixed number such as $3\frac{1}{2}$, the number 12 can be multiplied by $\frac{1}{2}$ and by 3; and then these results can be added, getting $6 + 36$, or 42. Or $3\frac{1}{2}$ can be changed to the mixed number $\frac{7}{2}$, and then 12 can be multiplied by $\frac{7}{2}$. In most of the examples on page 17, the pupils should first change each mixed number to an improper fraction.

Key: **2.** $\frac{1}{4}$; 28; $82\frac{1}{2}$; 6. **3.** 6; $1\frac{1}{6}$; $22\frac{1}{2}$; $9\frac{1}{10}$. **4.** $3\frac{3}{8}$; 27; $7\frac{1}{2}$; $4\frac{3}{8}$. **5.** $\frac{1}{10}$; $1\frac{1}{4}$; $20\frac{1}{4}$; $1\frac{1}{2}$. **6.** $\frac{1}{3}$; $\frac{3}{8}$; 6; $4\frac{2}{3}$. **7.** $7\frac{1}{2}$; 5; 12; 15. **8.** $\frac{1}{3} \times 7\frac{1}{2}$ bu. $= 2\frac{1}{2}$ bu.

Page 18

Aim: To review division by a fraction

Suggestions: Make sure that the pupils understand what division by a fraction means. They should understand that $6 \div \frac{3}{4}$ asks the question: "How many $\frac{3}{4}$'s are there in 6?" The result, which is 8, is obtained by using the rule: *To divide by a fraction, invert the divisor and multiply.* This result can be verified by drawing a line 6 in. long and then marking off on this line lengths that equal $\frac{3}{4}$ in. The line will contain 8 of the $\frac{3}{4}$-inch lengths. It is suggested that the pupils use diagrams to verify the answers to several examples of this type in order to develop a thorough understanding of the meaning of the process.

The inversion of the divisor can be explained this way:

$$6 \div \frac{3}{4} = \frac{6}{\frac{3}{4}} = \frac{6 \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}} = \frac{6 \times \frac{4}{3}}{1} = 6 \times \frac{4}{3}$$

This explanation makes use of the basic principle of fractions in ex. 1 on page 15, and the concept that a fraction indicates division.

Key: **2.** $5\frac{1}{4}$; 54; 8; $6\frac{3}{10}$. **3.** $1\frac{1}{3}$; 24; 4; $5\frac{1}{4}$. **4.** $\frac{8}{15}$; 80; $13\frac{1}{3}$; 6. **5.** 5; $17\frac{3}{5}$; $7\frac{1}{2}$; $5\frac{5}{6}$. **6.** $\frac{3}{5}$; 24; 9; $2\frac{3}{4}$. **7.** $2\frac{1}{2}$; $21\frac{1}{3}$; $3\frac{3}{4}$; 6. **8.** $4\frac{4}{5}$; $22\frac{1}{2}$; $12\frac{1}{2}$; $2\frac{1}{4}$. **9.** $1\frac{7}{20}$; 72; $2\frac{2}{3}$; 6. **10.** $4\frac{1}{2}$ lb. $\div \frac{3}{8}$ lb. $= 12$ (bags). **11.** $6\frac{3}{4}$ mi. $\div \frac{3}{4} = 9$ mi.

Page 19

Aim: To review division by whole numbers and mixed numbers

Suggestions: Remind the pupils that when a fraction or a mixed number is divided by a whole number, such as 7, the divisor is first written as $\frac{7}{1}$; and that when $\frac{7}{1}$ is inverted, it becomes $\frac{1}{7}$.

In the exercises on this page the pupils should first change the mixed numbers to improper fractions.

Key: **2.** 30; $\frac{2}{3}$; $1\frac{1}{4}$; $1\frac{2}{7}$. **3.** 8; $\frac{2}{9}$; $1\frac{1}{5}$; $1\frac{1}{5}$. **4.** $10\frac{2}{3}$; $\frac{1}{10}$; $2\frac{1}{2}$; $1\frac{1}{2}$. **5.** $7\frac{1}{2}$; $\frac{10}{21}$; $\frac{2}{5}$; $1\frac{1}{3}$. **6.** 15; $\frac{1}{2}$; $1\frac{7}{8}$; $\frac{4}{5}$. **7.** $2\frac{2}{3}$; $\frac{1}{6}$; $\frac{4}{5}$; $\frac{2}{5}$. **8.** 12; $\frac{4}{15}$; $2\frac{2}{5}$; 4.

Page 20

Aim: To review place value and the meaning and use of exponents

Suggestions: Pupils should have considerable practice in comparing the values represented by the different digits in a number. Point out the difference between

the size of a digit and the value it represents because of its place value.

As numbers become larger, the use of exponents to analyze them becomes more advantageous.

Key: **3.** $10 \times 10 \times 10 \times 10 \times 10$; 8×8; $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$; $5 \times 5 \times 5$; $4 \times 4 \times 4 \times 4$. **5.** $729 = (7 \times 10^2) + (2 \times 10^1) + (9 \times 1)$; $4956 = (4 \times 10^3) + (9 \times 10^2) + (5 \times 10^1) + (6 \times 1)$; $8253 = (8 \times 10^3) + (2 \times 10^2) + (5 \times 10^1) + (3 \times 1)$; $46{,}824 = (4 \times 10^4) + (6 \times 10^3) + (8 \times 10^2) + (2 \times 10^1) + (4 \times 1)$.

Page 21

Aim: To review the addition and subtraction of decimals

Suggestions: When decimals that are to be added or subtracted *represent measurements,* they should be given with the same number of decimal places. If the decimal 38.6 represents a length in inches correct to the nearest tenth of an inch, and 19.83 represents a length in inches correct to the nearest hundredth of an inch, their sum can be correct only to the nearest tenth of an inch; for in this situation the second decimal place of 38.6 cannot be assumed to be zero.

The sum or difference of \$8 and \$4.39 can, however, be correct to the nearest cent; for \$8 can usually be assumed to be \$8.00.

Key: **1.** 15.8 mi.

Pages 22–23

Aim: To review writing numbers in systems having bases other than ten; to review changing numbers to base ten

Suggestions: The experience of using bases other than ten to write numbers will increase pupils' understanding of our decimal system. Any difficulty in understanding the meaning of a numeral should be resolved by the use of objects or diagrams. Counting and grouping objects to show the meaning of a numeral will be very helpful to pupils.

Key: **1.** 2 hundreds, 3 hundredths, 8 thousands. **2.** 3 hundreds, 7 tens, 6 ones; 2 thousands, 4 hundreds, 8 tens, 6 ones; 4 hundreds, 7 tens, 2 ones, 6 tenths, 8 hundredths; 4 ten thousands; 5 hundreds, 7 tens, 2 hundredths, 9 thousandths.

4.

Base Ten	Base Five	Base Ten	Base Five	Base Ten	Base Five	Base Ten	Base Five
1	1	8	13	15	30	22	42
2	2	9	14	16	31	23	43
3	3	10	20	17	32	24	44
4	4	11	21	18	33	25	100
5	10	12	22	19	34	26	101
6	11	13	23	20	40	27	102
7	12	14	24	21	41	28	103

Base Ten	Base Five	Base Ten	Base Five	Base Ten	Base Five	Base Ten	Base Five
29	104	35	120	41	131	47	142
30	110	36	121	42	132	48	143
31	111	37	122	43	133	49	144
32	112	38	123	44	134	50	200
33	113	39	124	45	140	51	201
34	114	40	130	46	141	52	202

5.

Base Ten	Base Five	Base Ten	Base Five	Base Ten	Base Five	Base Ten	Base Five
5	10	80	310	155	1110	230	1410
10	20	85	320	160	1120	235	1420
15	30	90	330	165	1130	240	1430
20	40	95	340	170	1140	245	1440
25	100	100	400	175	1200	250	2000
30	110	105	410	180	1210	255	2010
35	120	110	420	185	1220	260	2020
40	130	115	430	190	1230	265	2030
45	140	120	440	195	1240	270	2040
50	200	125	1000	200	1300	275	2100
55	210	130	1010	205	1310	280	2110
60	220	135	1020	210	1320	285	2120
65	230	140	1030	215	1330	290	2130
70	240	145	1040	220	1340	295	2140
75	300	150	1100	225	1400	300	2200

7. 4 fives and 2 ones; 4 fives and 4 ones; 3 twenty-fives, 2 fives, 4 ones; 2 twenty-fives, 0 fives, 3 ones; 1 one hundred twenty-five, 4 twenty-fives, 0 fives, 0 ones; 2 one hundred twenty-fives, 3 twenty-fives, 1 five, 4 ones. **8.** 1 seven and 2 ones; 7 eights and 3 ones; 1 sixteen, 1 four, 3 ones; 2 hundreds, 9 tens, 5 ones; 1 eight, 1 four, 1 two, 1 one; 4 two hundred sixteens, 2 thirty-sixes, 5 sixes, 3 ones. **9.** 2 nines and 7 ones; 3 fours and 3 ones; 4 forty-nines, 0 sevens, 4 ones; 5 sixty-fours, 2 eights, 7 ones; 9 thousands, 7 hundreds, 4 tens, 6 ones; 1 eight, 1 four, 0 twos, 0 ones. **11.** 22; 24; 89; 53; 225; 334. **12.** 59; 25; 23; 200; 969; 12. **13.** 3; 4 (even); 5; 6 (even); 7; 57; 9; 13.

Pages 24–25

Aim: To show how to add, subtract, and multiply numbers in bases other than ten without changing these numbers to base ten

Suggestions: It is not the purpose of these pages to make pupils rapid computers with numbers written in bases other than ten. The purpose is to increase the pupil's understanding of the operations addition, subtraction, and multiplication by making a careful analysis of these operations with such numbers. Pupils

should, therefore, not change numbers to base ten before performing the required operation. They may, however, occasionally check an example by changing the given numbers and the answer to base ten.

Pupils will enjoy this work if they understand it thoroughly. Therefore, before assigning independent work, give careful explanations of model examples. You may wish to make up additional examples for practice. Remember when writing a number that a digit equal to or larger than the base cannot be used.

In ex. 8 on page 25, have pupils check the products in their chart by using the commutative, associative, and distributive principles. For example: $4 \times 4 = 4 \times (2 + 2) = (4 \times 2) + (4 \times 2) = 10_8 + 10_8 = 20_8$.

Key: 1. 11_5; 7_8; 11_7; 13_4; 13_6. **2.** 213_5; 225_6. **3.** 111_5; 63_8; 1011_4; 5066_8; 1010_2; 5606_7; 11122_6. **4.** 2; 6; 5; 2; 7. **6.** 22_7; 25_8; 14_7; 34_8; 10_2; 203_4; 255_8. **7.** $5 \times 6 = 6 + 6 + 6 + 6 + 6 = 6 + (2 + 4) + (4 + 2) + 6 + 6 = (6 + 2) + (4 + 4) + (2 + 6) + 6 = 36_8$. **8.** Row 0: 0, 0, 0; row 1: 2, 4, 6; row 2: 0, 6, 12, 16; row 3: 3, 14, 22; row 4: 0, 10, 24, 34; row 5: 5, 17, 36; row 6: 0, 14, 30, 52; row 7: 7, 25, 43, 52. **10.** 672_8; 1224_8; 1414_8; 12004_8.

11. (a)

BASE FIVE					
x	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

(b) 341_5; 1344_5; 1122_5; 11231_5.

Page 26

Aim: To review the multiplication of decimals

Suggestion: To give the pupils additional understanding of the rule stated on this page, you may wish to verify it by working some examples in the following manner: $.3 \times .4 = \frac{3}{10} \times \frac{4}{10} = \frac{12}{100} = .12$.

Key: 2. \$13.41. **3.** 110.7; 128.8; .6392; 1.1256. **4.** .402; 69.72; 4.5927; .23306. **5.** 45.75; 131.4; 12.212; .015625. **6.** 4.776; 17.08; .0168; .06888.

Page 27

Aim: To review the division of a decimal by a whole number

Suggestions: Some pupils may need a re-explanation of why the decimal point in the quotient is placed directly above the decimal point in the dividend when the divisor is a whole number. Review the meaning of division with an example such as $4\overline{)26.8}$, and then ask the pupils to estimate the quotient. This estimate will show where the decimal point in the quotient must be placed.

Key: **2.** 13.7; .093; .52; 15.8; 139.5; 39.625. **3.** 10.4; 1.29; 19.3; 12.5; 24.5; 122.4. **4.** .064; .042; .54; 8.125; 74.25; 52.5. **5.** .878; 37.25; 12.76; 6.5; 47.875; 98.75. **6.** 18.25 mi. **7.** 8.24 mi. **8.** \$.9375 or \$.93$\frac{3}{4}$.

Page 28

Aim: To reteach changing fractions to decimals

Suggestions: One of the meanings of a fraction is that it indicates division; that is, $\frac{5}{8}$ means $5 \div 8$. This meaning of a fraction is used when you change fractions to decimals. In some cases the division will terminate; but in other cases the division will never terminate, no matter how far it is carried. When the division does not terminate, you *always* have a *repeating* decimal if the division is carried out far enough; that is, the digits will start to repeat either singly or by groups. For example, in $\frac{1}{3} = .333333 \ldots$ the 3 begins to repeat at once. In $\frac{4}{11} = .363636 \ldots$ the pair of digits 36 repeat, and in $\frac{3}{7} = .428571428571 \ldots$ the group of digits 428571 repeat. Some of your pupils will enjoy the study of repeating decimals.

When the quotient is to be found to the nearest tenth, hundredth, or thousandth, the pupils should understand that the division must be carried out one additional place. The result is then rounded off, using the rule given at the bottom of page 28.

Key: **4.** .16; .17.

Pages 29–30

Aim: To review division by a decimal

Suggestions: Ex. 5 on page 29 gives an important principle which states that both the divisor and the dividend can be multiplied by the same number without changing the value of the quotient. This principle corresponds to the fundamental principle of fractions which states that you can multiply both numerator and denominator of a fraction by the same number without changing the value of the fraction. In a division example the dividend and the divisor correspond to the numerator and the denominator of a fraction.

If the divisor is a decimal, the first step in working the example is to change the divisor to a whole number by multiplying both the divisor and the dividend by 10, 100, or 1000.

In ex. 1 on page 30 two methods of handling the decimal points in a division example when the divisor is a decimal are illustrated. In each of these methods the divisor is changed to a whole number by multiplying both the divisor and the dividend by 100. In Method A the example can be rewritten with the divisor as a whole number and the decimal point of the dividend moved 2 places to the right; or the decimal points in the original example can be erased and moved 2 places to the right, without, of course, rewriting the decimal point in the divisor. In Method B the original decimal points are kept, and the new positions of the decimal points are indicated by carets. When Method B is

used, it is important to emphasize the fact that the decimal point of the quotient is placed directly above the *caret* in the dividend.

Key: Page 29 **2.** 63, 48.7, 3.32, \$65.00, \$278.00, \$3.40, 8, .635; 630, 487, 33.2, \$650, \$2780, \$34, 80, 6.35; 6300, 4870, 332, \$6500, \$27,800, \$340, 800, 63.5. **4.** \$2.50, .27, .032, \$12.60, .53, \$245.80, .07, .005; \$.25, .027, .0032, \$1.26, .053, \$24.58, .007, .005; \$.025, .0027, .00032, \$.126, .0053, \$2.458, .0007, .00005. **7.** 342; 105; 3.4; 55.9.

Page 30 **2.** 25.90; 59.85; 2.57; 21.63. **3.** 19.79; 433.72; 152.13; 1.54. **4.** .08; 728.42; 6.82; 26.22.

Page 31

Aim: To present an interesting project which uses large numbers and requires the rounding off of large numbers

Key: **1.** (1) 744,834,000 + 829,874,000 + 1,010,270,000 + 1,022,139,000 + 1,019,164,000 = 4,626,281,000 (bbl.); 4,626,281,000 bbl. ÷ 5 = 925,256,200 bbl. (Texas); (2) 332,942,000 + 327,607,000 + 354,561,000 + 359,450,000 + 365,085,000 = 1,739,645,000 (bbl.); 1,739,645,000 bbl. ÷ 5 = 347,929,000 bbl. (California). **2.** Round off 925,256,200 to 925,300,000, and 347,929,000 to 347,900,000; 925,300,000 ÷ 347,900,000 = 2.65, or about 3 (times). **3.** 974,275,000 + 355,865,000 + 185,851,000 = 1,515,991,000 (bbl.); 2,314,988,000 bbl. − 1,515,991,000 bbl. = 798,997,000 bbl. **4.** 2,314,988,000 rounds off to 2315 million; 5,006,205,000 rounds off to 5006 million; 2315 ÷ 5006 = .462, or .46. **5.** \$6,424,930,000 rounds off to \$6425 million; \$6425 ÷ 2315 = \$2.775, which is called \$2.78.

Pages 32–35

Aim: To give an introduction to the metric system and its uses

The Metric System. The metric system is now used by most countries of Europe, Asia, and South America. The only prominent countries not using this system are England, Canada, and the United States; yet even in these countries the metric system is used in scientific work and in manufacturing many types of goods that are to be sold in countries using the metric system.

It is important to know the metric system as a matter of general information, since metric measures are often found in news items in our papers. For example, if a French airplane establishes a new speed record, that speed may be given in American papers as so many *kilometers* per hour rather than as so many miles per hour. Metric terms are used also in connection with the radio, such as a *wave length* of 422 meters. In the Olympic games, where contestants from many parts of the world participate, almost all distances are given in metric measures.

Suggestions: On page 32 it is important at the beginning that the pupils memorize the prefixes given in ex. 3. These prefixes and their meanings are as follows: *deci* = $\frac{1}{10}$, *centi* = $\frac{1}{100}$, *milli* = $\frac{1}{1000}$, *deka* = 10, *hekto* = 100, and *kilo* = 1000.

An easy way to remember that *centi* means $\frac{1}{100}$ is to note its resemblance to the word *cent*, which is $\frac{1}{100}$ of a dollar. Likewise, *milli*, which means $\frac{1}{1000}$, is like *mill*, which is $\frac{1}{1000}$ of a dollar. To recall that *deci* means $\frac{1}{10}$ and *deka* means 10, it is helpful to associate these prefixes with *dime*, which means $\frac{1}{10}$ of a dollar, and to notice that all three of these expressions begin with the letter *d*. To remember that *hekto* means 100, notice that both *hekto* and *hundred* begin with the letter *h*. It is interesting to know that the prefixes *deci*, *centi*, and *milli* are derived from the Latin, while *deka*, *hekto*, and *kilo* are Greek. These prefixes always represent the same numerical values, whether they are prefixed to the word *meter*, *liter*, or *gram*. See page 35 of the text.

On page 33 the equivalents given in ex. 1 are the ones ordinarily used in exercises when approximate results are satisfactory. In ex. 2 and 6, use 1.1 yd. for 1 meter. On page 35 the equivalents of the kilogram and the liter, given in ex. 1, are sufficiently accurate for everyday purposes.

You can find in *The World Almanac* additional world records in metric units to supplement the work of page 32.

Key: Page 32 **4.** 10 mm. = 1 cm.
10 cm. = 1 dm.
10 dm. = 1 m.
1 m. = .1 dekameter
1 dekameter = .1 hm.
1 hm. = .1 km.

5. 10 cm.; 100 cm.; 10 mm.

Page 33 **2.** 121 yd.; 1760 yd. **3.** (1) 3000 m. = 3 km., 3 × .6 mi. = 1.8 mi.; (2) 10,000 m. = 10 km., 10 × .6 mi. = 6 mi.; (3) 500,000 m. = 50 km., 50 × .6 mi. = 30 mi. **4.** (a) 4.70 × 39.37 in. = 185.039 in.; 185.039 in. ÷ 12 in. = 15 ft. 5.039 in., which is called 15 ft. 5.0 in.; (b) 2.16 × 39.37 in. = 85.0392 in.; 85.0392 in. ÷ 12 in. = 7 ft. 1.0392 in., which is called 7 ft. 1.0 in.; (c) 85.71 × 39.37 in. = 3374.4027 in.; 3374.4027 in. ÷ 12 in. = 281 ft. 2.4027 in., which is called 281 ft. 2.4 in.; (d) 16.81 × 39.37 in. = 661.8097 in.; 661.8097 in. ÷ 12 in. = 55 ft. 1.8097 in., which is called 55 ft. 1.8 in.; (e) 59.18 × 39.37 in. = 2329.9166 in.; 2329.9166 in. ÷ 12 in. = 194 ft. 1.9166 in., which is called 194 ft. 1.9 in.; (f) 8.12 × 39.37 in. = 319.6844 in.; 319.6844 in. ÷ 12 in. = 26 ft. 7.6844 in., which is called 26 ft. 7.7 in. **5.** 3420 × .6 mi. = 2052 mi. **6.** 25 × 1.1 yd. = 27.5 yd., or $27\frac{1}{2}$ yd.

Page 34 **1.** 46,394 m. = 46.394 km.; 46.394 × .6 mi. = 27.8364 mi., which is called 27.8 mi. **2.** (1) 1500 × 1.1 yd. = 1650 yd.; (2) 18 min. 5.9 sec. = about 18.1 min.; 1650 yd. ÷ 18.1 = 91.16 yd., or about 91 yd. **3.** 13,812 m. = 13.812 km.; 13.812 × .6 mi. = 8.2872 mi., which is called 8.3 mi. **4.** (1) 26,429 m. = 26.429 km.; 26.429 × .6 mi. = 15.8574 mi., which is called 15.9 mi.; (2) 15.8574 mi. ÷ 2 = 7.9287 mi., or about 7.9 mi.

Page 35 **2.** (1) 1 milliliter (ml.) = $\frac{1}{1000}$ liter (l.)
1 centiliter (cl.) = $\frac{1}{100}$ liter
1 deciliter (dl.) = $\frac{1}{10}$ liter

1 dekaliter = 10 liters
1 hektoliter (hl.) = 100 liters
1 kiloliter (kl.) = 1000 liters
(2) 1 milligram (mg.) = $\frac{1}{1000}$ gram (g.)
1 centigram (cg.) = $\frac{1}{100}$ gram
1 decigram (dg.) = $\frac{1}{10}$ gram
1 dekagram = 10 grams
1 hektogram (hg.) = 100 grams
1 kilogram (kg.) = 1000 grams

3. 45.4×2.2 lb. = 99.88 lb. **4.** 121 lb. ÷ 2.2 lb. = 55 (kg.). **5.** 3 qt. **6.** 1 gal. = about 4 liters; 15×4 liters = 60 liters. **7.** (1) 1 pt. olive oil; (2) 700 g. = .7 kg., $.7 \times 2.2$ lb. = 1.54 lb., or $1\frac{1}{2}$ lb. nuts; (3) $1\frac{1}{2} \times 2.2$ lb. = 3.3 lb., or about $3\frac{1}{3}$ lb. coffee. **8.** (1) 30×2.2 lb. = 66 lb.; (2) 20×2.2 lb. = 44 lb.

Page 36

Aim: To present a project on the cost of operating an automobile

Suggestions: In ex. 3 on this page the term *depreciation* should be fully explained. It must be remembered that one of the items of cost in ownership of a car is this element of depreciation.

Key: **1.** \$65.75 + \$9.00 + \$3.00 + \$15.00 + \$5.50 + \$49.65 = \$147.90. **2.** 12 × \$12.50 = \$150.00; \$230.95 + \$16.40 + \$150.00 = \$397.35. **3.** \$2900 − \$2300 = \$600. **4.** \$147.90 + \$397.35 + \$600 = \$1145.25. **5.** \$1145.25 ÷ 12,972 = \$.0882, which is called \$.088, or 8.8¢. **6.** 12,972 mi. ÷ 775 = 16.7 mi. **7.** (1) \$230.95 ÷ 775 = \$.298, or 29.8¢; (2) 775 × \$.09 = \$69.75.

Page 37

Aim: To present Problem Test 1

Workbook Reference: Arithmetic Workshop, Book 8, page 9

Key: **1.** 6 hr. 36 min. = 6.6 hr.; 2500 mi. ÷ 6.6 = 378.78 mi., or 378.8 mi. **2.** 426.8 + 428.4 + 429.1 + 431.5 + 425.6 + 427.8 + 430.3 = 2999.5 (lb.); 2999.5 lb. ÷ 7 = 428.5 lb. **3.** $6\frac{1}{2}$ × \$.79 = \$5.13$\frac{1}{2}$, which is called \$5.14; \$10.00 − \$5.14 = \$4.86. **4.** Estimate: 8 × \$1.00 = \$8.00; exact: $7\frac{7}{8}$ × \$.98 = \$7.71$\frac{3}{4}$, which is called \$7.72. **5.** $2\frac{3}{5}$ mi. ÷ $\frac{1}{5}$ mi. = 13 (fifth miles); 13 − 1 = 12; 12 × 5¢ = 60¢; 60¢ + 25¢ = 85¢. **6.** 7 in. **7.** 8015 × .6 mi. = 4809 mi. **8.** 8945 × \$.098 = \$876.61. **9.** 480 + 320 = 800 (tickets), 960 − 800 = 160 (tickets); 480 × \$.25 = \$120.00, 320 × \$.35 = \$112.00, 160 × \$.50 = \$80.00, \$120.00 + \$112.00 + \$80.00 = \$312.00; \$312.00 − \$78.70 = \$233.30; \$233.30 ÷ 5 = \$46.66. **10.** 424 ÷ 24 = 17.66, or 17.7 miles.

Page 38

Aim: To provide a diagnostic test with page references for practice exercises

Suggestions: Page 38 is a diagnostic test covering computational skills reviewed in Chapter 1. It is important to observe how this test is constructed. All the

examples in a row are of the same general type. If a pupil secures the correct answer to all but one of the examples in a row, it may be that the wrong answer was caused by an accidental error. But if the pupil misses two or more examples in a row, something fundamental is probably wrong, and you should try to discover the reason for the errors. After correcting fundamental difficulties, additional practice should be assigned on the page or pages listed at the end of the row.

Key: **1.** $9\frac{2}{3}$; $8\frac{1}{8}$; $11\frac{11}{12}$; $10\frac{7}{12}$; $8\frac{13}{24}$. **2.** 16.09; 20.125; 16.708; \$59.77. **3.** $6\frac{2}{3}$; $5\frac{1}{3}$; $1\frac{1}{2}$; $2\frac{5}{16}$; $1\frac{3}{4}$. **4.** 7.8; 62.5; 1.625; \$21.54. **5.** $13\frac{1}{2}$; $1\frac{1}{8}$; $22\frac{1}{2}$. **6.** $\frac{3}{4}$; 32; $2\frac{1}{10}$. **7.** $\frac{1}{6}$; 24; 4. **8.** 1.68; .0375; .084. **9.** 5.375; 8.7; 5.94; 28.5. **10.** 3.2; .356; 21.6; 11.860. **11.** 2.68; 38.56; 23.41; 1.49.

Chapter 2

Aims of Chapter 2. The major aims of Chapter 2 are to:

1. Review the meaning of *per cent.*
2. Review finding a per cent of a number.
3. Extend the use of per cents to per cents ending in decimals.
4. Introduce per cents less than 1%.
5. Show how to take, score, and record improvement tests.
6. Reteach finding what per cent one number is of another number.
7. Teach finding a per cent correct to the nearest tenth and hundredth of 1%.
8. Extend the study of per cents of increase and decrease.
9. Reteach discount as used in retail stores for consumers.
10. Teach discount as used by wholesale merchants.
11. Reteach finding the amount and the rate of commission.
12. Teach finding a number when a per cent of it is known.
13. Teach the percentage formula $p = br$.
14. Reteach the relationship of profit and loss to selling price, cost, and expenses.
15. Extend the study of profit, loss, cost, and expenses as per cents of the selling price.
16. Teach finding the selling price when profit and expenses are given as per cents of the selling price.
17. Provide applications including all three types of percentage problems.

The Teaching of Percentage. From the standpoint of computation, the subject of percentage is merely another application of decimals. Hence, there are no new operations to be learned. The knowledge of decimals which the pupil has already acquired will serve all his needs in percentage. In fact, most of the actual work that the pupil will do in percentage will involve easier computation with decimals than much of that which he has already done.

From the standpoint of language, however, percentage is *entirely new.* In fact, the teaching of percentage is largely a series of language lessons. Percentage is new also with respect to its symbolism; that is, with respect to its method of expressing *hundredths* by means of the symbol %. Success in percentage requires that the pupil fully appreciate the fact that 37% is another way of writing .37 or $\frac{37}{100}$. Whenever a pupil sees 37%, he must understand that he can substitute .37 for it. Likewise, when he sees .37, he must be able to change it to 37%. The ability to make this rapid change from a decimal representing *hundredths* to the per cent symbolism, and vice versa, is one of the important skills that must be acquired early in the study of percentage.

In teaching percentage you must clearly understand that your task is not solely to teach pupils to *compute* with decimals that represent per cents. You must teach them also to *interpret* per cents and to understand their significance. Such interpretations are often necessary in situations where no computation at all is required. For example, in reading a newspaper you are frequently required

to interpret a statement such as "All radios are to be sold at 25% off" or "The population has increased 5%"; yet in these instances you make no computations whatever. In fact, for many people, their knowledge of percentage is usually applied in *interpreting* what they read.

Pages 39–41

Aim: To review the meaning of *per cent* as taught in Grades 6 and 7

Suggestions: The exercises on page 39 should be used to motivate a review of the meaning of *per cent* and of finding a per cent of a number. The diagrams on page 40 are provided to illustrate visually the meaning of certain per cents of a quantity. To develop his understanding the pupil should make the diagrams suggested in ex. 5–7.

The work on page 41 reviews the relationship of per cents to fractions and decimals. Make clear that 39% is *another way* of writing $\frac{39}{100}$ or .39.

Key: Page 39 **2.** (1) $\frac{1}{5} \times \$7.50 = \$1.50, \$7.50 - \$1.50 = \$6.00$; (2) $\frac{1}{5} \times \$6.00 = \$1.20, \$6.00 - \$1.20 = \$4.80$.

Page 40 **1.** 1%. **2.** 7%; 93 small squares; subtract 7 from 100; 93%. **3.** B: 28% red, 72% white; C: 65% red, 35% white. **5.** 81%; 65%; 40%. **6.** 75%; 88%.

Page 41 **2.** 16%; 2%; 81%; 100%. **3.** 61%; 3%; 29%; 93%. **4.** .25; .72; .04; .19; .07; .01; .21; .08; .84; .23; .06. **6.** $\frac{3}{4}$; $\frac{7}{20}$; $\frac{17}{20}$; $\frac{18}{25}$; $\frac{1}{4}$; $\frac{3}{10}$; $\frac{4}{5}$; $\frac{3}{20}$; $\frac{1}{5}$.

Page 42

Aim: To review finding a per cent of a number

Suggestions: Make sure that the pupils understand the meaning of the terms *base, rate,* and *percentage.* In ex. 6–11 the answers are to be found correct to the nearest cent. The rule for doing this is as follows: *To find an answer correct to the nearest cent, if the digit in the third decimal place is 5 or more, drop it but make the digit before it 1 larger; if the digit in the third decimal place is less than 5, drop it.*

Workbook Reference: Arithmetic Workshop, Book 8, pages 10–12

Key: **2.** \$95. **3.** 139,834,500 sq. mi. **4.** \$15.25. **6.** \$9.50; \$.31; \$1425. **7.** \$57; \$2.29; \$1080. **8.** \$9.46; \$3.68; \$1562.50. **9.** \$6.60; \$.29; \$1600. **10.** \$14.25; \$3.74; \$278.75. **11.** \$20.15; \$5.76; \$1183.33.

Page 43

Aim: To extend the use of per cents to per cents which include decimals

Suggestions: The work in percentage in *American Arithmetics* has been most carefully graded with respect to the various difficulties. In the sixth and seventh grades the pupil learned how to compute with such per cents as 6%, 13%, $3\frac{1}{2}$%, and $5\frac{3}{4}$%. He learned also how to use the fractional equivalents of the more common per cents, such as 25% and $33\frac{1}{3}$%. Per cents which include decimals,

such as 9.7% and 15.17%, are now introduced for the first time on page 43. The pupil's success in working with these new per cents will depend on his ability to change them quickly to decimals and vice versa. For this reason considerable practice should be given in working exercises like ex. 2 and 4 on page 43.

Workbook Reference: Arithmetic Workshop, Book 8, page 13

Key: **2.** .173; .249; .925; .4523; .026; .0387. **4.** 53.1%; 97.3%; 2.1%; 93.5%; 1.5%; 7.73%. **5.** 15,204 persons. **6.** 127; 82; 633. **7.** 45; 3; 405. **8.** 41; 6; 296.

Page 44

Aim: To review estimating answers in certain types of percentage problems

Suggestions: An estimate of the answer to a problem will often help a pupil detect errors in his computation. For example, by using his estimate he can usually see at once a mistake in the placement of a decimal point in the answer. Estimates of answers are useful also in reading mathematical statements in the newspaper. Therefore, you should encourage pupils to make estimates of answers to problems whenever possible.

Key: **3.** Think of 48% as about $\frac{1}{2}$; $\frac{1}{2} \times 75$ pupils = $37\frac{1}{2}$ pupils, which is called 38 pupils. **4.** Think of \$5100 as about \$5000 and of 12% as about $12\frac{1}{2}\%$, or $\frac{1}{8}$; $\frac{1}{8} \times \$5000 = \$625$. **5.** Think of 133,600,000 as about 134,000,000, and of 57% as about 60%, or $\frac{3}{5}$; $\frac{3}{5} \times 134{,}000{,}000 = 80{,}400{,}000$ (telephones). **6.** Think of 33% as $\frac{1}{3}$, and of \$9.25 as about \$9.30; $\frac{1}{3} \times \$9.30 = \$3.10$. **7.** \$100, \$104.58; \$600, \$585; \$50, \$47.76. **8.** \$300, \$288.96; \$300, \$295.26; \$90, \$91.80. **9.** \$20, \$21.56; \$300, \$296; \$300, \$295.96.

Page 45

Aim: To introduce per cents less than 1% expressed either in fractional or in decimal form

Suggestions: Per cents less than 1%, such as $\frac{1}{4}\%$, $\frac{1}{2}$ of 1%, and 0.3%, are often difficult for pupils. Such per cents, therefore, need very careful explanation. The per cent $\frac{1}{4}\%$, which means $\frac{1}{4}$ of 1%, is often confused with the fraction $\frac{1}{4}$, which equals 25%. Likewise, 0.3%, which means $\frac{3}{10}$ of 1%, is often misinterpreted to mean 3%. Forms such as $\frac{3}{4}$ of 1% and $\frac{1}{10}$ of 1% are often preferred to $\frac{3}{4}\%$ and $\frac{1}{10}\%$, since they are less likely to be misinterpreted.

Workbook Reference: Arithmetic Workshop, Book 8, page 14

Key: **1.** 15 accidents. **2.** 5 accidents. **3.** .75%, 0.75%, $\frac{3}{4}$ of 1%; .5%, 0.5%, $\frac{1}{2}\%$; .7%, $\frac{7}{10}$ of 1%, $\frac{7}{10}\%$; .25%, 0.25%, $\frac{1}{4}\%$; 0.9%, $\frac{9}{10}$ of 1%, $\frac{9}{10}\%$; .4%, 0.4%, $\frac{4}{10}$ of 1%. **4.** \$4.50. **6.** \$1.54. **7.** \$1.50; \$15; \$7. **8.** \$2.10; \$7.34; \$11.10. **9.** \$1.84; \$14.80; \$5.04. **10.** \$.40; \$3.60; \$1.60. **11.** \$5; \$.12; \$7. **12.** \$1.62; \$.28; \$20.

Pages 46–47

Aim: To present a project on cotton which applies the new work in percentage

Suggestion: The facts regarding cotton production given in this project are authentic.

Workbook Reference: Arithmetic Workshop, Book 8, page 49

Key: **1.** .305 × 46,840,000 bales = 14,286,200 bales. **2.** .332 × 14,286,200 bales = 4,743,018.4 bales, or about 4,743,018 bales. **3.** (1) 28.8% + 11.5% + 9.9% + 5.3% + 4.5% + 3.7% + 2.7% + 2.1% = 68.5%; (2) 100% − 68.5% = 31.5%. **4.** (1) 45.1% + 14.7% + 10.8% + 8.3% + 6.0% + 5.7% = 90.6%; (2) 100% − 90.6% = 9.4%. **5.** U.S., 15,559,500 bales; India, 5,071,500 bales; China, 3,726,000 bales; Russia, 2,863,500 bales; Egypt, 2,070,000 bales; Brazil, 1,966,500 bales; total, 31,257,000 bales. **6.** 13,696,000 is 13,700,000 to the nearest hundred thousand; $2,301,212,000 is $2,301,200,000 to the nearest hundred thousand; $2,301,200,000 ÷ 13,700,000 = $167.97, or $168 to the nearest dollar. **7.** 7844 lb. ÷ 24 = 326.8 lb., which is called 327 lb. **8.** .065 × 3,708,000 farms = 241,020 farms, or 241,000 farms, to the nearest thousand. **9.** (1) Mississippi: .193 × 241,000 farms = 46,513 farms; (2) Arizona: .008 × 241,000 farms = 1928 farms. **10.** 13.0 billion; 10.1 billion; (1) 46.8 million; (5) 34.5 million; (6) 13.7 million, $2.3 billion; (8) 3.7 million.

Pages 48–49

Aim: To show how to take and score improvement tests

Purpose of Improvement Tests. The teacher of mathematics in the eighth grade has three large tasks which have to be completed during the year. These tasks are:

(1) To review the four operations with whole numbers sufficiently to maintain and to develop further the computational skills that the pupils have already learned

(2) To review the four operations with common fractions and decimals

(3) To teach the new work of Grade 8, which includes further study of our number system and other number systems, new methods and applications in percentage, informal geometry, areas and volumes, equations, inequalities, and important social uses of arithmetic

Unless you plan your work carefully, you may find that tasks (1) and (2) take so much time that they greatly interfere with task (3). In order to reduce task (1) to a minimum, the authors have devised a series of improvement tests which give the pupils the necessary practice on whole numbers in less than 30 min. a week. These tests are described on pages 48–49 and 375–377 of *American Arithmetic, Grade 8.*

These improvement tests have been used successfully by many hundreds of teachers who are unanimous in their verdict that they are the most effective means yet found of improving the pupil's skill in computation. One important

fact about these tests is that the pupils themselves grow more enthusiastic about them the longer they use them, whereas with many other types of tests the interest of the pupils soon dies out. These improvement tests were first introduced in *American Arithmetic, Grade 6;* they were given also in *American Arithmetic, Grade 7* and are continued throughout this text.

Description of Improvement Tests. There are 72 improvement tests in this book, and these tests are distributed uniformly over the year's work. The tests are arranged in sets of 3 tests each; hence, there are 24 sets of such tests. They cover the addition, subtraction, multiplication, and division of whole numbers. For a complete list of these tests, see the Index under "Improvement Tests." A preliminary orientation concerning these tests is given on page 48 of the text, which the pupils should read carefully at this time. A set of improvement tests consisting of 3 addition tests, is shown on page 49. The nature of these tests is evident by an examination of the 3 tests. Each test has 8 exercises, all the exercises being of equal difficulty. If a pupil gets a score of 9 on Test 1a and a score of 10 on Test 1b, it is evident that he has done better on Test 1b than on Test 1a, since the tests are of equal difficulty.

Though there are 3 tests on page 49, these 3 tests should not all be given on the same day. Only one test of a set is to be taken on a given day. The reason for this caution is to avoid fatigue and to minimize the time devoted to practice on whole numbers. In general, one test should be given every second or third day; hence, the 3 tests on page 49 will be spread over a period of 5 to 7 days.

The number of exercises in each test is indicated by a small number in a circle, which is printed near the last exercise in the test. For example, at the end of Test 1a on page 49 you find ⑧, which means that there are 8 exercises in this test. On the other hand, in Subtraction Test 1a, given on page 65, there are 15 exercises in each test.

How to Give Improvement Tests. The method of giving improvement tests is described in detail on page 375 of the text. These directions should be studied carefully. In timing the tests, use a watch with a second hand so that the pupils can be stopped promptly at the end of the time assigned to the test. In giving the addition tests on page 49, allow exactly 4 minutes for each test so that the results on these tests may be comparable. If you allow 4 minutes for Test 1a but permit the time for Test 1b to be $4\frac{1}{2}$ minutes, the results on Test 1b cannot be reliably compared with those on Test 1a. The purpose of these tests is to measure improvement in computing with whole numbers, and this measurement can be made only if the tests are given under the same conditions each time.

In taking an addition test such as one of those on page 49, the pupils should not copy the examples before working them. Instead, they are to write the answers along the edge of a piece of folded paper, as described on page 375 of the text. Folded paper is used only for the tests on addition and subtraction. For tests on multiplication and division, such as those given on pages 75 and 87, the exercises should be copied on a sheet of paper *before* the test is given. In

copying the exercises, the pupils should spread them out over the sheet to allow ample space for the work. The time assigned for a test on multiplication or division is the time actually allowed for taking the test *after* the copying has been completed.

Scoring Tests. All improvement tests are scored on a scale of 10, and the actual scoring is done by the aid of a Scoring Table, which is shown on page 376. The pupils must be carefully instructed in using this table to find their scores. Detailed instructions for finding scores are given below the Scoring Table.

Keeping Records. Each pupil should keep a graphic record of his scores on each improvement test, as explained on page 377 of the text. This record should include all tests taken during the school year. A Record Book can easily be made from a few sheets of squared paper. If the school does not have squared paper, a pad of such paper should be purchased and a few sheets distributed to each pupil. It will be seen in the graph on page 377 that 3 scores are kept for each set of tests. Since there are 24 sets of tests in all in this book, each pupil need have only enough squared paper to provide for the scores on 24 sets.

The plan of having each pupil keep a graphic record of his scores has a very favorable psychological effect upon him. If he finds that his score on the second test of a set is not above that on the first test, he immediately puts forth all his energy to improve his record on the third test. The pupil is always trying to better his previous performance, and this element maintains a high degree of interest in this work throughout the school year. The pupil also gains considerable satisfaction in studying his scores from time to time during the year. In the middle of the year his scores should be better, in general, than those at the first of the year; and at the end of the year he should be doing still better.

A pupil will find it helpful to compare his scores on subtraction tests with those on addition tests. If he finds that he does not do so well on the subtraction tests as on the addition tests, he will see that he needs more practice on subtraction. Similar comparisons may be made with respect to the tests on multiplication and division.

Key: Page 49 **1.** 430; 406; 467; 545; 499; 600; 422; 530. **2.** 618; 453; 509; 532; 604; 505; 571; 493. **3.** 555; 505; 461; 561; 576; 541; 571; 653.

Page 50

Aims: To reteach large per cents and to extend this topic to include large per cents ending in decimals

Workbook Reference: Arithmetic Workshop, Book 8, pages 15 and 16

Key: **1.** 3; 7; 4. **3.** 243 rooms. **5.** 3.00; 9.00; 4.32; 3.48; 2.20; 12.00. **6.** 193%; 265%; 1515%; 591%; 400%; 112.5%. **7.** 324%; 413%; 2704%; 387%; 1700%; 237.5%. **8.** 405. **9.** 2583 people. **10.** $756; $454.40; $230.35. **11.** $312.50; $775.10; $486.68. **12.** $686; $698.19; $140.63.

Page 51

Aim: To reteach finding what per cent one number is of another number

Suggestions: In this text the previous work on percentage has been confined to the finding of a per cent of a number, which is the most frequent application of percentage. This work is sometimes called Case 1 of percentage. Another case of percentage, sometimes called Case 2, is to find what per cent one number is of another, as in finding what per cent 13 is of 20. This work is explained on page 51 of the text. Some teachers feel that this type of example is not very important; but it occurs more frequently than many persons realize. We often compare two numbers by means of a per cent. For example, when we find that 13 is 65% of 20, we are comparing 13 with 20.

Ex. 1 on page 51 shows a quick way to find what per cent 13 is of 20. In this case, $\frac{13}{20}$ is easily changed to $\frac{65}{100}$, which equals 65%. This easy method cannot be used in ex. 2, however, since $\frac{23}{40}$ is not quickly changed to another fraction with a denominator of 100. Instead, $\frac{23}{40}$ is changed to a two-place decimal. The pupils should study carefully the method of working ex. 2, since it represents the usual method of finding what per cent one number is of another.

Workbook Reference: Arithmetic Workshop, Book 8, page 18

Key: **3.** 30; 2; $33\frac{1}{3}$. **4.** 55; $7\frac{1}{2}$; $46\frac{1}{4}$. **5.** $32\frac{1}{2}$; 4; $66\frac{2}{3}$. **6.** 20%; $32\frac{1}{2}$%; 44%; 100%; 125%. **7.** 75%; $73\frac{1}{3}$%; $16\frac{1}{4}$%; 12%; 150%. **8.** 92%; 90%; $31\frac{1}{4}$%; $4\frac{1}{2}$%; 140%.

Page 52

Aim: To reteach finding a per cent correct to the nearest whole per cent

Suggestions: The model solution given in ex. 1 should be carefully studied. In this type of percentage problem it is very important to be able to give the answers to the nearest whole per cent, to the nearest tenth of 1%, and to the nearest hundredth of 1%. As this work develops, make sure that the pupils see that they always carry out the quotient to one more place than required in the answer, and then round it off.

Workbook Reference: Arithmetic Workshop, Book 8, page 19

Key: **2.** 13 da. ÷ 28 da. = .464, which is called .46, or 46%. **3.** \$5 ÷ \$37 = .135, which is called .14, or 14%. **4.** 15%; 33%; 20%; 10%; 13%; 108%; 322%. **5.** 47%; 6%; 29%; 1%; 25%; 251%; 706%. **6.** (1) 4 ÷ 17 = .235, which is called .24, or 24%; (2) 72 ÷ 79 = .911, which is called .91, or 91%; (3) 48 ÷ 133 = .360, which is called .36, or 36%; (4) 24 ÷ 21 = 1.142, which is called 1.14, or 114%; (5) 109 ÷ 73 = 1.493, which is called 1.49, or 149%. **7.** 6%; 53%; 18%; 155%; 235%. **8.** 24%; 52%; 58%; 81%; 124%. **9.** 10%; 39%; 38%; 57%; 138%.

Pages 53–55

Aims: To teach finding a per cent correct to the nearest tenth of 1% and the nearest hundredth of 1% and to apply this skill in computing baseball standings

Suggestions: The pupils should be made familiar with the expression "to the nearest tenth of 1%" since it is used frequently throughout this book. They should study very carefully the explanation of finding an answer correct to the nearest tenth of 1%, given in ex. 1 on page 53.

The computation of the standings of baseball teams, given on pages 54 and 55 of the text, is an excellent example of the computation to the nearest tenth of 1%. Usually, all teams in the major leagues play the same number of games during the season. However, in the season for which the data on page 54 are given this was not true, as you can see. When the standings of two teams are nearly equal, it is sometimes necessary to compute the standings to an extra decimal place; that is, to the nearest hundredth of 1%. This situation is illustrated in ex. 2 on page 55.

Encourage the pupils to look up the standings of baseball teams as given in the daily newspapers and to check such standings by computing them themselves. If the baseball season is over when this topic is studied, perhaps some pupil can find an old newspaper containing baseball records. *The World Almanac* also has baseball records which can be used for illustrations and problems.

Workbook Reference: Arithmetic Workshop, Book 8, page 21

Key: Page 53 **2.** 487 pupils ÷ 742 pupils = .6563, which is called .656, or 65.6%. **3.** Lucy is right. **4.** 89.5%; 71.7%; 10.3%; 22.3%; 20.9%. **5.** 0.1%; 58.1%; 30.0%; 83.6%; 23.5%. **6.** (1) (a) 96 ÷ 189 = .5079, which is called .508, or 50.8%; (b) 99 ÷ 278 = .3561, which is called .356, or 35.6%; (2) (a) 745 ÷ 888 = .8389, which is called .839, or 83.9%; (b) 856 ÷ 915 = .9355, which is called .936, or 93.6%. **7.** (1) (a) 88 ÷ 173 = .5086, which is called .509, or 50.9%; (b) 93 ÷ 206 = .4514, which is called .451, or 45.1%; (2) (a) 587 ÷ 950 = .6178, which is called .618, or 61.8%; (b) 556 ÷ 910 = .6109, which is called .611, or 61.1%.

Page 54 **2.** (1) Minnesota: 91 ÷ 162 = .5617, or .562; (2) Los Angeles: 86 ÷ 162 = .5308, or .531; (3) Detroit: 85 ÷ 161 = .5279, or .528; (4) Chicago: 85 ÷ 162 = .5246, or .525; (5) Cleveland: 80 ÷ 162 = .4938, or .494; (6) Baltimore: 77 ÷ 162 = .4753, or .475; (7) Boston: 76 ÷ 160 = .4750, or .475; (8) Kansas City: 72 ÷ 162 = .4444, or .444; (9) Washington: 60 ÷ 161 = .3726, or .373.

Page 55 **1.** Team A: 38 ÷ 95 = .4000; Team B: 37 ÷ 93 = .3978. To the nearest whole per cent, the standing of each team would be .40; hence, they would be equal. To the nearest tenth of 1%, the standings are .400 and .398; so Team A is ahead. **2.** Team A: 50 ÷ 98 = .51020; Team B: 51 ÷ 100 = .51000; so Team A was ahead with .5102 as against the .5100 of Team B. **3.** New York: 58 ÷ 112 = .51785; Chicago: 59 ÷ 114 = .51754; so New York was ahead with .5179 as against the .5175 of Chicago. **4.** Pittsburgh: 65 ÷ 112 = .58035; St. Louis: 69 ÷ 119 = .57983; so Pittsburgh was ahead with .5804 as against the .5798 of St. Louis. **5.** 159 ÷ 590 = .2694, or .269. **6.** 25 ÷ 29 = .8620, which is called .862. **7.** 188 ÷ 533 = .3527, which is called .353. **8.** (1) 129 ÷ 478 = .2698, which is called .270; (2) 168 ÷ 619 = .2714, which is called .271;

(3) Green had the better record. **9.** (1) 198 ÷ 615 = .3219, which is called .322; (2) 195 ÷ 585 = .3333, which is called .333; (3) Mays had the better record.

Page 56

Aim: To reteach per cents of increase and decrease

Suggestions: Ex. 1 on this page illustrates the method of finding the per cent of increase, while ex. 2 illustrates the method of finding the per cent of decrease. These methods are summed up in this statement: *To find the per cent of increase or decrease, find the difference between the two numbers; then find what per cent this difference is of the original number.* You will have to emphasize the fact that the amount of increase or decrease is compared with the *original* number.

Workbook Reference: Arithmetic Workshop, Book 8, page 22

Key: **2.** 60%. **3.** 109 lb. − 97 lb. = 12 lb.; 12 lb. ÷ 97 lb. = .123, or 12%. **4.** (1) 702 − 624 = 78; 78 ÷ 624 = .125, or 13%, increase; (2) 3696 − 3520 = 176; 176 ÷ 3520 = .050, or 5%, increase; (3) \$76.65 − \$40.88 = \$35.77; \$35.77 ÷ \$40.88 = .875, or 88%, increase. **5.** (1) 270 − 138 = 132; 132 ÷ 270 = .488, or 49%, decrease; (2) 2444 − 1833 = 611; 611 ÷ 1833 = .333, or 33%, increase; (3) \$64.50 − \$60.85 = \$3.65; \$3.65 ÷ \$64.50 = .056, or 6%, decrease.

Pages 57–58

Aims: To extend per cents of increase and decrease to large per cents and to use such work in problem solving

Suggestions: You will need to explain carefully that when a number increases by a large per cent such as 300%, the *increase* is 300% of the original value. Hence, it becomes its original value *plus* 300% of its original value. Otherwise some pupils may make the error of thinking that when a number increases 300% it becomes 300% of its original value.

Key: Page 57 **1.** \$20; \$40. **2.** \$30; \$50. **4.** More than 100%; \$25. **5.** \$60 − \$50 = \$10; \$10 ÷ \$60 = .16$\frac{2}{3}$, or 16$\frac{2}{3}$%. **6.** \$0; no; yes. **8.** (1) \$115 − \$50 = \$65; \$65 ÷ \$50 = 1.30, or 130%; (2) \$50 − \$42 = \$8; \$8 ÷ \$50 = .16, or 16%. Page 58 **1.** 900 − 75 = 825; 825 ÷ 75 = 11.00, or 1100%. **2.** 7500 cars − 6375 cars = 1125 cars; 1125 ÷ 7500 = .15, or 15%. **3.** \$12.00 − \$8.00 = \$4.00; \$4 ÷ \$8 = .50, or 50%. **4.** 28,000 − 12,500 = 15,500; 15,500 ÷ 12,500 = 1.24, or 124%. **5.** \$110 − \$95 = \$15; \$15 ÷ \$95 = .157, which is called 16%. **6.** 403 − 369 = 34; 34 ÷ 369 = .092, which is called 9%. **7.** (1) 756 − 270 = 486; 486 ÷ 270 = 1.80, or 180%, increase; (2) 8372 − 3640 = 4732; 4732 ÷ 3640 = 1.30, or 130%, increase; (3) \$133.90 − \$26.00 = \$107.90; \$107.90 ÷ \$26.00 = 4.15, or 415%, increase. **8.** (1) 833 − 238 = 595; 595 ÷ 238 = 2.50, or 250%, increase; (2) 4368 − 1400 = 2968; 2968 ÷ 1400 = 2.12, or 212%, increase; (3) \$104.50 − \$47.50 = \$57.00; \$57.00 ÷ \$47.50 = 1.20, or 120%, increase. **9.** (1) 243 − 162 = 81; 81 ÷ 243 = .33$\frac{1}{3}$, or 33$\frac{1}{3}$%, decrease; (2) 5445 − 1980 = 3465; 3465 ÷ 1980 = 1.75, or 175%, increase; (3) \$195.25 − \$117.15 = \$78.10; \$78.10 ÷ \$195.25 = .40, or 40%, decrease.

Page 59

Aim: To reteach discount in the form in which it is used in retail stores

Suggestions: This page presents the subject of discount as it applies to the retail customer; hence, discounts offered at sales are emphasized. In this work make sure that the pupils understand and use properly such terms as *discount, marked price,* and *net price.* In connection with this work, encourage the pupils to bring to class advertisements of sales from newspapers. Use these advertisements as a source of supplementary problems.

Key: **2.** (1) \$147 − \$98 = \$49; (2) \$49 ÷ \$147 = $\frac{1}{3}$, or $33\frac{1}{3}\%$. **3.** \$3.76 − \$3.29 = \$.47; \$.47 ÷ \$3.76 = .122, or 12%. **4.** (1) \$60 − \$40 = \$20, \$20 ÷ \$60 = $\frac{1}{3}$, or $33\frac{1}{3}$; (2) $\frac{1}{3}$ × \$45 = \$15, \$45 − \$15 = \$30. **5.** (1) \$21.00 − \$10.50 = \$10.50, \$10.50 ÷ \$21.00 = $\frac{1}{2}$, or 50%; (2) \$150 − \$90 = \$60, \$60 ÷ \$150 = $\frac{2}{5}$, or 40%; (3) \$20 − \$16 = \$4, \$4 ÷ \$20 = $\frac{1}{5}$, or 20%; (4) \$10 − \$7 = \$3, \$3 ÷ \$10 = $\frac{3}{10}$, or 30%.

Pages 60–61

Aim: To teach discount in the form in which it is used when a retail store buys merchandise from a wholesale merchant or manufacturer

Suggestions: The work on these pages is usually called *trade discount* to distinguish it from the discounts that retail stores allow their customers. In all this work the terms *list price* and *net price* must be clearly explained. The work on page 61, which explains a second method of finding net prices, is very important since the principle used here has other applications besides those relating to discounts.

Key: Page 60 **3.** .25 × \$185 = \$46.25; \$185 − \$46.25 = \$138.75. **4.** 25 × \$24.80 = \$620.00; $\frac{1}{4}$ × \$620 = \$155; \$620 − \$155 = \$465. **5.** .40 × \$5.00 = \$2.00, \$5 − \$2 = \$3; .20 × \$3.50 = \$.70, \$3.50 − \$.70 = \$2.80; \$3.00 − \$2.80 = \$.20; so the pens listed at \$3.50 with 20% discount cost \$.20 less for each one. **6.** \$223.50, \$521.50; \$138.60, \$554.40; \$347, \$2429. **7.** \$247.20, \$370.80; \$24.50, \$220.50; \$567, \$1053. **8.** \$192, \$128; \$241, \$482; \$1806, \$3010. **9.** \$74.40, \$421.60; \$114, \$342; \$350, \$1400. **10.** \$145, \$435; \$92, \$460; \$348, \$1972.
Page 61 **2.** (1) 100% − 20% = 80%; (2) .80 × \$75 = \$60. **3.** 25% equals $\frac{1}{4}$; $\frac{1}{4}$ × \$3.00 = \$.75; \$3.00 − \$.75 = \$2.25; it is easier to find $\frac{1}{4}$ of \$3.00 and subtract than it is to find 75% of \$3.00. **4.** (1) It is easier to take 10% of \$50, which gives \$5, and then to subtract \$5 from \$50, which gives \$45; (2) it is easier to subtract 15% from 100%, which gives 85%, and then to find 85% of \$45, which gives \$38.25. **5.** (1) .05 × \$10 = \$.50, \$10 − \$.50 = \$9.50; 100% − 5% = 95%, .95 × \$10 = \$9.50; (2) .30 × \$360 = \$108, \$360 − \$108 = \$252; 100% − 30% = 70%, .70 × \$360 = \$252; (3) .40 × \$632 = \$252.80, \$632 − \$252.80 = \$379.20; 100% − 40% = 60%; .60 × \$632 = \$379.20; (4) .05 × \$255 = \$12.75, \$255 − \$12.75 = \$242.25; 100% − 5% = 95%; .95 × \$255 = \$242.25. **6.** \$23.25; \$360; \$630; \$298.41. **7.** \$72; \$360.80; \$277.50; \$414.64. **8.** \$60; \$770; \$645; \$648.15. **9.** \$51; \$162; \$150; \$146.40. **10.** \$66; \$140; \$130; \$225.

Pages 62–63

Aim: To reteach the work on finding the amount and the rate of commission

Suggestions: These pages represent the less technical phases of commission, with problems of types that may easily be appreciated by boys and girls. The pupils should become familiar with such terms as *commission*, *agent*, and *net proceeds*. Ask the pupils to find out the rate of commission charged by real-estate agents in your locality.

Workbook Reference: Arithmetic Workshop, Book 8, page 28

Key: Page 62 **1.** (1) $.05 \times \$15{,}000 = \$750$; (2) $\$15{,}000 - \$750 = \$14{,}250$. **2.** (1) (a) $.05 \times \$9500 = \$475$, (b) $\$9500 - \$475 = \$9025$; (2) (a) $.05 \times \$12{,}900 = \$645$, (b) $\$12{,}900 - \$645 = \$12{,}255$; (3) (a) $.05 \times \$17{,}500 = \$875$, (b) $\$17{,}500 - \$875 = \$16{,}625$; (4) (a) $.05 \times \$24{,}000 = \$1200$, (b) $\$24{,}000 - \$1200 = \$22{,}800$. **3.** (1) (a) $.05 \times \$45{,}000 = \$2250$, (b) $\$45{,}000 - \$2250 = \$42{,}750$; (2) (a) $.05 \times \$73{,}500 = \$3675$, (b) $\$73{,}500 - \$3675 = \$69{,}825$; (3) (a) $.05 \times \$94{,}000 = \$4700$, (b) $\$94{,}000 - \$4700 = \$89{,}300$.

Page 63 **1.** (1) $\$21{,}000 - \$20{,}000 = \$1000$, $.05 \times \$20{,}000 = \$1000$, $.03 \times \$1000 = \$30$, $\$1000 + \$30 = \$1030$; (2) $\$22{,}500 - \$20{,}000 = \$2500$, $.05 \times \$20{,}000 = \$1000$, $.03 \times \$2500 = \$75$, $\$1000 + \$75 = \$1075$; (3) $.05 \times \$18{,}500 = \$925$; (4) (a) $\$21{,}000 - \$1030 = \$19{,}970$; (b) $\$22{,}500 - \$1075 = \$21{,}425$; (c) $\$18{,}500 - \$925 = \$17{,}575$. **2.** (1) $.03 \times \$300 = \$9$, $\$35 + \$9 = \$44$; (2) $.03 \times \$425 = \$12.75$, $\$35 + \$12.75 = \$47.75$; (3) $.03 \times \$550 = \$16.50$, $\$35 + \$16.50 = \$51.50$; (4) $.03 \times \$785 = \$23.55$, $\$35 + \$23.55 = \$58.55$. **3.** (1) $40 \times \$3 = \$120$, $\frac{1}{4} \times \$120 = \$30$; (2) $\$120 - \$30 = \$90$. **4.** (1) $\$875 + \$16{,}625 = \$17{,}500$; (2) $\$875 \div \$17{,}500 = .05$, or 5%. **5.** \$18, \$282; \$322, \$483; \$200, \$3800; \$1.65, \$81. **6.** \$45.80, \$870.20; \$72.60, \$169.40; \$390, \$6110; \$2.54, \$60.96. **7.** \$46.60, \$186.40; \$145, \$435; \$312, \$7488; \$2.27, \$35.58. **8.** (1) $\$8 \div \$50 = .16$, or 16%; (2) $\$72 \div \$600 = .12$, or 12%; (3) $\$185 \div \$740 = .25$, or 25%; (4) $\$400 \div \$8000 = .05$, or 5%. **9.** $33\frac{1}{3}\%$; $7\frac{1}{2}\%$; 15%; 4%. **10.** 25%; 8%; 20%; 6%.

Page 64

Aim: To teach finding a number when a per cent of it is given

Suggestions: In connection with the suggestions for teaching page 51 of the text, we have discussed the first two types of percentage problems. The third type of percentage problem, which is sometimes called Case 3, is presented on page 64. The explanation of this new work, as given in ex. 1 on page 64, should be carefully studied. This is a simple explanation of this type of percentage problem based upon the meaning of per cent. Another method for solving such problems, which makes use of the percentage formula $p = br$, is presented on page 67.

The applications of this work to the other exercises on this page should be easily understood by the pupils.

Workbook Reference: Arithmetic Workshop, Book 8, pages 29 and 30

Key: **2.** 8% of salary = \$360; 1% of salary = \$360 ÷ 8, or \$45; 100% of salary = 100 × \$45, or \$4500. **3.** \$850. **4.** (1) 7% of sales = \$38.50; 1% of sales = \$38.50 ÷ 7, or \$5.50; 100% of sales = 100 × \$5.50, or \$550; (2) \$600; (3) \$750; (4) \$900; (5) \$950; (6) \$875. **5.** (1) 6% of number = 15; 1% of number = 15 ÷ 6, or 2.5; 100% of number = 100 × 2.5, or 250; (2) 9% of the amount = \$2.88; 1% of the amount = \$2.88 ÷ 9, or \$.32; 100% of the amount = 100 × \$.32, or \$32. **6.** 400; \$44. **7.** 225; \$68.

Page 65

Aim: To give the first set of improvement tests in subtraction of whole numbers

Suggestion: This set of improvement tests in subtraction should be administered in the same manner as the set of improvement tests in addition which was given on page 49 of the text.

Key: **1.** 54,557; 10,282; 13,904; 23,788; 36,754. **2.** 22,147; 19,876; 18,376; 9056; 19,063. **3.** 21,869; 51,975; 41,942; 23,466; 2279. **4.** 17,418; 2464; 50,857; 61,723; 15,934. **5.** 8467; 23,187; 39,819; 42,663; 19,259. **6.** 45,668; 32,618; 3577; 48,791; 33,042. **7.** 178; 19,584; 39,784; 25,890; 43,695. **8.** 17,678; 26,579; 60,825; 27,369; 53,878. **9.** 17,575; 36,958; 26,268; 8156; 26,934.

Page 66

Aim: To give additional practice in finding a number when a per cent of it is known

Key: **1.** 125 votes. **2.** 80% of votes = 92; 1% of votes = 92 ÷ 80, or 1.15; 100% of votes = 100 × 1.15, or 115 (votes). **3.** 24% of cost = \$624; 1% of cost = \$624 ÷ 24, or \$26; 100% of cost = 100 × \$26, or \$2600. **4.** 60% of games = 9; 1% of games = 9 ÷ 60, or .15; 100% of games = 100 × .15, or 15 (games). **5.** (1) \$714.29; (2) 7% of sales = \$55; 1% of sales = \$55 ÷ 7, or \$7.85714; 100% of sales = 100 × \$7.85714, or \$785.71; (3) 7% of sales = \$65; 1% of sales = \$65 ÷ 7, or \$9.28571; 100% of sales = 100 × \$9.28571, or \$928.57; (4) 7% of sales = \$70; 1% of sales = \$70 ÷ 7, or \$10; 100% of sales = 100 × \$10, or \$1000. **6.** 12% of amount = \$17.49; 1% of amount = \$17.49 ÷ 12, or \$1.4575; 100% of amount = 100 × \$1.4575, or \$145.75. **7.** 300; 350; 425. **8.** 620; 275; 240. **9.** 275; 68; 1025. **10.** 80; 225; 950. **11.** 125; 160; 508.

Page 67

Aim: To generalize percentage relationships into a formula that can be used for any type of percentage problem

Suggestions: Emphasize which parts of the formula are factors and which is the product. Have pupils tell how to find the product when both factors are known, and how to find an unknown factor when the product and the other factor are known. When the pupil fully understands how to use this formula, he can solve any percentage problem by remembering this *one* relationship instead of three

separate procedures. Thus, this page is an important conclusion to his study of the meaning of percentage.

Key: **2.** (1) 400, base; 23%, rate; (2) $p = .30 \times 1250 = 375$; 136; 1246; 305.5; 1664.4; 33.93. **3.** $8 = r \times 40$, $8 \div 40 = r$, $r = .20$, or 20%; 96%; 16%; 43.2%; 150%. **4.** (1) Yes; (2) $70 = .35 \times b$, $70 \div .35 = b$, $b = 200$; 7200; 1700.

Pages 68–73

Aims: To reteach the relationship of profit and loss to selling price, cost, and expenses; to extend the study of profit, loss, cost, and expenses as per cents of the selling price; and to teach finding the selling price when profit and expenses are given as per cents of the selling price

Profit and Loss. The topic of profit and loss, like percentage, has its technical language which must be carefully learned if this topic is to be understood. Hence, it is important for the pupils to become familiar with the meanings of the terms *expenses, profit, cost,* and *selling price.* The term *overhead* is sometimes used for expenses. The word *profit,* as used here, means the *actual profit,* or what is sometimes called the *net profit.* The *cost* is the price actually paid for merchandise, including any freight and delivery charges, while the *selling price* is the price received for the merchandise when it is sold.

Success in the work on profit and loss depends upon a full understanding of the following relationship:

Selling Price = Cost + Expenses + Profit

The pupils must remember that *profit* means *actual profit.*

It is the general practice in business today to compute profit as a per cent of the *selling price.* Formerly, profit was frequently computed as a per cent of the *cost* of the goods, but that practice has gradually been replaced by the use of the *selling price* as the basis for computing profit.

In modern business the selling price is used also as a basis for computing other items. For example, *expenses* are computed as a per cent of the selling price. When clerks are paid a commission, this *commission* is computed as a per cent of the amount of the clerks' sales, which means, of course, that it is computed on the selling price. Discounts allowed to customers are also computed on the selling price. Certain taxes likewise are computed on the sales, or the selling price. In view of all these situations, it has been found most convenient in modern business to use the selling price as a common base for all computations. In this book, it will be made clear in each exercise that the profit, or the loss, is to be computed as a per cent of the selling price.

Suggestions: The purpose of pages 68 and 69 is to acquaint the pupil with the use of the relationship

Selling Price = Cost + Expenses + Profit

No per cents are involved in this particular work.

On page 70, ex. 1 shows how Mr. King finds out what his *profit* is for the year. In ex. 2, Mr. King studies his sales, costs, expenses, and profits for an entire year to get data for expressing his cost, expenses, and profit as per cents of the selling price. Mr. King bases his conclusions on his own experience. A store in another neighborhood might have higher rent and other expenses which would make the per cents differ from those of Mr. King; see ex. 4 on page 70.

Page 71 explains the sales dollar, which is usually illustrated as a circle graph. Graphs of sales dollars often appear in newspapers and magazines; hence, it is important that the pupils understand their meaning.

Attention is called to the fact that the sales dollar represents every dollar that is received from sales; that is, it represents the *selling price*. Furthermore, all per cents given on circle graphs of the sales dollar are computed on the selling price as a basis. Thus we see that the sales dollar is another illustration of the practice of computing profits on the selling price rather than on the cost.

Page 72 gives practice in finding the cost, the expenses, and the profit or loss when they are given as per cents on the selling price.

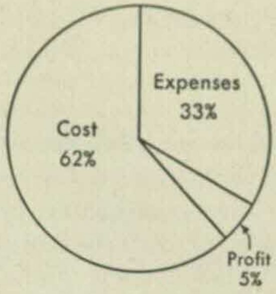
The work on page 73 shows how Mr. Ware determines the price at which he must sell a table and still cover his expenses and make a desired profit. The model solution given in ex. 1 should be carefully studied. This solution shows that in order to find the selling price it is necessary to use Case 3 of percentage, which was first presented on page 64 of the text. Case 3 is required in this situation because a per cent (54%) of a whole amount is known ($72.90), and we want to find the whole amount (selling price).

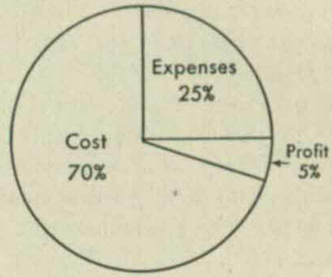
Workbook Reference: Arithmetic Workshop, Book 8, pages 32–35

Key: Pages 68–69 **2.** (1) $2600 + $1000 = $3600, $4000 − $3600 = $400 (profit); (2) $2600 + $1400 = $4000, $4000 − $4000 = $0 (no profit); (3) $2600 + $1600 = $4200, $4200 − $4000 = $200 (loss). **5.** $3100 + $1500 = $4600, $5000 − $4600 = $400. **6.** $600 + $300 = $900; $1000 − $900 = $100 (profit). **7.** $770 + $126 = $896; $1400 − $896 = $504 (expenses). **8.** $760 + $240 = $1000; $2000 − $1000 = $1000 (cost). **9.** $992 + $528 + $80 = $1600 (selling price). **10.** $780 + $516 = $1296; $1296 − $1200 = $96 (loss). **11.** $1800 − $1134 = $666; $666 + $54 = $720 (expenses). **12.** $4030 + $1885 = $5915; $6500 − $5915 = $585 (profit). **13.** $5166 + $3690 = $8856; $8856 − $656 = $8200 (selling price).

Page 70 **1.** $18,000 + $9000 = $27,000; $30,000 − $27,000 = $3000. **2.** (1) $9000 ÷ $30,000 = .30, or 30%; (2) $3000 ÷ $30,000 = .10, or 10%; (3) $18,000 ÷ $30,000 = .60, or 60%. **5.** (6) 60%, 30%, 10%; (7) 55%, 36%, 9%; (8) 50%, 38%, 12%; (9) 62%, 33%, 5%; (10) 65%, 43%, 8% (loss); (11) 63%, 40%, 3% (loss); (12) 62%, 29%, 9%; (13) 63%, 45%, 8% (loss).

Page 71 **2.** Expenses, 28%; cost, 40%. **3.** (1) 5% + 33% = 38%, 100% − 38% = 62%; (2) .05 × 360° = 18°; .33 × 360° = 118.8°, which rounds off to 119°; .62 × 360° = 223.2°, which rounds off to 223°.





4. Profit: $.05 \times 360° = 18°$
Expenses: $\frac{1}{4} \times 360° = 90°$
Cost: $.70 \times 360° = 252°$

Page 72 **1.** (1) $.10 \times \$40 = \$4$; (2) $\frac{1}{4} \times \$40 = \$10$; (3) $.65 \times \$45 = \$26$. **2.** \$40. **3.** (1) In ex. 1, 65% of the S.P. represents the cost; hence, 65% of \$40, or \$26, represents the cost; (2) another way is to find the amounts representing the profit and the expenses and to subtract their sum from \$40. **4.** (1) $.05 \times \$50 = \$2.50$ (loss); (2) $.60 \times \$50 = \$30$ (cost); (3) $.45 \times \$50 = \$22.50$ (expenses). **5.** (1) $.60 \times \$25 = \$15$; (2) $.33 \times \$25 = \$8.25$; (3) $.07 \times \$25 = \$1.75$. **6.** \$61.75; \$42.75; \$9.50 (loss). **7.** \$63.25; \$40.25; \$11.50. **8.** \$78.75; \$40; \$6.25. **9.** \$132; \$92.40; \$4.40 (loss).

Page 73 **3.** (1) $15\% + 30\% = 45\%$, $100\% - 45\% = 55\%$ (cost); 55% of S.P. = \$11.55; 1% of S.P. = \$11.55 ÷ 55, or \$.21; 100% of S.P. = 100 × \$.21, or \$21; (2) 55% of S.P. = \$29.15; 1% of S.P. = \$29.15 ÷ 55, or \$.53; 100% of S.P. = 100 × \$.53, or \$53; (3) 55% of S.P. = \$3.74; 1% of S.P. = \$3.74 ÷ 55, or \$.068; 100% of S.P. = 100 × \$.068, or \$6.80. **4.** (1) $25\% + 13\% = 38\%$, $100\% - 38\% = 62\%$; 62% of S.P. = \$31.00; 1% of S.P. = \$31.00 ÷ 62, or \$.50; 100% of S.P. = 100 × \$.50, or \$50; (2) $29\% + 13\% = 42\%$; $100\% - 42\% = 58\%$; 58% of S.P. = \$522; 1% of S.P. = \$522 ÷ 58, or \$9; 100% of S.P. = 100 × \$9, or \$900. **5.** (1) $30\% + 7\% = 37\%$, $100\% - 37\% = 63\%$; 63% of S.P. = \$30.24; 1% of S.P. = \$30.24 ÷ 63, or \$.48; 100% of S.P. = 100 × \$.48, or \$48; (2) $26\% + 8\% = 34\%$, $100\% - 34\% = 66\%$; 66% of S.P. = \$429; 1% of S.P. = \$429 ÷ 66, or \$6.50; 100% of S.P. = 100 × \$6.50, or \$650. **6.** (1) $19\% + 11\% = 30\%$, $100\% - 30\% = 70\%$; 70% of S.P. = \$96.60; 1% of S.P. = \$96.60 ÷ 70, or \$1.38; 100% of S.P. = 100 × \$1.38, or \$138; (2) $29\% + 6\% = 35\%$, $100\% - 35\% = 65\%$; 65% of S.P. = \$481; 1% of S.P. = \$481 ÷ 65, or \$7.40; 100% of S.P. = 100 × \$7.40, or \$740. **7.** \$85; \$1250. **8.** \$42; \$350. **9.** (1) \$118; (2) $35\% + 8\% = 43\%$, $100\% - 43\% = 57\%$; 57% of S.P. = \$536;

1% of S.P. = \$536 ÷ 57, or \$9.40350; 100% of S.P. = 100 × \$9.40350, which is called \$940.35 or about \$940.

Page 74

Aim: To teach how to find the net price when two or more discounts are allowed in succession

Suggestions: Sometimes a buyer is allowed two or more discounts when he makes a purchase. Such discounts are called *successive* or *chain discounts*. Ex. 1 and 3 on this page illustrate a typical situation in which several discounts are allowed on a single purchase. From these exercises it is seen that a series of discounts, such as 20%, 10%, 2%, may arise naturally: the discount of 20% is the regular catalogue discount, the extra 10% is given because the order is very large, and the additional 2% is allowed for prompt payment of the bill.

In this work explain to the pupils that *a chain discount of 20%, 10% on \$150 is not the same as a single discount of 30%*. In the chain discount of 20%, 10%, the discount of 20% is first computed on \$150 and then subtracted for a remainder of \$120. The discount of 10% is then computed on \$120, which is a smaller base than \$150. A single discount of 30% would all be computed on \$150. It is evident that the single discount of 30% produces a larger total discount than a chain discount of 20%, 10%.

Ex. 5 on page 74 illustrates an interesting fact relative to chain discounts; namely, that the order in which a series of discounts is deducted does not affect the final net price. If you have a series of discounts such as 20%, 10%, 5%, with a list price of \$250, you can deduct the discounts in the order in which they are given, or in any other order, and in each case the net price will be the same. In other words, a chain discount of 20%, 10%, 5% is the same as a discount of 10%, 20%, 5%, or of 5%, 10%, 20%, or of 10%, 5%, 20%, or of any other order of discounts. Pupils who are especially interested in mathematics will find it a challenging exercise to verify this fact.

Workbook Reference: Arithmetic Workshop, Book 8, pages 26 and 27

Key: **2.** .20 × \$100 = \$20, \$100 − \$20 = \$80, .10 × \$80 = \$8, \$80 − \$8 = \$72; .30 × \$100 = \$30, \$100 − \$30 = \$70. **4.** .30 × \$500 = \$150; \$500 − \$150 = \$350; .10 × \$350 = \$35; \$350 − \$35 = \$315; .04 × \$315 = \$12.60; \$315 − \$12.60 = \$302.40. **5.** .10 × \$500 = \$50; \$500 − \$50 = \$450; .30 × \$450 = \$135; \$450 − \$135 = \$315; .04 × \$315 = \$12.60; \$315 − \$12.60 = \$302.40. **6.** (1) .30 × \$400 = \$120, \$400 − \$120 = \$280, .10 × \$280 = \$28, \$280 − \$28 = \$252; (2) .20 × \$800 = \$160, \$800 − \$160 = \$640, .10 × \$640 = \$64, \$640 − \$64 = \$576, .05 × \$576 = \$28.80, \$576 − \$28.80 = \$547.20. **7.** \$408; \$279.30. **8.** \$540; \$138.32. **9.** \$98; \$175.50.

Page 75

Aims: To present the first set of improvement tests in multiplication and to review important arithmetical terms

Suggestions: A set of improvement tests in multiplication is given at the top of this page. In giving Test 1a, first have the pupils copy the exercises on a sheet of paper, spreading them over the paper in order to provide ample room for working each exercise. The time assigned for this test, which is $3\frac{1}{2}$ min., is the time for giving the test *after* it has been copied. The method of giving, scoring, and recording this test is the same as that for the improvement tests in addition. See pages 375–377 of the text.

Success in arithmetic depends upon understanding the language of arithmetic; hence, the pupils should be able to give the correct words for the blank spaces in ex. 4–10.

Key: **1.** 341,388; 425,568; 105,704; 389,364; 58,672; 499,410. **2.** 153,853; 203,628; 59,422; 216,460; 438,882; 91,200. **3.** 480,128; 51,917; 715,650; 451,134; 227,682; 108,560. **4.** Expenses; profit. **5.** Discount; net price. **6.** Meter. **7.** Loss. **8.** Round number. **9.** Commission. **10.** Agent.

Page 76

Aim: To provide a review of problems that involve percentage

Key: **1.** 14.8% = .148; .148 × 1,793,500 mi. = 265,438 mi. **2.** (1) .01 × 1200 lb. = 12 lb. (hay); (2) .03 × 1200 lb. = 36 lb. (silage). **3.** (1) 30% + 10% = 40%, 100% − 40% = 60%; 60% of S.P. = \$45; 1% of S.P. = \$45 ÷ 60, or \$.75; 100% of S.P. = 100 × \$.75, or \$75; (2) .10 × \$75 = \$7.50 (profit). **4.** 35¢ − 25¢ = 10¢; 10¢ ÷ 25¢ = .40, or 40%. **5.** (1) 30 ÷ 35 = .857, or 86%; (2) 40 ÷ 45 = .888, or 89%. **6.** 15% of total amount = \$16.50; 1% of total amount = \$16.50 ÷ 15, or \$1.10; 100% of total amount = 100 × \$1.10, or \$110. **7.** (1) 12 × \$35 = \$420; .10 × \$420 = \$42; \$420 − \$42 = \$378; .10 × \$378 = \$37.80; \$378 − \$37.80 = \$340.20 (net cost); (2) \$340.20 ÷ 12 = \$28.35 (cost of 1 radio). **8.** 60% of S.P. = \$28.35; 1% of S.P. = \$28.35 ÷ 60, or \$.4725; 100% of S.P. = 100 × \$.4725, or \$47.25. **9.** (1) .20 × \$44 = \$8.80; .10 × \$44 = \$4.40, \$44 − \$4.40 = \$39.60, .10 × \$39.60 = \$3.96, \$4.40 + \$3.96 = \$8.36; so a discount of 20% is better; (2) \$44 − \$8.80 = \$35.20; (3) \$44 − \$8.36 = \$35.64.

Page 77

Aim: To present an interesting project which requires the use of percentage

Suggestion: The production of rubber is measured in *long tons*. Therefore, this unit of measure is used on this page. The long ton is used in this country only to a limited extent, but it is the standard ton in Great Britain.

Key: **1.** 2,040,000 × 2240 lb. = 4,569,600,000 lb. **2.** (1) 2000 lb.; (2) 2240 lb. − 2000 lb. = 240 lb., 240 lb. ÷ 2000 lb. = .12, or 12%. **3.** (1) 1,870,900 ÷ 2,040,000 = .9171, or 91.7% (Far East); (2) 28,100 ÷ 2,040,000 = .0137, or 1.4% (Tropical America); (3) 141,000 ÷ 2,040,000 = .0691, or 6.9% (Africa). **4.** (1) 1,842,480 long tons; (2) 635,000; 895,000; 313,000; 1,842,000; (3) 635,000 ÷ 1,842,000 = .344, or 34%; (4) 895,000 ÷ 1,842,000 = .485, or 49%; (5) 313,000

÷ 1,842,000 = .169, or 17%. **5.** (1) 97,000,000; (2) 15,000,000; (3) 97,000,000 + 15,000,000 = 112,000,000; 97,000,000 ÷ 112,000,000 = .866, or 87%; (4) 15,000,000 ÷ 112,000,000 = .133, or 13%.

Page 78

Aim: To review the work of Chapter 2

Key: **1.** (1) 14 − 9 = 5, 5 ÷ 9 = .555, or 56% (increase); (2) 57 − 23 = 34, 34 ÷ 57 = .596, or 60% (decrease); (3) 226 − 217 = 9, 9 ÷ 217 = .041, or 4% (increase); (4) 6381 − 4215 = 2166, 2166 ÷ 6381 = .339, or 34% (decrease). **2.** (1) 15 − 11 = 4, 4 ÷ 15 = .266, or 27% (decrease); (2) 53 − 42 = 11, 11 ÷ 42 = .261, or 26% (increase); (3) 416 − 150 = 266, 266 ÷ 150 = 1.773, or 177% (increase); (4) 5992 − 2925 = 3067, 3067 ÷ 2925 = 1.048, or 105% (increase). **3.** \$253.00, \$12,397.00; \$2.17, \$84.53; \$104.56, \$313.69. **4.** \$561.25, \$10,663.75; \$4.87, \$43.82; \$20.64, \$254.61. **5.** \$1.43, \$12.82; \$1.06, \$7.44; \$33.00, \$242.00. **6.** \$3.99, \$11.96; \$3.25, \$6.50; \$57.90, \$135.10. **7.** (1) \$4.50 + \$2.25 = \$6.75, \$7.50 − \$6.75 = \$.75 (profit); (2) \$124 + \$62 = \$186, \$200 − \$186 = \$14 (profit). **8.** (1) \$3.78 + \$1.80 = \$5.58, \$6.00 − \$5.58 = \$.42 (profit); (2) \$210 + \$75 = \$285, \$300 − \$285 = \$15 (profit). **9.** (1) \$5.20 + \$2.40 = \$7.60, \$8.00 − \$7.60 = \$.40 (profit); (2) \$183 + \$99 = \$282, \$300 − \$282 = \$18 (profit). **10.** (ex. 7) (1) \$.75 ÷ \$7.50 = .10, or 10%, (2) \$14 ÷ \$200 = .07, or 7%; (ex. 8) (1) \$.42 ÷ \$6.00 = .07, or 7%; (2) \$15 ÷ \$300 = .05, or 5%; (ex. 9) (1) \$.40 ÷ \$8.00 = .05, or 5%; (2) \$18 ÷ \$300 = .06, or 6%. **11.** (1) 25% + 5% = 30%, 100% − 30% = 70%; 70% of S.P. = \$8.40; 1% of S.P. = \$8.40 ÷ 70, or \$.12; 100% of S.P. = 100 × \$.12, or \$12; (2) 30% + 6% = 36%, 100% − 36% = 64%; 64% of S.P. = \$7.68; 1% of \$7.68 = \$7.68 ÷ 64, or \$.12; 100% of S.P. = 100 × \$.12, or \$12. **12.** (1) 31% + 6% = 37%, 100% − 37% = 63%; 63% of S.P. = \$5.67; 1% of S.P. = \$5.67 ÷ 63, or \$.09; 100% of S.P. = 100 × \$.09, or \$9; (2) 29% + 7% = 36%, 100% − 36% = 64%; 64% of S.P. = \$9.60; 1% of S.P. = \$9.60 ÷ 64, or \$.15; 100% of S.P. = 100 × \$.15, or \$15. **13.** (1) 31% + 7% = 38%, 100% − 38% = 62%; 62% of S.P. = \$4.34; 1% of S.P. = \$4.34 ÷ 62, or \$.07; 100% of S.P. = 100 × \$.07, or \$7; (2) 27% + 8% = 35%, 100% − 35% = 65%; 65% of S.P. = \$9.10; 1% of S.P. = \$9.10 ÷ 65, or \$.14; 100% of S.P. = 100 × \$.14, or \$14.

Page 79

Aim: To present Problem Test 2

Workbook Reference: Arithmetic Workshop, Book 8, page 37

Key: **1.** 50 × 936.3 lb. = 46,815 lb. **2.** 4778 − 2683 = 2095; 2095 ÷ 2683 = .7808, or 78.1%. **3.** 21 words − 15 words = 6 words; 6 × 6.5¢ = 39¢; \$1.45 + \$.39 = \$1.84; .10 × \$1.84 = \$.184, which is called \$.18; \$1.84 + \$.18 = \$2.02. **4.** $\frac{1}{4}$ of original price = \$2.40; $\frac{4}{4}$ of price = 4 × \$2.40, or \$9.60. **5.** (1) .91 × 500 lb. = 455 lb. (water); (2) .03 × 500 lb. = 15 lb. (protein); (3) .048 × 500 lb. = 24 lb. (carbohydrates); (4) .005 × 500 lb. = 2.5 lb. (fat); (5) .007 × 500 lb. = 3.5 lb.

(ash). **6.** (1) 750 × .6 mi. = 450 mi.; (2) 20 × 2.2 lb. = 44 lb. **7.** $12\frac{1}{2}\% = \frac{1}{8}$; $\frac{1}{8}$ × \$95 = \$11.875, which is called \$11.88. **8.** $1\frac{1}{4}$ × \$.60 = \$.75; 5 × \$.75 = \$3.75; \$37.50 ÷ \$3.75 = 10 (wk.). **9.** 3 × \$1.17 = \$3.51; \$5.00 − \$3.51 = \$1.49. **10.** 47 lb. − 40 lb. = 7 lb.; .005 × \$124 = \$.62; 7 × \$.62 = \$4.34.

Page 80

Aim: To provide a diagnostic test with page references for remedial work

Suggestion: This diagnostic test includes all three types of percentage problems. You may find that some pupils will need a re-explanation of one or more of these types of problems as well as more practice.

Workbook Reference: Arithmetic Workshop, Book 8, page 38

Key: **1.** \$.36; \$3.84; \$1.71. **2.** \$3.75; \$1.88; \$35.33. **3.** \$.10; \$2.76; \$30.42. **4.** \$.02; \$.60; \$.33. **5.** \$6; \$20; \$294. **6.** 75; $132\frac{1}{2}$. **7.** 85; 69. **8.** 36.5; 80.7. **9.** \$6880; \$644; \$92.50. **10.** 6%; 7%; $6\frac{1}{2}\%$. **11.** 6%; 8%; 7%. **12.** \$50; \$400. **13.** \$75; \$200.

Chapter 3

Aims of Chapter 3. The major aims of Chapter 3 are to:

1. Review the meaning of the geometric terms *line, ray,* and *segment.*
2. Review the formula for finding the area of a triangle.
3. Review the formulas for finding the area of a rectangle and of a square.
4. Reteach the formula for finding the area of a parallelogram.
5. Teach the formula for finding the area of a trapezoid.
6. Reteach the formula for finding the circumference of a circle.
7. Reteach the drawing of a hexagon.
8. Teach the drawing of a dodecagon.
9. Teach the formula for finding the area of a circle.
10. Provide applications of the formulas for finding areas.
11. Show how to make and read graphs of formulas.

Pages 81–83

Aim: To review and use the formula for finding the area of a triangle

Suggestions: In reviewing the formula for the area of a triangle, which is given in ex. 1 on page 82, remind the pupils that the formula is one of the *most useful* mathematical devices that we have. People in all parts of the world use formulas to express briefly the rules of mathematics and the laws of science. Formulas are particularly useful to businessmen, engineers, electricians, navigators, and surveyors. Without formulas it would be impossible to guide a ship safely at sea or to build a bridge or a skyscraper.

The pupil should memorize the formula $A = \frac{1}{2}bh$ and learn how to substitute numerical values in it. You should emphasize the fact that when two letters are written *side* by *side,* they are to be *multiplied.* This rule is important in algebra. There is no such convention in arithmetic, since multiplication is not indicated when two digits are written side by side. For example, 35 does *not* mean 3×5. When, however, a letter is written beside a numeral, or several letters are written side by side, multiplication is indicated. Thus, $3a$ means $3 \times a$, and abc means $a \times b \times c$.

On page 83 emphasize the fact that the height of a triangle is the length of a segment drawn from the vertex perpendicular to the base. Also remind the pupils that the base and the height must both be given in, or changed to, the same unit of measure; that is, both must be in inches, or in feet, and so on. See ex. 1 on page 83.

Key: Page 81 **3.** (1) $A = \frac{1}{2} \times 10 \times 16 = 80$ (sq. ft.); (2) $81 - 80 = 1$ (sq. ft.); so Fred's sail is larger.

Page 82 **3.** $A = \frac{1}{2} \times 9 \times 10 = 45$ (sq. ft.). **4.** 135 sq. in. **5.** $A = \frac{1}{2} \times 110 \times 200 = 11{,}000$ (sq. ft.); $\frac{11000}{43560} = \frac{25}{99}$ (A.), or about $\frac{1}{4}$ acre. **6.** 63 sq. in. **7.** 34 sq. ft. **8.** 68 sq. in. **9.** 30 sq. yd. **10.** 225 sq. ft. **11.** $123\frac{1}{2}$ sq. in. **12.** 14.07 sq. in. **13.** 22.63 sq. ft.

Page 83 **2.** (1) 54 sq. ft.; (2) 150 sq. in.; (3) 24 sq. yd.; (4) 1 ft. 6 in. = $1\frac{1}{2}$ ft., $\frac{1}{2} \times 1\frac{1}{2} \times 2 = 1\frac{1}{2}$ (sq. ft.). **3.** (1) 6 ft. = 2 yd., $\frac{1}{2} \times 3 \times 2 = 3$ (sq. yd.); (2) 7.2 sq. ft.; (3) 2 ft. 6 in. = $2\frac{1}{2}$ ft., $\frac{1}{2} \times 4 \times 2\frac{1}{2} = 5$ (sq. ft.); (4) $1\frac{7}{8}$ sq. mi. **4.** 42 sq. in. **5.** 67.2 sq. ft. **6.** $103\frac{1}{2}$ sq. in. **7.** 735 sq. ft. **8.** 2.56 sq. yd. **9.** $232\frac{1}{2}$ sq. ft. **10.** $68\frac{7}{8}$ sq. ft. **11.** 4 yd. 2 ft. = 14 ft.; $\frac{1}{2} \times 14 \times 10 = 70$ (sq. ft.). **12.** 6 ft. 3 in. = $6\frac{1}{4}$ ft.; $\frac{1}{2} \times 6\frac{1}{4} \times 14 = 43\frac{3}{4}$ (sq. ft.). **13.** 8 ft. 8 in. = $8\frac{2}{3}$ ft.; $\frac{1}{2} \times 8\frac{2}{3} \times 15 = 65$ (sq. ft.).

Page 84

Aim: To review the meaning of the geometric terms *line*, *ray*, and *line segment*

Suggestions: In order to give experience with the terms and their representation, provide several examples and have the pupils identify which are lines, line segments, rays, and endpoints. After considerable practice in identification, have pupils draw examples of lines, rays, and segments.

Sometimes a double arrowhead is placed above the letters that identify a line, a single arrowhead above the letters that identify a ray, and a line segment above the letters that identify a line segment, thus: $\overleftrightarrow{AB}$, $\overrightarrow{AB}$, and $\overline{AB}$. The arrowhead for the ray should be above the letter which is not the endpoint. C is the endpoint of the ray $\overrightarrow{CD}$. After identification of examples with proper symbols, have the pupils draw examples for several given symbols.

Key: **1.** The arrowheads suggest that the line does not stop at a point. **4.** Lines: extensions of line segments without apparent ends, such as a cross-country highway; rays: beams of light from a flashlight or headlight, one's line of sight, the side of an angle; segment: the edge of a box, the segment where the wall and the floor meet, the side of any polygon, the edge of this book.

Page 85

Aims: To review the formulas for the area of a rectangle and of a square, and to present problems that use areas of triangles and rectangles

Suggestions: In ex. 5 and 6 on this page, remind the pupils that the waterproof liquid is sold in quart cans and that each quart covers only 25 sq. ft. of canvas. Hence, to cover 104 sq. ft. of canvas, it is necessary to buy 5 cans of liquid, which means that some liquid will be left over.

Key: **2.** (1) 24 sq. in.; (2) 63 sq. ft.; (3) 9 in. = $\frac{3}{4}$ ft., $1 \times \frac{3}{4} = \frac{3}{4}$ (sq. ft.); (4) 1 yd. = 3 ft., $3 \times 2 = 6$ (sq. ft.). **4.** 169 sq. ft.; 529 sq. in.; 8836 sq. ft.; 13,225 sq. ft.; 196 sq. yd.; 5625 sq. ft. **5.** (1) $8 \times 5 = 40$ (sq. ft.), 2×40 sq. ft. = 80 sq. ft., area of sides; $\frac{1}{2} \times 6 \times 4 = 12$ (sq. ft.), 2×12 sq. ft. = 24 sq. ft., area of ends; 80 sq. ft. + 24 sq. ft. = 104 sq. ft., total area; (2) $104 \div 25 = 4\frac{4}{25}$ (qt.), so he will need 5 qt.; (3) $5 \times \$1.09 = \$5.45$. **6.** (1) $\frac{1}{2} \times 9 \times 8 = 36$ (sq. ft.), 4×36 sq. ft. = 144 sq. ft., total area; $144 \div 25 = 5\frac{19}{25}$ (qt.), so he will need 6 qt.; (2) $6 \times \$1.17 = \$7.02$.

Page 86

Aims: To use the formula for the area of a parallelogram in solving problems and to show experimentally that the formula is correct

Suggestions: Make clear that the *height* of a parallelogram is the length of a segment perpendicular to the base and is *not* the length of one of its slanting sides. In fact, the *height* is always measured perpendicular to the base whether the figure be a rectangle, a triangle, a parallelogram, or a trapezoid. The experiment described in ex. 2 should be performed by each pupil. You may wish to perform it also with the class, using two large identical parallelograms made of cardboard.

Key: **3.** 240 sq. ft. **4.** 60 sq. in. **5.** 15 ft. = 5 yd.; $6 \times 5 = 30$ (sq. yd.). **6.** 36 sq. ft. **7.** 175 sq. ft. **8.** 781 sq. in. **9.** 10 in. = $\frac{5}{6}$ ft.; $\frac{5}{6} \times 1 = \frac{5}{6}$ (sq. ft.). **10.** 32 in. = $2\frac{2}{3}$ ft.; $2\frac{2}{3} \times 2 = 5\frac{1}{3}$ (sq. ft.). **11.** 2 yd. = 6 ft.; $6 \times 4\frac{1}{2} = 27$ (sq. ft.). **12.** 6 ft. = 2 yd.; $3\frac{1}{3} \times 2 = 6\frac{2}{3}$ (sq. yd.).

Page 87

Aims: To present the first set of improvement tests in division, and to give a review of preceding work

Suggestions: A set of improvement tests in division is given at the top of this page. In giving Test 1a, first have the pupils copy the exercises on a sheet of paper, spreading them out over the paper in order to provide ample room for working each exercise. The time assigned for this test, which is 5 min., is the time for giving the test *after* it has been copied. The method of giving, scoring, and recording this test is the same as that for the improvement tests in addition. See pages 375–377 of the text. When giving tests in division, always tell the pupils how you wish the remainders to be handled. In these examples it is suggested that the pupils write the quotient with a fraction when there is a remainder.

Key: **1.** 347; 2936; $2740\frac{17}{36}$. **2.** $538\frac{5}{8}$; 336; 1253. **3.** 559; 2147; $562\frac{9}{32}$. **4.** $2175\frac{7}{11}$; 287; 3257. **5.** $342\frac{13}{98}$; 1264; 983. **6.** 3179; 947; $1379\frac{1}{4}$. **7.** 13,783,444; 32%; $9\frac{3}{5}$. **8.** 597; \$5.48; 12. **9.** 1304; \$345; $13\frac{1}{3}$. **10.** \$162; $1\frac{5}{16}$; $16\frac{2}{3}$. **11.** \$3.63; 150; $5\frac{1}{6}$. **12.** 86; \$150.72; $1\frac{5}{8}$. **13.** \$1.25; \$4.06; $6\frac{2}{3}$. **14.** 1,514,955; $4\frac{2}{3}$; .1. **15.** \$192.75; \$4104; $6\frac{2}{5}$.

Page 88

Aim: To present problems that require the areas of rectangles and parallelograms

Suggestions: In ex. 4 you will find it interesting to ask the pupils to find out what different kinds of hard-surfaced driveways cost per square yard in your community, and to use these figures when solving this problem. In ex. 5 call attention to the fact that the given dimensions, which are 36′ and 42′, apply to the inside rectangle. Since the walk is 3′ wide, the dimensions of the outside rectangle are 42′ and 48′. Notice that the area of the walk can be found in

two ways: (1) by finding the area of the walk directly, using the fact that the walk is 3 ft. wide, and (2) by subtracting the area of the inside rectangle from that of the outside rectangle. If method (1) is used, the area of the entire walk is made up of 4 rectangles, each 3′ wide. These rectangles have the dimensions 3′ × 48′; 3′ × 36′, 3′ × 48′, 3′ × 36′.

Workbook Reference: Arithmetic Workshop, Book 8, pages 40 and 42

Key: **1.** (1) 42 × 27 = 1134 (sq. ft.); (2) 1134 ÷ 9 = 126 (sq. yd.); or 42 ft. = 14 yd., 27 ft. = 9 yd., 14 × 9 = 126 (sq. yd.). **2.** 1134 ÷ 600 = 1.89 (gal.); so 2 gal. will be needed. **3.** (1) Parallelograms; (2) A = 250 × 125 = 31,250 (sq. ft.); (3) B = 250 × 100 = 25,000 (sq. ft.); (4) 31,250 + 25,000 = 56,250 (sq. ft.), 56,250 ÷ 43,560 = 1.291 (A.), which is called 1.29 A. **4.** 10 ft. = $3\frac{1}{3}$ yd.; 45 ft. = 15 yd.; $3\frac{1}{3}$ × 15 = 50 (sq. yd.); 50 × \$2.25 = \$112.50. **5.** (1) 36 ft. = 12 yd., 42 ft. = 14 yd., 12 × 14 = 168 (sq. yd.), area of garden; 42 ft. + 3 ft. + 3 ft. = 48 ft., or 16 yd., length of outside rectangle, 36 ft. + 3 ft. + 3 ft. = 42 ft., or 14 yd., width of outside rectangle; 16 × 14 = 224 (sq. yd.), area of walk and garden together; 224 sq. yd. − 168 sq. yd. = 56 sq. yd.; (2) 56 × \$4.75 = \$266.00. **6.** 9 ft. = 3 yd., 12 ft. = 4 yd., 18 ft. = 6 yd.; 3 × 4 = 12 (sq. yd.), 3 × 12 sq. yd. = 36 sq. yd.; 4 × 6 = 24 (sq. yd.), 2 × 24 sq. yd. = 48 sq. yd.; 36 sq. yd. + 48 sq. yd. = 84 sq. yd.; 84 × \$9.75 = \$819.00.

Pages 89–90

Aim: To develop and apply the formula for finding the area of a trapezoid

Suggestions: On page 89 the experiment described in ex. 2 should be performed in class when you develop the formula for the area of a trapezoid. Remind the pupils that the *height* of a trapezoid is the length of a segment perpendicular to the base; the height is *not* the length of one of the slanting sides of the trapezoid.

Workbook Reference: Arithmetic Workshop, Book 8, page 43

Key: Page 90 **1.** $\frac{1}{2}$ × 4 × 9 = 18 (sq. ft.). **2.** 18 × $2\frac{1}{2}$ lb. = 45 lb. **3.** $\frac{1}{2}$ × 15 × 50 = 375 (sq. ft.). **4.** No; (2) trapezoid; (3) 100 ft.; (4) Lot 1: $\frac{1}{2}$ × 100 × 270 = 13,500 (sq. ft.), Lot 2: $\frac{1}{2}$ × 100 × 330 = 16,500 (sq. ft.), Lot 3: $\frac{1}{2}$ × 100 × 390 = 19,500 (sq. ft.); (5) 13,500 + 16,500 + 19,500 = 49,500 (sq. ft.), 49,500 ÷ 43,560 = 1.13 (A.), which is called 1.1 A. **5.** $\frac{1}{2}$ × 425 × 700 = 148,750 (sq. ft.); 148,750 ÷ 43,560 = 3.41 (A.), which is called 3.4 A. **6.** $\frac{1}{2}$ × 40 × 58 = 1160 (sq. in.). **7.** 144 sq. yd. **8.** 6800 sq. ft. **9.** 1080 sq. yd. **10.** $\frac{1}{2}$ × 10 × 11 = 55 (sq. ft.). **11.** 750 sq. in. **12.** $\frac{1}{2}$ × 18 × 15 = 135 (sq. ft.). **13.** 11 ft. = $3\frac{2}{3}$ yd., $\frac{1}{2}$ × $3\frac{2}{3}$ × 18 = 33 (sq. yd.); or 10 yd. = 30 ft., 8 yd. = 24 ft., $\frac{1}{2}$ × 11 × 54 = 297 (sq. ft.).

Page 91

Aim: To reteach the formula for finding the circumference of a circle

Suggestions: The formula for finding the circumference of a circle was first developed in *American Arithmetic, Grade 7;* hence, the work on page 91 of the pres-

ent text is a reteaching of that work. It is important to be familiar with the terms *radius, diameter, circumference,* and *arc.* Draw a circle on the board, using a piece of string for a radius. Then draw a diameter of the circle, emphasizing that a diameter always passes through the center of the circle. Also show by measuring with a yardstick that the diameter is the longest distance across a circle. This fact is a helpful one when the center of the circle is not known. If you wish to find the diameter of the circular top of a wastebasket, you make use of this fact because the center of the circular top is not known.

For ordinary purposes you can use either 3.14 or $3\frac{1}{7}$ for π. If you change $3\frac{1}{7}$ to a decimal, rounded off to 5 places, you get 3.14286. It is interesting to note that 3.14 is slightly smaller and $3\frac{1}{7}$ slightly larger than 3.14159, which is the value of π correct to 5 decimal places.

Workbook Reference: Arithmetic Workshop, Book 8, page 45

Key: **4.** 78.5 ft.; 94.2 in.; 31.4 mi.; 109.9 in. **5.** 392.5 ft.; 50.24 in.; 628 ft.; 15.7 yd.

Page 92

Aim: To reteach the drawing of designs based on a hexagon inscribed in a circle

Suggestions: The method of drawing an inscribed hexagon, described in ex. 1, is the best and the most rapid way to draw a hexagon since it requires the drawing of only two arcs. The method more commonly used, in which you step around the circle with the compasses, requires the drawing of six arcs, the last arc being a check on the accuracy of the work. When this method is used, it is often found that the last arc does not pass through the original starting point, which it should do; that is, the work does not check. Both methods of drawing a hexagon are described at length on page 186 of *American Arithmetic, Grade 7.*

After the pupils have copied the designs on this page, you may wish to have them create some designs based on a hexagon inscribed in a circle.

Page 93

Aim: To show that the formula for the circumference of a circle is a reasonable one

Suggestions: On page 309 of *American Arithmetic, Grade 7,* certain experiments were suggested by which the formula for the circumference of a circle can be developed approximately. If time permits, you may care to repeat those experiments at this time. In addition, the experiment described in ex. 1 on page 93 of this text may be carried out. In this experiment it is seen in the figure in ex. 1 that the perimeter of the hexagon is 6 in. long, or exactly 3 times the diameter. It is also evident that the circumference in this figure is a little longer than the perimeter of the hexagon; hence, the circumference is a little longer than 3 times the diameter. The pupil should then be told that "a little longer than 3 times the diameter" is more accurately stated by saying that the circumference is 3.14 or π times the diameter. When the pupil studies geometry in senior high school,

he will learn more about π. Ex. 5 suggests an interesting and practical way to measure longer distances approximately.

Key: **4.** (1) $\frac{22}{7} \times 28 = 88$ (in.); (2) 88 in. ÷ 12 in. = $7\frac{1}{3}$ (ft.). **5.** (1) $182 \times 7\frac{1}{3}$ ft. = $1334\frac{2}{3}$ ft.; (2) $\frac{1}{4} \times 5280$ ft. = 1320 ft.; so the distance is about $\frac{1}{4}$ mi.

Pages 94–95

Aim: To teach the drawing of designs based on a circle divided into 12 equal parts

Suggestions: In ex. 7 you may wish to encourage the pupils to create several designs of their own. Some of the best designs can be selected for a bulletin-board display.

Pages 96–97

Aims: To present the formula for the area of a circle and to show experimentally that the formula is reasonable

Suggestions: On page 96 make clear to the pupils that the formula $A = \pi r^2$ means $A = \pi \times r \times r$. If $r = 4$, then $A = 3.14 \times 4 \times 4$, or 50.24. In ex. 4 remind the pupils that in measuring the diameter of a round table or of a plate, they should take the greatest distance across the circle as the diameter, since the center is not known. On page 97 a careful study should be made of the circle that is drawn on squared paper. Ask the pupils to count carefully the squares in each row of the colored part of the circle, remembering to drop a part of a square that is less than a half square and to count as a whole square a part of a square that is larger than a half square. A half square should be counted as a half square. If the pupils count the squares correctly, they will get the numbers shown in the vertical column in the upper right-hand corner of the figure. It is desirable that each pupil also perform his own experiment on squared paper.

Workbook Reference: Arithmetic Workshop, Book 8, page 46

Key: Page 96 **3.** $3.14 \times 60 \times 60 = 11{,}304$ (sq. ft.). **5.** 12,474 sq. in.; 7546 sq. yd.; 9856 sq. in.; 15,400 sq. ft.; 18,634 sq. in.; 3850 sq. yd. **6.** 15,386 sq. ft.; 7850 sq. yd.; 1256 sq. in.; 20,096 sq. yd.; 5024 sq. ft.; 31,400 sq. ft.

Page 98

Aim: To review problem solving connected with areas

Suggestion: In ex. 3 on this page it is necessary to use the fact that 1 acre = 43,560 sq. ft. This fact is found in the table of square measure given on page 373 of the text.

Key: **1.** (1) The roof and the two parts of the back are rectangles, and the sides are trapezoids; (2) $7 \times 8 = 56$ (sq. ft.), $7 \times 9 = 63$ (sq. ft.), $3 \times 7 = 21$ (sq. ft.), $\frac{1}{2} \times 8 \times 10 = 40$ (sq. ft.), 2×40 sq. ft. = 80 sq. ft., $56 + 63 + 21 + 80 = 220$ (sq. ft.); $220 \div 25 = 8\frac{4}{5}$ (qt.), so he will need 9 qt.; (3) 9 qt. = 2 gal. 1 qt., so he should buy 2 gallon cans and 1 quart can; (4) $2 \times \$4.49 = \$8.98$, $\$8.98 + \$1.29$

= \$10.27. **2.** $25 \times 18 = 450$ (sq. in.); $3 \times 5 = 15$ (sq. in.); $450 \div 15 = 30$ (cards). **3.** (1) $605 \times 180 = 108{,}900$ (sq. ft.), $108{,}900 \div 43{,}560 = 2.5$ (A.); (2) \$425 $\div$ 2.5 = \$170. **4.** (1) She can use 2 strips of linoleum 6 ft. wide and 14 ft. long; (2) $2 \times 14 = 28$ (running feet); (3) $28 \times \$.69 = \$19.32$. **5.** $9 \times 9 = 81$ (sq. ft.); $81 \div \frac{1}{2} = 162$ (bulbs).

Page 99

Aim: To present the second set of improvement tests in addition

Suggestions: The second set of improvement tests in addition, which is given on this page, is to be administered in the same manner as the first set of such tests which was given on page 49 of the text. Remember that an improvement test in addition is not to be copied on paper. Instead, the answers are to be written on folded paper, as explained on page 375.

Key: **1.** 503; 611; 637; 506; 625; 634; 542; 565. **2.** 554; 506; 630; 655; 616; 632; 549; 505. **3.** 654; 576; 613; 797; 603; 546; 410; 640.

Page 100

Aim: To provide more problems on finding the areas of circles

Suggestions: In ex. 2 remind the pupils that the formula gives the area of a circle in terms of its radius. Thus, if the diameter of a circle is given, it is necessary to find the radius before using the formula. Ex. 6 is an illustration of the type of problem in which a required area is found indirectly by using two areas that can be found easily by formula. This approach to problem solving is important and should be carefully developed.

Key: **1.** 113.04 sq. ft. **2.** 50.24 sq. ft. **3.** $28 \times 28 = 784$ (sq. ft.); $\frac{22}{7} \times 14 \times 14 = 616$ (sq. ft.); 784 sq. ft. − 616 sq. ft. = 168 sq. ft.; so the square garden is 168 sq. ft. larger. **4.** (1) $3.14 \times 15 \times 15 = 706.5$ (sq. ft.); (2) $3.14 \times 10 \times 10 = 314$ (sq. in.); (3) $3.14 \times 25 \times 25 = 1962.5$ (sq. ft.); (4) $3.14 \times 12 \times 12 = 452.16$ (sq. mi.). **5.** (1) $3.14 \times 30 \times 30 = 2826$ (sq. ft.); (2) $3.14 \times 8 \times 8 = 200.96$ (sq. yd.); (3) $3.14 \times 3\frac{1}{2} \times 3\frac{1}{2} = 38.465$ (sq. in.); (4) $3.14 \times 18 \times 18 = 1017.36$ (sq. in.). **6.** (1) $\frac{22}{7} \times 21 \times 21 = 1386$ (sq. ft.), area of small circle; $\frac{22}{7} \times 28 \times 28 = 2464$ (sq. ft.), area of large circle; 2464 sq. ft. − 1386 sq. ft. = 1078 sq. ft., area of walk; (2) $1078 \div 9 = 119\frac{7}{9}$ (sq. yd.); (3) $120 \times \$4.95 = \$594.00$. **7.** (1) 93 ft. − 2 ft. = 91 ft., radius of Dick's circle; $2 \times \frac{22}{7} \times 91 = 572$ (ft.); 5280 ft. $\div$ 572 ft. = 9.23 (times), or about $9\frac{1}{4}$ times; (2) $\frac{22}{7} \times 93 \times 93 = 27{,}182\frac{4}{7}$ (sq. ft.).

Page 101

Aim: To give a rule for choosing the correct value of π for use under certain conditions

Suggestions: If your class is below average in ability, or if your time for arithmetic is limited, the work on this page may be omitted. If properly done, this work requires careful study and considerable practice in using the several values of π in appropriate exercises.

In ex. 5 notice that the answer is to be rounded off to the nearest hundredth of an inch. In problems of this type it is undesirable to give answers with more decimal places than the original measurements. Thus, it is customary to round off the answers after they have been computed.

Key: **2.** 3.142; 3.14. **5.** 3.142 × 8.65 in. = 27.17830 in., or 27.18 in. **6.** (1) 3.14 × 2.5 ft. = 7.850 ft., or 7.85 ft.; (2) 3.142 × 325 ft. = 1021.150 ft., or 1021.15 ft.; (3) 3.1416 × 45.25 in. = 142.157400 in., or 142.16 in.; (4) 2 × 3.14159 × 145.27 ft. = 912.7575586 ft., or 912.76 ft. **7.** 3.14 × 23 × 23 = 1661.06 (sq. in.), or 1661.1 sq. in.

Pages 102–104

Aim: To show how to make tables from formulas and how to make and read graphs of formulas

Suggestions: Line graphs were presented in *American Arithmetic, Grade 7* on pages 251–254. Pupils were taught to read them and to make them from tables of values. You may review this material by having pupils interpret some line graphs you have selected from current newspapers and magazines. The new work on these pages concerns the formula. A table of values is obtained from a formula, and the line graph is then made from this table. Thus you have a *graph of the formula*. Only simple formulas that make graphs with straight lines are presented here. Perhaps some of the pupils have already seen and recognized the more complicated graphs and may want to explore them on their own initiative.

In doing the work on pages 103 and 104, each pupil should be supplied with a few sheets of squared paper on which the graphs suggested in the exercises can be drawn. The work on pages 102–104 requires considerable time if it is done properly.

Key: Page 102 **2.** 6, 18; 7, 21; 8, 24.

3. $D = 8t$

Hours t	Miles D
0	0
1	8
2	16
3	24
4	32
5	40

4. $D = 40t$

Hours t	Miles D
0	0
1	40
2	80
3	120
4	160
5	200
6	240
7	280
8	320
9	360

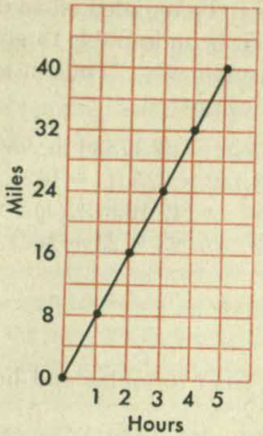
5. $D = 300t$

Hours t	Miles D
0	0
1	300
2	600
3	900
4	1200
5	1500
6	1800

6. $D = 55t$

Hours t	Miles D
0	0
1	55
2	110
3	165
4	220
5	275
6	330
7	385
8	440

Page 103 **2.**



Page 104 **2.** 60 mi.; 80 mi. **3.** 4 hr.; 3 hr.; 2 hr. **5.** 30 mi.; 70 mi.; 10 mi. **6.** 120 mi.; 140 mi.; 110 mi.; 130 mi.; 90 mi. **7.** 6 hr.; 7 hr.; $7\frac{1}{2}$ hr.; $3\frac{1}{2}$ hr.; $6\frac{1}{2}$ hr.; $4\frac{1}{2}$ hr.

Page 105

Aim: To give a review of computation with fractions, decimals, and percentage

Key: **1.** $22\frac{15}{16}$; $15\frac{2}{3}$. **2.** $32\frac{3}{5}$; $26\frac{3}{4}$. **3.** $1\frac{1}{2}$; $22\frac{1}{2}$; $10\frac{2}{3}$; $\frac{2}{3}$. **4.** 165; $\frac{3}{4}$; 6; $4\frac{1}{20}$. **5.** $6\frac{1}{2}$; $11\frac{9}{16}$; $1\frac{1}{2}$; $3\frac{13}{24}$. **6.** 1.14; 1.06; .05; 3.56; .37. **7.** 700; 31.72; 1637.5; 57.5; 24.79. **8.** 7.86; 29; 15.96; 3850; .37. **9.** (1) Think of 48% as about $\frac{1}{2}$ and of \$10.25 as about \$10, $\frac{1}{2} \times \$10 = \$5$ (estimate); $.48 \times \$10.25 = \$4.92$ (exact); (2) think of 12.4% as about $\frac{1}{8}$, $\frac{1}{8} \times \$80 = \$10$ (estimate); $.124 \times \$80 = \$9.92$ (exact); (3) think of 151% as about $1\frac{1}{2}$ and of \$79 as about \$80, $1\frac{1}{2} \times \$80 = \$120$ (estimate); $1.51 \times \$79 = \$119.29$ (exact). **10.** (1) Think of 11% as about 10% and of \$49 as about \$50, $.10 \times \$50 = \$5$ (estimate); $.11 \times \$49 = \$5.39$ (exact); (2) think of 19.6% as about 20%, or $\frac{1}{5}$; $\frac{1}{5} \times \$35 = \$7$ (estimate); $.196 \times \$35 = \$6.86$ (exact); (3) think of 198% as about 2 and of \$81 as about \$80, $2 \times \$80 = \$160$ (estimate); $1.98 \times \$81 = \$160.38$ (exact). **11.** (1) Think of 24% as about 25%, or $\frac{1}{4}$, and of \$28.75 as about \$28; $\frac{1}{4} \times \$28 = \$7$ (estimate); $.24 \times \$28.75 = \$6.90$ (exact); (2) think of 33.2% as about $33\frac{1}{3}$%, or $\frac{1}{3}$; $\frac{1}{3} \times \$75 = \$25$ (estimate); $.332 \times \$75 = \$24.90$ (exact); (3) think of 74% as about 75%, or $\frac{3}{4}$; $\frac{3}{4} \times \$60 = \$45$ (estimate); $.74 \times \$60 = \$44.40$ (exact). **12.** (1) Think of 19% as about 20%, or $\frac{1}{5}$, and of \$26 as about \$25; $\frac{1}{5} \times \$25 = \$5$ (estimate); $.19 \times \$26 = \$4.94$ (exact); (2) think of 37.4% as about $37\frac{1}{2}$%, or $\frac{3}{8}$; $\frac{3}{8} \times \$40 = \$15$ (estimate); $.374 \times \$40 = \$14.96$ (exact); (3) think of $49\frac{1}{2}$% as about 50%, or $\frac{1}{2}$; $\frac{1}{2} \times \$80 = \$40$ (estimate); $.495 \times \$80 = \$39.60$ (exact). **13.** $77\frac{1}{2}$; 60; 32. **14.** 19; 87; 21. **15.** 46.0; 84.5; 37.8. **16.** (1) 8% of the number = 28; 1% of the number = 28 ÷ 8, or 3.5; 100% of the number = 100 × 3.5, or 350; (2) 15% of the number = 36; 1% of the number = 36 ÷ 15, or 2.4; 100% of the number = 100 × 2.4, or 240; (3) 20% of the

number = 48; 1% of the number = 48 ÷ 20, or 2.4; 100% of the number = 100 × 2.4, or 240. **17.** 4100; 125; 375.

Page 106

Aim: To review the formulas that have been presented in this chapter

Suggestions: In ex. 1 the pupil finds the unknown factor d by dividing 198 by $3\frac{1}{7}$. In ex. 3 the pupil substitutes $b = 6$ and $A = 15$ in the formula $A = \frac{1}{2}bh$ and gets $15 = 3h$. He then finds the unknown factor h. Using a formula in this way not only reviews the formula but also extends its usefulness. Some pupils may need help and practice in finding an unknown factor of a product.

Key: **1.** 63 ft. **2.** 4 yd. **3.** 15 sq. ft. is the area of the triangle and equals $\frac{1}{2}bh$; then $bh = 30$ ft.; if the base is 6 ft., the height is 5 ft., since $6 \times 5 = 30$. **4.** (1) $\frac{1}{2} \times 15 \times 76 = 570$ (sq. ft.), 2×570 sq. ft. = 1140 sq. ft.; $\frac{1}{2} \times 24 \times 12 = 144$ (sq. ft.), 2×144 sq. ft. = 288 sq. ft.; 1140 sq. ft. + 288 sq. ft. = 1428 sq. ft.; (2) 1428 ÷ 33 = $43\frac{9}{33}$ (bundles), which is called 44 bundles; (3) 44 × \$3.20 = \$140.80. **5.** (1) $\frac{1}{2} \times 100 \times 300 = 15{,}000$ (sq. ft.); (2) 15,000 ÷ 1000 = 15 (thousands), 15 × 40 lb. = 600 lb.; (3) 600 lb. ÷ 100 lb. = 6 (bags). **6.** $A = bh$, so 75,000 sq. ft. = bh; if the height is 250 ft., the base is 300 ft., since 300 × 250 = 75,000. **7.** $C = \pi d$, so 628 ft. = πd; since $\pi = 3.14$, the diameter is 200 ft., because 3.14 × 200 = 628. **8.** $\frac{1}{2}h(a + b) = 200$ sq. ft., so $h(a + b) = 400$ sq. ft.; $a + b = 25$ ft.; then $h = 400 \div 25$, or 16 (ft.).

Page 107

Aim: To present Problem Test 3

Key: **1.** 17 ÷ 21 = .8095, which is called 81.0%; 38 ÷ 45 = .8444, which is called 84.4%; so 38 games out of 45 games is the better record. **2.** $\frac{1}{2} \times 8 \times 9 = 36$ (sq. ft.); 36 × 5 lb. = 180 lb. **3.** $\frac{1}{2} \times 6 \times 8 = 24$ (sq. ft.). **4.** .15 × \$240 = \$36, \$240 − \$36 = \$204; 100% − 15% = 85%, .85 × \$240 = \$204. **5.** 5% of S.P. = \$375; 1% of S.P. = \$375 ÷ 5, or \$75; 100% of S.P. = 100 × \$75, or \$7500. **6.** \$.02¼ = \$.0225; \$12.42 ÷ \$.0225 = 552 (papers). **7.** 20,437 − 11,243 = 9194 (increase); 9194 ÷ 11,243 = .817, which is called 82%. **8.** 60 × 3½ mi. = 210 mi. (per hour); 2½ × 210 mi. = 525 mi. **9.** 60% of S.P. = \$6.90; 1% of S.P. = \$6.90 ÷ 60, or \$.115; 100% of S.P. = 100 × \$.115, or \$11.50. **10.** .01 × \$300 = \$3.00; \$300 + \$3 = \$303.

Page 108

Aim: To provide a diagnostic test and suggested practice pages

Suggestion: Use the results of this test as the basis of practice assignments.

Key: **1.** 57 sq. ft.; $33\frac{3}{4}$ sq. ft. **2.** 96 sq. ft.; 56 sq. ft. **3.** 255 sq. ft.; $1\frac{2}{3}$ sq. ft. **4.** 36 sq. ft.; 64 sq. in.; $\frac{1}{4}$ sq. yd.; 90.25 sq. ft. **5.** 35 sq. ft.; 162 sq. in. **6.** 26 sq. ft.; 645 sq. in. **7.** 62.8 yd.; 141.3 ft.; 34.54 in.; 942 ft. **8.** Round off answers to nearest tenth: 615.4 sq. in.; 314 sq. ft.; 19.6 sq. yd.; 1384.7 sq. in. **9.** 2464 sq. ft.; $346\frac{1}{2}$ sq. ft.; $1257\frac{1}{7}$ sq. in.; 154 sq. yd.

Chapter 4

Aims of Chapter 4. The major aims of Chapter 4 are to:

1. Review how to open a checking account, write checks, and keep a record of bank balances.
2. Show how service charges on checking accounts are determined.
3. Reteach how to compute interest for 1 yr. and $\frac{1}{2}$ yr.
4. Review and extend the use of the formula for computing interest.
5. Give the procedure for borrowing money from a bank, and teach bank discount.
6. Discuss small loans and teach how to find the rate of interest that is actually paid on small loans.
7. Discuss installment buying and show how to find the carrying charge.
8. Show that the carrying charge in installment buying is the equivalent of an interest charge, and teach how to find the rate of interest this charge represents.

Page 109

Aim: To describe the common services of a commercial bank

Page 110

Aim: To review the procedure for opening a checking account and making out deposit slips

Suggestions: A knowledge of certain of the more common procedures in banking is of importance to everyone. Pupils should be taught to make out deposit slips, to write and endorse checks correctly, and to keep a record of bank balances on checkbook stubs. These things can be learned properly only by filling out the actual forms. If possible, the school should provide the pupils with blank checks and deposit slips.

Key: **1.** \$136.79. **2.** \$178.83. **3.** \$1859.05. **4.** \$925.82. **5.** \$559.12.

Pages 111–112

Aim: To review writing checks and keeping a record of bank balances on checkbook stubs

Suggestions: On page 111 the check illustrated in ex. 1 should be carefully studied. In the illustration point out that the check is the part to the right of the vertical dotted line, while the stub is the part to the left of that line. After the check is written, it is torn out of the checkbook, but the stub remains in the book and serves as a record. In a check the amount is written in figures immediately after the dollar sign and as close to the sign as possible. If a space is left between the dollar sign and the amount, a dishonest person might raise the amount by inserting another number. The amount is written in words also; in this

case the writing should begin at the extreme left-hand end of the line of writing so that the amount cannot be raised. If the amount is less than one dollar, such as $0.75, it may be written in words, as "Seventy-five cents" or "Only seventy-five cents," in which case the word *Dollars* is crossed out. The amount "$\frac{75}{100}$" may be written at the extreme left-hand end of the line used for writing the amounts in words, in which case the word *Dollars* is not crossed out. If the amount written in figures differs from the amount written in words, the bank usually returns the check to the depositor, who informs the writer of the check of the discrepancy.

The stub of the check should be filled out at the time the check is written; otherwise the writer may forget that he wrote the check, or he may forget the amount of the check and to whom it was written. In endorsing a check the endorsement should be written across the back of the *left end* of the check.

In ex. 2 on page 111 the advantage in paying the rent by check rather than in cash is that the check serves as a receipt. After the bank pays the check, it is marked "Paid" and is returned periodically by the bank to the writer of the check.

Workbook Reference: Arithmetic Workshop, Book 8, page 50

Key: Page 111 **5.** No; endorsement in full.

Page 112 **2.** A stub of a check should be filled out when the check is written so that one can keep a record of his balance in order not to overdraw his account. If checks are numbered, one can easily keep track of them.

3.

No. 11
Date Dec. 5, 19—
To Paul Baker
For Wages

Bal. forward	841	79
Deposited		
Total		
This check	68	00
Bal. forward	773	79

4.

No. 12
Date Dec. 8, 19—
To Hall Brothers
For 2 Chairs

Bal. forward	773	79
Deposited	56	75
Total	830	54
This check	73	35
Bal. forward	757	19

Page 113

Aim: To present the second set of improvement tests in subtraction

Suggestions: The second set of improvement tests in subtraction is given on this page. Since these are subtraction tests, the answers are to be written on folded paper. See page 375 of the text.

Key: **1.** 279,854; 28,860; 805,278; 148,731. **2.** 553,699; 834,678; 544,998; 517,547. **3.** 20,754; 663,994; 36,855; 263,583. **4.** 96,279; 169,494; 444,643; 221,437. **5.** 287,581; 81,798; 216,728; 138,095. **6.** 514,747; 537,839; 368,426; 69,866. **7.** 657,558; 101,451; 348,683; 87,148. **8.** 492,198; 293,489; 85,769; 575,276. **9.** 110,291; 567,979; 627,586; 759,676.

Pages 114–115

Aim: To explain service charges on checking accounts and the reason for them

Suggestions: The practice of banks with respect to service charges varies widely. Have the pupils inquire about the practice of your local banks as to service charges, and report to the class. You will find it effective to use local service charges in solving some problems similar to ex. 6 and 8 on page 115. Have the pupils inquire also as to the practice in your locality of handling small accounts by making a charge for each check drawn, as described in ex. 7 on page 115.

Key: **4.** $6 \times \$.06 = \$.36$; $10 \times \$.04 = \$.40$; $\$.50 + \$.36 + \$.40 = \$1.26$; a balance of \$325 calls for a credit of \$.30; $\$1.26 - \$.30 = \$.96$. **6.** A balance of \$259 calls for 20 free checks; $23 - 20 = 3$ (checks); $3 \times 5¢ = 15¢$. **8.** $3 \times 6¢ = 18¢$, charge for deposits; $9 \times 4¢ = 36¢$, charge for checks; $\$.50 + \$.18 + \$.36 = \$1.04$; $10 \times \$.05 = \$.50$; $\$1.04 - \$.50 = \$.54$.

Page 116

Aim: To show why a checking account is useful

Suggestions: Ex. 4 on this page states that Mr. Bell had "overdrawn" his account. This statement means that he did not have enough money on deposit in the bank to pay all the checks he had drawn. Since the bank asked Mr. Bell to deposit \$50 at once, Mr. Bell's bank balance needed \$50 more to cover checks drawn by him and received by the bank that day. Banks charge a penalty for each check overdrawn.

Key: **2.** By his endorsement. **6.** The statement is true if all amounts you receive are deposited in the bank and if all amounts you spend are paid by check. Most people pay cash for many items that are purchased instead of paying for them by check; under such circumstances, the statement is not true.

Page 117

Aim: To give oral practice in using mathematical terms and information and in performing certain computations

Key: **1.** Perimeter; area. **2.** Trapezoid. **3.** Circle; $3\frac{1}{7}$ or 3.14. **4.** Parallelogram. **5.** Rectangles; squares. **6.** Triangle. **7.** $C = 2\pi r$ or $C = \pi d$. **8.** Multiplied. **9.** \$8; \$20; \$5; \$10; \$40; \$60; \$80; \$30. **10.** 10%; 20%; 50%; 100%; 200%; 600%. **11.** 4; 25; 64; 16; 81. **12.** (By 10) 2160, 21.6, 2.16, 216, .216; (by 100) 21,600, 216, 21.6, 2160, 2.16; (by 1000) 216,000, 2160, 216, 21,600, 21.6. **13.** $\frac{9}{12}$; $\frac{15}{20}$; $\frac{54}{32}$; $\frac{36}{48}$; $\frac{6}{8}$; $\frac{12}{16}$. **14.** \$1.20; \$.14; \$45.00; \$1.00; \$.06; \$.17.

Pages 118–119

Aim: To reteach how to compute interest for periods of 1 yr. and $\frac{1}{2}$ yr.

Suggestions: In most situations the interest on ordinary loans *seldom runs beyond a year* without being paid. Problems involving time beyond one year usually serve to let one know in advance what the total amount of interest would be

for such a long period, rather than to indicate that all this interest is paid at one time. On loans running for a year or more, *the interest is usually paid quarterly, semiannually, or annually.*

You should emphasize the fact that the rate of interest always means the rate *for one year* unless there is some statement to the contrary. For example, if money is borrowed at 5%, the rate is 5% for one year. In making small loans, interest may be charged at the rate of 2% a month, but in such cases it must be clearly stated that this rate applies for a month. Otherwise the rate of 2% would be understood to apply for an entire year.

Workbook Reference: Arithmetic Workshop, Book 8, page 51

Key: Page 118 **2.** \$24; \$424. **3.** (1) $.05 \times \$2400 = \$120$; (2) $.03\frac{1}{2} \times \$1750 = \$61.25$; (3) $.03\frac{1}{2} \times \$2350 = \$82.25$. **4.** \$94.50; \$118.67; \$33.75. **5.** \$50; \$107.25; \$94.90. **6.** \$114; \$80.75; \$201.25. **7.** \$32; \$16; \$816. **8.** (1) $.06 \times \$638 = \$38.28$, $\frac{1}{2} \times \$38.28 = \$19.14$; (2) $.065 \times \$3500 = \$227.50$, $\frac{1}{2} \times \$227.50 = \$113.75$; (3) $.05 \times \$1200 = \$60$, $\frac{1}{2} \times \$60 = \$30$. **9.** \$18.54; \$42.90; \$18.75. **10.** \$26.25; \$55.48; \$27.90. **11.** $\frac{1}{4}$ yr.; \$8.

Page 119 **1.** $.045 \times \$3000 = \$135$; $\frac{1}{2} \times \$135 = \$67.50$. **2.** Oct. 1. **3.** $.045 \times \$2000 = \$90$; $\frac{1}{2} \times \$90 = \$45$. **4.** $.045 \times \$1000 = \$45$; $2 \times .045 \times \$2000 = \$180$; $\$45 + \$180 = \$225$.

Page 120

Aim: To show (1) how to compute interest for part of a year when the time is given in months and (2) how to use the interest formula

Suggestions: The method of finding interest by using the formula is the best method for a person to use in his everyday activities. The formula gives him *one* method for all situations, it is easy to remember, and the required computation can often be simplified by cancellation. Point out to the pupil that when using the formula, the rate of interest is expressed as a fraction or as a decimal, and the time is expressed in years.

Attention is called to the suggestion in ex. 4, which states that the fraction of a year, which represents the time, should first be changed to lowest terms in order to avoid extra cancellation.

For additional practice, the examples on page 118 can be solved by using the formula.

Workbook Reference: Arithmetic Workshop, Book 8, page 52

Key: **4.** \$7.50. **5.** (1) $i = \frac{\$300}{1} \times \frac{4}{100} \times \frac{2}{3} = \$8$; (2) $i = \frac{\$800}{1} \times \frac{6}{100} \times \frac{7}{12} = \$28$; (3) $i = \frac{\$900}{1} \times \frac{3}{100} \times \frac{5}{12} = \$11.25$. **6.** (1) $i = \frac{\$450}{1} \times \frac{3}{100} \times \frac{1}{6} = \$2.25$; (2) $i = \frac{\$500}{1} \times \frac{4}{100} \times \frac{3}{4} = \$15$; (3) $i = \frac{\$800}{1} \times \frac{4}{100} \times \frac{7}{12} = \$18.666$, or \$18.67.

7. \$10; \$3.85; \$1.50. **8.** \$16.67; \$6.25; \$11.67. **9.** \$29.75; \$7; \$5.83.

Page 121

Aim: To show how to find the exact number of days between dates

Suggestions: In finding the exact number of days between dates, it is necessary to remember the number of days in each month. The following rhyme, which many of the pupils already know, is useful for this purpose. It might be read to the class.

Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
To which we twenty-eight assign
Till leap year gives it twenty-nine.

Most pupils will probably remember that a leap year occurs every four years when the year is exactly divisible by 4. But you will have to remind them of the exception to this rule. The centenary years are *not* leap years unless they are exactly divisible by 400. Thus, 1900 was not a leap year, but 2000 will be a leap year.

Key: **2.** (1) 22 da. (Aug.) + 30 da. (Sept.) + 31 da. (Oct.) + 30 da. (Nov.) + 7 da. (Dec.) = 120 da.; (2) 2 da. (June) + 7 da. (July) = 9 da. **3.** (1) 23 da. (Jan.) + 13 da. (Feb.) = 36 da.; (2) 20 da. (Sept.) + 31 da. (Oct.) + 12 da. (Nov.) = 63 da. **4.** (1) 3 da. (Mar.) + 30 da. (Apr.) + 31 da. (May) + 11 da. (June) = 75 da.; (2) 13 da. (Mar.) + 30 da. (Apr.) + 31 da. (May) + 19 da. (June) = 93 da. **5.** (1) 6 da. (May) + 30 da. (June) + 31 da. (July) + 23 da. (Aug.) = 90 da.; (2) 3 da. (Apr.) + 31 da. (May) + 30 da. (June) + 31 da. (July) + 31 da. (Aug.) + 30 da. (Sept.) + 31 da. (Oct.) + 29 da. (Nov.) = 216 da. **6.** 24 da. (Jan.) + 29 da. (Feb.) + 1 da. (Mar.) = 54 da.

Page 122

Aim: To find the interest on a given amount of money for an exact number of days

Suggestions: It is important to emphasize, as stated in ex. 1, that in computing interest, banks usually count 360 days to the year and the exact number of days between dates. The practice of counting 360 days to the year gives more interest than would be obtained by counting 365 days to the year. This is shown in ex. 7 on this page.

Workbook Reference: Arithmetic Workshop, Book 8, page 53

Key: **1.** $\frac{1}{15}$. **3.** (1) 28 da. (May) + 8 da. (June) = 36 da., $\frac{36}{360} = \frac{1}{10}$, $i = \frac{\$600}{1} \times \frac{6}{100} \times \frac{1}{10} = \$3.60$; (2) 25 da. (July) + 20 da. (Aug.) = 45 da., $\frac{45}{360} = \frac{1}{8}$, $i = \frac{\$2000}{1} \times \frac{6}{100} \times \frac{1}{8} = \$15$. **4.** (1) 28 da. (June) + 31 da. (July) + 13 da. (Aug.)

$= 72$ da., $\frac{72}{360} = \frac{1}{5}$, $i = \frac{\$125}{1} \times \frac{6}{100} \times \frac{1}{5} = \$1.50$; (2) 12 da. (Sept.) + 18 da. (Oct.) $= 30$ da., $\frac{30}{360} = \frac{1}{12}$, $i = \frac{\$1200}{1} \times \frac{6}{100} \times \frac{1}{12} = \$6$. **5.** (1) 29 da. (July) + 31 da. (Aug.) + 30 da. (Sept.) + 3 da. (Oct.) = 93 da., $\frac{93}{360} = \frac{31}{120}$, $i = \frac{\$280}{1} \times \frac{6}{100} \times \frac{31}{120} = \$4.34$; (2) 1 da. (Nov.) + 5 da. (Dec.) = 6 da., $\frac{6}{360} = \frac{1}{60}$, $i = \frac{\$3500}{1} \times \frac{6}{100} \times \frac{1}{60} = \$3.50$. **6.** (1) 15 da. (May) + 30 da. (June) + 31 da. (July) + 4 da. (Aug.) = 80 da., $\frac{80}{360} = \frac{2}{9}$, $i = \frac{\$900}{1} \times \frac{6}{100} \times \frac{2}{9} = \$12$; (2) 24 da. (Apr.) + 31 da. (May) + 30 da. (June) + 31 da. (July) + 19 da. (Aug.) = 135 da., $\frac{135}{360} = \frac{3}{8}$, $i = \frac{\$2400}{1} \times \frac{6}{100} \times \frac{3}{8} = \$54$. **7.** $29.59; $30; 360 da. to 1 yr. gives $.41 more.

Pages 123–124

Aim: To present a short method of computing interest

Suggestions: Although using the formula is the basic method of computing interest, the "6% method" is an interesting method and saves work in computation in certain situations. The 6% method may be omitted if time is limited. It should not be taught until the pupils have mastered the basic method. However, if there is time for the 6% method and if it is carefully explained, the pupils will enjoy the number relationships involved in it and will be able to save time with this method in those situations where it is especially useful.

The 6% method of computing interest, which is called also the "60-day method" and the "6% 60-day method," is based on the fact that the interest at 6% for 60 days is equal to 1% of the principal. It is important to remember that the 6% method counts 360 days to the year; hence, this method should be used *only* in those problems where the year is counted in this way. Since banks count 360 days to the year, this method may be used in computing the interest on bank loans.

The 6% method is fully described in ex. 1 on page 123. Ex. 1–5 on page 124 show how the time is broken up into combinations of 60 da. and 6 da. Ex. 10 on page 124 shows how to use the 6% method for rates of interest other than 6%, such as 3% and 2%. In most cases, banks use interest tables or computing machines to find interest instead of the formula or this method.

Workbook Reference: Arithmetic Workshop, Book 8, pages 54 and 55

Key: Page 123 **2.** $7; $12; $5; $6.95; $.80; $22.40. **3.** $3.75; $11.45; $1; $2.80; $.65; $11.70. **4.** $.70, $1.20, $.50, $.70, $.08, $2.24; $.38, $1.15, $.10, $.28, $.07, $1.17.

Page 124 **3.** (1) \$5.50 (60 da.) + \$.55 (6 da.) + \$.55 (6 da.) = \$6.60 (72 da.); (2) \$5.50 (60 da.) + \$.55 (6 da.) = \$6.05 (66 da.). **4.** (1) 15 da. = $\frac{1}{4}$ of 60 da.; (2) 20 da. = $\frac{1}{3}$ of 60 da.; (3) 40 da. = $\frac{2}{3}$ of 60 da.; (4) 45 da. = $\frac{3}{4}$ of 60 da. **5.** \$3.60; \$3.20. **6.** (1) \$5.90 (60 da.) + \$.59 (6 da.) = \$6.49 (66 da.); (2) \$4.50 (60 da.) + \$.45 (6 da.) + \$.45 (6 da.) + \$.45 (6 da.) = \$5.85 (78 da.); (3) $\frac{1}{2}$ × \$3.40 = \$1.70 (30 da.), \$3.40 (60 da.) + \$1.70 (30 da.) + \$.34 (6 da.) = \$5.44 (96 da.); (4) \$7.10 (60 da.) + \$.71 (6 da.) + \$.71 (6 da.) = \$8.52 (72 da.). **7.** (1) \$8.60 (60 da.) + \$.86 (6 da.) + \$.86 (6 da.) = \$10.32 (72 da.); (2) $\frac{1}{2}$ × \$6.00 = \$3.00, \$3.00 (30 da.) + \$.60 (6 da.) + \$.60 (6 da.) = \$4.20 (42 da.); (3) $\frac{3}{4}$ × \$2.40 = \$1.80 (45 da.); (4) \$.51 (6 da.) + \$.51 (6 da.) = \$1.02 (12 da.). **8.** (1) \$.29 (6 da.) + \$.29 (6 da.) = \$.58 (12 da.); (2) $\frac{1}{2}$ × \$4.40 = \$2.20 (30 da.), \$4.40 (60 da.) + \$2.20 (30 da.) = \$6.60 (90 da.); (3) $\frac{1}{2}$ × \$9.50 = \$4.75 (30 da.), \$9.50 (60 da.) + \$4.75 (30 da.) = \$14.25 (90 da.); (4) \$3.90 (60 da.) + \$.39 (6 da.) = \$4.29 (66 da.). **9.** \$3.84; \$1.55; \$5.60; \$1.94. **11.** \$3.20. **12.** (1) (a) $\frac{1}{2}$ × \$3.00 = \$1.50; (b) \$4.80; (c) \$3.60; (d) \$2.70; (e) \$7.20; (f) \$10.20; (2) (a) $\frac{1}{3}$ × \$3.00 = \$1.00; (b) \$3.20; (c) \$2.40; (d) \$1.80; (e) \$4.80; (f) \$6.80; (3) (a) $\frac{2}{3}$ × \$3.00 = \$2.00; (b) \$6.40; (c) \$4.80; (d) \$3.60; (e) \$9.60; (f) \$13.60.

Page 125

Aim: To present the second set of improvement tests in multiplication and also in division

Suggestions: In giving the improvement tests on this page, remember to have each test copied on paper before the test begins. The time assignment for each test is the time actually allowed for taking it *after* the copying has been completed.

Key: **1.** 2,760,780; 1,777,930; 1,672,128; 936,924; 3,864,770. **2.** 2,500,989; 1,093,032; 580,944; 2,105,899; 3,377,400. **3.** 2,623,127; 2,184,840; 4,601,856; 917,320; 5,190,128. **4.** 7059; $824\frac{5}{76}$; 3745. **5.** $1270\frac{1}{2}$; 908; 2371. **6.** $705\frac{2}{85}$; 3768; 1940. **7.** 2013; $2971\frac{1}{3}$; 817. **8.** 4607; 997; $840\frac{7}{76}$. **9.** $3879\frac{1}{2}$; 907; 1765.

Pages 126–127

Aim: To give the procedure in borrowing money from a bank

Suggestions: The pupils should become acquainted with the usual procedure in borrowing money from a bank. They should also have some practice in writing promissory notes in correct form and in using properly such technical terms as *face*, *bank discount*, *proceeds*, and *date of maturity*. It should be made clear that *bank discount* is nothing more than another term for *interest collected in advance*. This procedure is the more common one when banks lend money. In cases where interest is not collected in advance by a bank, but at the end of the period when the money is due, it is called *interest* and not *bank discount*. Bank loans are usually made for short periods of time, such as 60 da. or 90 da.

Ex. 2 on page 126 points out that the note shown in ex. 1 makes no mention of interest because the interest was collected *in advance* on March 10 when Mr. Harris borrowed the money. The question may be asked as to what hap-

pens if Mr. Harris does not pay the $800 at the end of 60 da. when it is due. One way this situation is handled is to have Mr. Harris write a new note on the day the first note is due, this new note to run for as many days as may be agreed upon. In this case the new note is discounted in the usual manner. Another way to handle this situation is to provide for it in advance by including in the original note a phrase which reads "with interest at 6% after maturity," this phrase to follow the word *Dollars* in the note. In this case Mr. Harris pays interest at 6% for the additional time beyond the 60 days that he keeps the money. The interest for the 60 days was collected in advance on March 10 when he borrowed the money.

Workbook Reference: Arithmetic Workshop, Book 8, pages 58 and 59

Key: Page 126 **2.** $8; $792.

Page 127 **3.** (1) $1.20 (60 da.) + $.12 (6 da.) = $1.32 (66 da.), bank discount; (2) 26 da. (June) + 31 da. (July) + 9 da. (Aug.) = 66 da., so the date of maturity is Aug. 9; (3) $120 − $1.32 = $118.68, amount received June 4; (4) $120. **4.** (1) (a) 14 da. (July) + 1 da. (Aug.) = 15 da., or $\frac{1}{4}$ of 60 da.; $\frac{1}{4} \times$ $3.60 = $.90; (b) $360 − $.90 = $359.10; (2) (a) 21 da. (Oct.) + 15 da. (Nov.) = 36 da.; $\frac{1}{2} \times$ $16.00 = $8.00 (30 da.); $8.00 (30 da.) + $1.60 (6 da.) = $9.60 (36 da.); (b) $1600 − $9.60 = $1590.40. **5.** (1) (a) 19 da. (Jan.) + 11 da. (Feb.) = 30 da., $\frac{1}{2} \times$ $5.40 = $2.70 (30 da.); (b) $540 − $2.70 = $537.30; (2) (a) 30 da. (May) + 30 da. (June) + 30 da. (July) = 90 da., $\frac{1}{2} \times$ $12.50 = $6.25 (30 da.), $12.50 + $6.25 = $18.75 (90 da.); (b) $1250 − $18.75 = $1231.25. **6.** (1) (a) Mar. 6 to Mar. 26 is 20 da., $\frac{1}{3} \times$ $9.00 = $3; (b) $900 − $3 = $897; (2) (a) 13 da. (Nov.) + 2 da. (Dec.) = 15 da., $\frac{1}{4} \times$ $26.40 = $6.60; (b) $2640 − $6.60 = $2633.40. **7.** (1) (a) 20 da. (Aug.) + 13 da. (Sept.) = 33 da., $\frac{1}{2} \times$ $5.60 = $2.80, $2.80 + $.28 = $3.08; (b) $560 − $3.08 = $556.92; (2) (a) 28 da. (Jan.) + 17 da. (Feb.) = 45 da., $\frac{3}{4} \times$ $18.80 = $14.10; (b) $1880 − $14.10 = $1865.90. **8.** (1) (a) 2 da. (Apr.) + 31 da. (May) + 27 da. (June) = 60 da., so interest is $8.80; (b) $880 − $8.80 = $871.20; (2) (a) 12 da. (June) + 31 da. (July) + 23 da. (Aug.) = 66 da., $20 (60 da.) + $2 (6 da.) = $22; (b) $2000 − $22 = $1978. **9.** (1) $2.75; (2) $275 − $2.75 = $272.25. **10.** (1) $\frac{1}{2} \times$ $6.00 = $3.00 (30 da.), $3.00 − $.30 = $2.70 (27 da.); (2) $6.00 − $2.70 = $597.30.

Page 128

Aim: To review fractions

Suggestions: The examples on this page review the four fundamental processes with fractions. Be sure to review the procedure given on page 16 for finding the least common denominator. If time permits, have the pupils make up some everyday problems similar to ex. 1 and 2, which involve computation with fractions.

Workbook Reference: Arithmetic Workshop, Book 8, page 5

Key: **1.** $4\frac{1}{2}$ mi. + $\frac{3}{4}$ mi. + $16\frac{1}{2}$ mi. = $20\frac{7}{4}$ mi., or $21\frac{3}{4}$ mi. **2.** $2 \times 1\frac{3}{4}$ ft. = $3\frac{1}{2}$ ft.;

6 ft. − $3\frac{1}{2}$ ft. = $2\frac{1}{2}$ ft. **3.** 6; $10\frac{7}{10}$; $13\frac{1}{3}$; $15\frac{1}{2}$; $25\frac{5}{12}$; $11\frac{19}{24}$; $10\frac{1}{2}$. **4.** $6\frac{1}{2}$; $12\frac{11}{12}$; $7\frac{1}{8}$; $17\frac{1}{8}$; 49; $15\frac{11}{12}$; 11. **5.** $1\frac{1}{2}$; $1\frac{5}{8}$; $2\frac{3}{5}$; $1\frac{3}{10}$; $4\frac{2}{3}$; $2\frac{1}{2}$; $5\frac{3}{5}$. **6.** $3\frac{1}{2}$; $2\frac{5}{12}$; $4\frac{1}{4}$; 2; $4\frac{7}{8}$; $2\frac{5}{12}$; $3\frac{11}{16}$. **7.** $4\frac{3}{8}$; 12; 12; $\frac{3}{32}$; 100. **8.** $\frac{3}{8}$; $\frac{15}{32}$; $\frac{7}{16}$; $\frac{1}{2}$; $22\frac{1}{2}$. **9.** $17\frac{1}{2}$; $6\frac{2}{3}$; $3\frac{1}{2}$; $1\frac{1}{2}$; $\frac{5}{8}$. **10.** 8; $3\frac{1}{3}$; $2\frac{1}{2}$; $2\frac{1}{4}$; $20\frac{5}{6}$. **11.** 30; $1\frac{1}{8}$; 9; 3; $13\frac{1}{3}$. **12.** $\frac{5}{24}$; $\frac{5}{6}$; $\frac{1}{2}$; $2\frac{1}{2}$; $8\frac{1}{4}$.

Page 129

Aim: To present a short project to illustrate the use of a bank loan

Suggestions: Remind the pupils that in the usual situation the interest on a bank loan is collected in advance. The promissory note in ex. 1 will be written for \$3000, and the bank will discount this note at 5%.

Key: **1.** (1) 26 da. (Mar.) + 30 da. (Apr.) + 31 da. (May) + 30 da. (June) + 31 da. (July) + 31 dá. (Aug.) + 30 da. (Sept.) + 1 da. (Oct.) = 210 da.; (2) interest on \$3000 at 5% for 60 da. = $\frac{5}{6}$ × \$30, or \$25; \$25 (60 da.) + \$25 (60 da.) + \$25 (60 da.) + \$12.50 (30 da.) = \$87.50 (210 da.); (3) \$3000 − \$87.50 = \$2912.50; (4) \$3000. **2.** 30 da. (Oct.) + 24 da. (Nov.) = 54 da.; interest on \$500 for 54 da. at 5% = \$3.75; \$500 + \$3.75 = \$503.75. **3.** 90 da. = 12 da. (Dec.) + 31 da. (Jan.) + 28 da. (Feb.) + 19 da. (Mar.); so the date of maturity is March 19, 1959.

Pages 130–131

Aim: To discuss personal or small loans

Suggestions: Most of the states in this country now have laws controlling the size of small loans and the rates of interest that may be charged for them. Originally a small loan was considered to be one which did not exceed \$300. Now this limit has been changed to \$500 or \$1000 in a substantial number of states. With this change in the size of a small loan, many states have adopted multiple interest rates. That is, the interest rates are different for different parts of the loan, as described in ex. 6. The companies that specialize in making small loans are sometimes called "personal finance companies." If there is a small-loan company in your locality, you can easily obtain from it the interest rates charged for small loans. Some of these companies will be glad to give you booklets which provide useful information on small-loan laws, rates, and payments.

In recent years the banks in some of our cities have opened departments for making small personal loans upon which the interest charged is similar to that charged by the personal finance companies. If there is a bank in your community that makes such small loans, have the pupils secure information concerning the conditions and rates charged for such loans.

Workbook Reference: Arithmetic Workshop, Book 8, page 61

Key: **6.** 3% of \$100 = \$3.00; 2% of \$200 = \$4.00; $\frac{3}{4}$% of \$150 = \$1.125, which is called \$1.13; \$3.00 + \$4.00 + \$1.13 = \$8.13. **7.** 30%; 42%.

Aim: To teach how to find the rate of interest that is really paid on small loans

Suggestions: On page 132 a method of handling a small loan *when the interest rate is given* is explained in ex. 1. This example should have careful study; and working it out, step by step, with the pupils will be profitable. The monthly interest charges could be paid as indicated in the schedule. Then the first monthly payment would be \$11.00; the second, \$10.80; and so on. However, it is the usual practice to pay off the principal and interest of small loans by making *equal* monthly payments. Therefore, the *average* monthly interest, which is \$3.00 ÷ 5, or \$.60, is found and added to the \$10 monthly payment on the principal. This sum makes each monthly payment equal to \$10.60.

The companies and banks that deal in small loans often make no mention of the interest rate that they charge for such loans; instead, they give the equal monthly payments that must be made to pay off, with interest, a loan of a given amount in various lengths of time.

In ex. 1 on page 133, Mr. Burns is told that if he wishes to borrow \$150, he must pay back 6 monthly payments of \$26.75 each. If Mr. Burns wants to know the interest rate that he is paying on this loan, he has to figure it out for himself. A method of finding the rate is explained in detail for this example. Notice that, in this method, you first find the *average interest per month* and the *average monthly amount* of the loan. Then, using these two values, you find the *monthly* rate of interest. After you know the monthly rate of interest, you find the annual rate by multiplying by 12.

Pages 132 and 133 are designed to give background for the formula presented on page 134. If you have limited time, you may omit pages 132 and 133 and go directly to the formula, taking special care to teach the pupils how to use it.

Key: Page 132 **2.** (1) $i = \$50 \times \frac{24}{100} \times \frac{5}{12} = \$5$; (2) Mr. Rice's principal kept decreasing.

3. (1)

	Principal Used	Monthly Interest
1st month	\$60	\$1.50
2d month	48	1.20
3d month	36	.90
4th month	24	.60
5th month	12	.30
		\$4.50, total int.

(2) \$4.50 ÷ 5 = \$.90, average interest per month; \$12.00 + \$.90 = \$12.90, monthly payment.

Page 133 **2.** (1) 8 × \$34.05 = \$272.40,
\$272.40 − \$240 = \$32.40,
total interest

Principal Used	
\$240.00	1st month

(2) \$32.40 ÷ 8 = \$4.05, average interest per month; \$34.05 − \$4.05 = \$30.00, monthly payment on principal; therefore, the borrower has the use of \$135.00 per month, for which he pays \$4.05 per month; so, \$4.05 ÷ \$135 = .03, or 3%, monthly rate of interest	210.00	2d month
	180.00	3d month
	150.00	4th month
	120.00	5th month
	90.00	6th month
	60.00	7th month
	30.00	8th month
(3) 12 × 3% = 36%, yearly rate of interest	8)\$1080.00	
	\$ 135.00	Average Principal Used per Month

3. (1) 7 × \$22 = \$154, \$154 − \$140 = \$14, total interest	Principal Used	
(2) \$14 ÷ 7 = \$2, average interest per month; \$22 − \$2 = \$20, monthly payment on principal; therefore, the borrower has the use of \$80 per month, for which he pays \$2 per month; so, \$2 ÷ \$80 = .025, or $2\frac{1}{2}$%, monthly rate of interest	\$140	1st month
	120	2d month
	100	3d month
	80	4th month
	60	5th month
	40	6th month
	20	7th month
	7)\$560	
	\$ 80	Average Principal Used per Month

Pages 134–135

Aims: To present a formula for finding the interest rate paid on small loans and to give practice in using it

Suggestions: It takes considerable time and careful thinking to find the interest rate on a small loan by the method explained in ex. 1 on page 133. The work for such a problem can be shortened and simplified by using the formula given in ex. 1 on page 134. The use of this formula needs to be explained carefully. Emphasize the meaning of each letter in the formula; otherwise, mistakes are likely to be made in substituting values for the letters in the formula. However, do not attempt to explain to the entire class the derivation of the formula. At this grade level most pupils can learn only to use it. Practice in using the formula is given in the other exercises on page 134 and in all the exercises on page 135.

You may be interested to know how the formula is derived. It can be derived as follows by applying the method explained on page 133 to the general problem expressed in letters. If P is the amount of a loan to be repaid with n equal monthly payments, and C is the total interest charge, then:

(1) The average interest per month is $\frac{C}{n}$.

(2) The amount of the loan during the first month is P and during the last month is $\frac{P}{n}$.

Thus the average monthly amount of the loan is $\frac{1}{2}\left(P + \frac{P}{n}\right)$ or $\frac{P(n+1)}{2n}$.

(Notice that in ex. 1 on page 133 the average monthly amount of the loan can be found by adding \$150 and \$25, and then dividing this sum by 2.)

(3) The average interest per month divided by the average monthly amount of the loan gives the *monthly* rate of interest:

$$\frac{C}{n} \div \frac{P(n+1)}{2n} = \frac{C}{n} \times \frac{2n}{P(n+1)} = \frac{2C}{P(n+1)}$$

(4) The *yearly* rate of interest, r, is found by multiplying the monthly rate of interest by 12:

$$r = 12 \times \frac{2C}{P(n+1)} = \frac{24C}{P(n+1)}$$

This formula is, as you can see, equivalent to the procedure given on page 133.

Key: Page 134 **2.** (1) \$32.40; (2) \$240; (3) 8; (4) $r = \frac{24 \times 32.40}{240(8+1)} = .36$, or 36% (yearly rate); (5) 36% ÷ 12 = 3% (monthly rate). **3.** (1) $r = \frac{24 \times 14}{140(7+1)} = \frac{336}{1120}$ = .30, or 30% (yearly rate); (2) 30% ÷ 12 = $2\frac{1}{2}$% (monthly rate). **4.** (1) 6 × \$13.50 = \$81.00, \$81.00 − \$75 = \$6.00, interest charge; C = \$6.00, P = \$75, $n = 6$; $r = \frac{24 \times \$6.00}{75(6+1)} = .2742$, or 27.4%, yearly rate; (2) 27.4% ÷ 12 = 2.28%, or about 2.3%, monthly rate. (3) Interest up to 4% monthly is legal in some states under the small loan laws. **5.** 30%; $2\frac{1}{2}$%.

Page 135 **1.** (a) 6 × \$54.02 = \$324.12; \$324.12 − \$300 = \$24.12, interest charge; C = \$24.12, P = \$300, $n = 6$; $r = \frac{24 \times 24.12}{300(6+1)} = .2756$, or 27.6% (yearly rate); (b) 12 × \$28.82 = \$345.84; \$345.84 − \$300 = \$45.84, interest charge; C = \$45.84, P = \$300, $n = 12$; $r = \frac{24 \times 45.84}{300(12+1)} = .2820$, or 28.2% (yearly rate); (c) 15 × \$23.80 = \$357; \$357 − \$300 = \$57, interest charge; C = \$57, P = \$300, $n = 15$; $r = \frac{24 \times 57}{300(15+1)} = .285$, or 28.5% (yearly rate). **2.** 12 × \$17.34 = \$208.08; \$208.08 − \$200 = \$8.08, interest charge; C = \$8.08, P = \$200, $n = 12$; $r = \frac{24 \times 8.08}{200(12+1)} = .0745$, or 7.5% (yearly rate). **3.** 24 × \$91.67 = \$2200.08; \$2200.08 − \$2000 = \$200.08, interest charge; C = \$200.08, P = \$2000, $n = 24$; $r = \frac{24 \times 200.08}{2000(24+1)} = .0960$, or 9.6% (yearly rate). **4.** 12 × \$12 = \$144; \$144

$- \$100 = \$44$, interest charge; $C = \$44$, $P = \$100$, $n = 12$; $r = \dfrac{24 \times 44}{100(12 + 1)}$ $= .8123$, or 81.2% (yearly rate). **5.** $C = \$18$, $P = \$300$, $n = 12$; $r = \dfrac{24 \times 18}{300(12 + 1)}$ $= .1107$, or 11.1% (yearly rate). **6.** $50 \times \$2.75 = \$137.50$; $\$137.50 - \$100$ $= \$37.50$, interest charge; $C = \$37.50$, $P = \$100$, $n = 50$; $r = \dfrac{104 \times 37.50}{100(50 + 1)} = .7647$, or 76.5% (yearly rate).

Pages 136–137

Aim: To discuss installment buying including carrying charges

Suggestions: The problems on these pages show how to find the total cost of an article if it is bought on the installment plan and how much more this installment price is than the cash price. The difference between these two prices is called the *carrying charge*. The pupils should be made familiar with this term.

Workbook Reference: Arithmetic Workshop, Book 8, page 62

Key: **1.** (1) $12 \times \$26.50 = \$318$; $\$318 + \$30 = \$348$, cost on installment plan; (2) $\$348 - \$330 = \$18$, extra charge. **3.** Yes. **4.** $8 \times \$5 = \$40$; $\$40 + \$4 = \$44$, cost on installment plan; $\$44 - \$40 = \$4$, carrying charge. **5.** $11 \times \$5 = \$55$; $\$55 + \$10 = \$65$, cost on installment plan; $\$65 - \$60 = \$5$, carrying charge. **6.** $18 \times \$15.75 = \$283.50$; $\$283.50 + \$27.50 = \$311.00$, cost on installment plan; $\$311 - \$280 = \$31$, carrying charge.

Page 138

Aim: To provide a review of problem solving

Key: **1.** 3×35 mi. = 105 mi.; 275 mi. − 105 mi. = 170 mi.; 3 hr. + $\frac{3}{4}$ hr. = $3\frac{3}{4}$ hr.; 8 hr. − $3\frac{3}{4}$ hr. = $4\frac{1}{4}$ hr.; 170 mi. ÷ $4\frac{1}{4}$ = 40 mi. **2.** $275 \div 16.5 = 16.6$ (gal.), which is called 17 gal. **3.** 1 hr. 40 min. = 100 min.; 100 min. ÷ 12 = $8\frac{1}{3}$ min. **4.** $\$14,500 - \$10,000 = \$4500$; $\$4500 \div \$10,000 = \frac{45}{100}$, or 45%. **5.** $.10 \times \$2000$ $= \$200$; $\$2000 - \$200 = \$1800$; $.02 \times \$1800 = \$36$, extra discount which he can get by borrowing the money; interest on \$800 at 6% for 60 da. is \$8; $\$36 - \$8 = \$28$, amount saved by borrowing money and paying promptly. **6.** (1) 60% of S.P. = \$39, 1% of S.P. = \$39 ÷ 60, or \$.65, 100% of S.P. = $100 \times \$.65$, or \$65; (2) 60% + 32% = 92%, 100% − 92% = 8%, $.08 \times \$65 = \$5.20$, profit. **7.** The diagram is similar to the one in ex. 6 on page 100 of the textbook. $A = 3\frac{1}{7} \times 14 \times 14$ $= 616$ (sq. ft.), area of flower garden; 28 ft. + 2 ft. + 2 ft. = 32 ft., diameter of outside circumference of walk; $A = 3\frac{1}{7} \times 16 \times 16 = 804\frac{4}{7}$ (sq. ft.), area of walk and garden together; $804\frac{4}{7}$ sq. ft. − 616 sq. ft. = $188\frac{4}{7}$ sq. ft., area of walk.

Page 139

Aim: To present another set of improvement tests in addition

Suggestion: This set of improvement tests should be administered in the same manner as previous improvement tests in addition.

Key: **1.** 3119; 4716; 5043; 4660; 6204; 6434. **2.** 6631; 5460; 6441; 5272; 5266; 6104. **3.** 5343; 5201; 6300; 6156; 6223; 5321.

Pages 140–141

Aims: To show that the carrying charge in installment buying is the equivalent of an interest charge and to show what rate of interest this charge represents

Suggestions: Ex. 1 and 2 on page 140 require careful study. The formula given in ex. 2 is the same as the one used on page 134, but the letters in the formula represent quantities different from those they represented on page 134. You should give the pupils careful instructions in substituting values in the formula. Emphasize the fact that P represents the *cash price less the down payment;* otherwise, some pupils will make the mistake of substituting for P the installment price less the down payment.

In most cases the rate of interest charged for installment buying is not stated. Therefore, it is especially important to be able to figure out the rate for yourself. If you want to decide whether to borrow money and pay cash for an article or to use the installment plan, you need to know what rate of interest the installment plan represents. The problems on page 141 give practice in using the formula. You may also wish to use some illustrations of installment buying from the stores in your community. For each article you must obtain *both* the cash price and the installment price. You can find some useful illustrations in the mail-order catalogues.

Key: Page 140 **4.** $10 \times \$17 = \$170$; $\$170 + \$25 = \$195$; $\$195 - \$175 = \$20$, carrying charge; $\$175 - \$25 = \$150$, unpaid balance; $C = \$20, P = \$150, n = 10$; $r = \frac{24 \times 20}{150(10+1)} = .290$, or 29% (yearly rate).
Page 141 **1.** $11 \times \$5 = \$55$; $\$55 + \$6 = \$61$, installment price; $\$61 - \$56 = \$5$, carrying charge; $\$56 - \$6 = \$50$, unpaid balance; $C = \$5$, $P = \$50$, $n = 11$; $r = \frac{24 \times 5}{50(11+1)} = .20$, or 20% (yearly rate). **2.** $8 \times \$9.50 = \$76$; $\$76 + \$15.50 = \$91.50$, installment price; $\$91.50 - \$84.00 = \$7.50$, carrying charge; $\$84.00 - \$15.50 = \$68.50$, unpaid balance; $C = \$7.50, P = \$68.50, n = 8; r = \frac{24 \times 7.50}{68.50(8+1)} = .291$, or 29% (yearly rate). **3.** $4 \times \$5 = \$20$; $\$20 + \$3 = \$23$, installment price; $\$23.00 - \$21.50 = \$1.50$, carrying charge; $\$21.50 - \$3.00 = \$18.50$, unpaid balance; $C = \$1.50$, $P = \$18.50$, $n = 4$; $r = \frac{24 \times 1.50}{18.50(4+1)} = .389$, or 39% (yearly rate). **4.** $14 \times \$16 = \$224$; $\$224 + \$32 = \$256$, installment price; $\$256 - \$240 = \$16$, carrying charge; $\$240 - \$32 = \$208$, unpaid balance; $C = \$16$, $P = \$208$, $n = 14$; $r = \frac{24 \times 16}{208(14+1)} = .123$, or 12% (yearly rate). **5.** $18 \times \$14.50 = \$261$; $\$261 + \$15 = \$276$, installment price; $\$276 - \$245 = \$31$, carrying charge; $\$245 - \$15 = \$230$, unpaid balance; $C = \$31, P = \$230, n = 18$; $r = \frac{24 \times 31}{230(18+1)}$

$= .170$, or 17% (yearly rate). **6.** $11 \times \$7 = \$77$; $\$77 + \$8 = \$85$, installment price; $\$85 - \$78 = \$7$, carrying charge; $\$78 - \$8 = \$70$, unpaid balance; $C = \$7$, $P = \$70$, $n = 11$; $r = \frac{24 \times 7}{70(11+1)} = .20$, or 20% (yearly rate). **7.** $13 \times \$22 = \$286$; $\$286 + \$31.50 = \$317.50$, installment price; $\$317.50 - \$292.00 = \$25.50$, carrying charge; $\$292.00 - \$31.50 = \$260.50$, unpaid balance; $C = \$25.50$, $P = \$260.50$, $n = 13$; $r = \frac{24 \times 25.50}{260.50(13+1)} = .167$, or 17% (yearly rate). **8.** (1) $8 \times \$7.35 = \$58.80$, installment price; $\$58.80 - \$48 = \$10.80$, carrying charge; $C = \$10.80$, $P = \$48$, $n = 8$; $r = \frac{24 \times 10.80}{48(8+1)} = .60$, or 60% (yearly rate); (2) $60\% \div 12 = 5\%$ (monthly rate);

(3) $.02 \times \$48 = \$\ .96$, int. for 1st month
$.02 \times \$42 = \$\ .84$, int. for 2d month
$.02 \times \$36 = \$\ .72$, int. for 3d month
$.02 \times \$30 = \$\ .60$, int. for 4th month
$.02 \times \$24 = \$\ .48$, int. for 5th month
$.02 \times \$18 = \$\ .36$, int. for 6th month
$.02 \times \$12 = \$\ .24$, int. for 7th month
$.02 \times \$\ 6 = \$\ .12$, int. for 8th month
$4.32, int. for 8 months

$\$48 + \$4.32 = \$52.32$, total cost of ring if $48 is borrowed. On the installment plan, total cost of ring would be $58.80. The second plan is cheaper.

Page 142

Aim: To review the work of Chapter 4

Key: **2.** Just before cashing it. **3.** A certified check is one which is stamped "certified" across the face of the check. This stamping is done by the bank and guarantees that the check is good. The bank would not certify the check unless Mr. Allen had at least $5000 in the bank. **4.** 360 da. **5.** In advance; bank discount. **6.** At the end of 6 mo. **7.** $300 to $1000, according to the state; 2% to 4%. **8.** The extra charge beyond the cash price. **9.** (1) $\frac{\$500}{1} \times \frac{5}{100} \times \frac{1}{4} = \$6.25$; (2) $\frac{\$650}{1} \times \frac{5}{100} \times \frac{1}{2} = \$16.25$; (3) $\frac{\$840}{1} \times \frac{5}{100} \times \frac{1}{6} = \$7.00$; (4) $\frac{\$1000}{1} \times \frac{5}{100} \times \frac{1}{4} = \$12.50$. **10.** $15; $11.25; $3.30; $6. **11.** $21; $22.50; $1; $5. **12.** (1) $5 \times \$10.75 = \$53.75$; $\$53.75 - \$50 = \$3.75$, interest charge; (2) $C = \$3.75$, $P = \$50$, $n = 5$; $r = \frac{24 \times 3.75}{50(5+1)} = .30$, or 30% (yearly rate). **13.** (1) $12 \times \$24 = \$288$; $\$288 + \$50 = \$338$, installment price; $\$338 - \$300 = \$38$, carrying charge; (2) $\$300 - \$50 = \$250$, unpaid balance; $C = \$38$, $P = \$250$, $n = 12$; $r = \frac{24 \times 38}{250(12+1)} = .280$, or 28% (yearly rate).

Page 143

Aim: To present Problem Test 4

Workbook Reference: Arithmetic Workshop, Book 8, page 63

Key: **1.** $i = \frac{\$150}{1} \times \frac{6}{100} \times \frac{3}{4} = \$6.75$. **2.** $9 \times \$18.75 = \$168.75$; $\$168.75 - \$150.00 = \$18.75$. **3.** (1) $9 \times \$18 = \$162$; $\$162 + \$75 = \$237$, installment price; $\$237 - \$225 = \$12$, carrying charge; (2) borrowing from Mr. Best would be cheapest. **4.** 3 ft. = 1 yd., 88 ft. = $29\frac{1}{3}$ yd.; $1 \times 29\frac{1}{3} = 29\frac{1}{3}$ (sq. yd.); $29\frac{1}{3} \times \$4.75 = \$139.33\frac{1}{3}$, or \$139.33. **5.** 3.1 billion − 2.7 billion = .4 billion; $.4 \div 2.7 = .148$, or 15%. **6.** 10 min. = $\frac{1}{6}$ hr.; in $\frac{1}{6}$ hr. he goes 6.5 mi., so in 1 hr. he goes 6×6.5 mi., or 39 mi. **7.** The interest on \$350 for 6 da. at 6% is \$.35; \$.35 (6 da.) + \$.35 (6 da.) + \$.35 (6 da.) = \$1.05. **8.** (1) 28 da. (Aug.) + 30 da. (Sept.) + 5 da. (Oct.) = 63 da.; (2) 27 da. (Jan.) + 3 da. (Feb.) = 30 da. **9.** $\frac{1}{4} \times \$480 = \$120$; $\$480 - \$120 = \$360$; $.03 \times \$360 = \$10.80$; $\$360 - \$10.80 = \$349.20$. **10.** $\$920 + \$17{,}480 = \$18{,}400$, sale price; $\$920 \div \$18{,}400 = .05$, or 5%.

Page 144

Aims: To provide a diagnostic test with page references for practice, and to review arithmetical terms and expressions

Workbook Reference: Arithmetic Workshop, Book 8, page 64

Key: **1.** 15 da.; 24 da. **2.** 90 da.; 120 da. **3.** \$36; \$6. **4.** \$55; \$206.25. **5.** \$16.20; \$22. **6.** \$9, \$991; \$3.50, \$1746.50; \$9.75, \$640.25. **7.** \$1.25; \$498.75.

Chapter 5

Aims of Chapter 5. The major aims of Chapter 5 are to:

1. Review the names of the different kinds of angles.
2. Review how to measure angles with a protractor.
3. Reteach and extend the work on perpendicular lines.
4. Reteach how to bisect an angle using compasses and how to use this skill in drawing a regular octagon.
5. Reteach and extend the work on parallel lines.
6. Teach the names of the different kinds of triangles.
7. Teach some of the important properties of triangles.
8. Teach how to draw a triangle when given:
   a. three sides.
   b. two sides and the included angle.
   c. two angles and the included side.
9. Show how to measure distances indirectly by making use of triangles.

Teaching Informal Geometry. In Grade 8, informal geometry is concerned with a study of the more common geometric figures having two and three dimensions, with the making of simple geometric constructions, and with indirect measurement. In order to succeed in this work, each pupil should be supplied with a ruler, a draftsman's triangle, a pair of compasses, and a protractor. If it is not easy to obtain a draftsman's triangle, one can be made from heavy cardboard. If the pupil has a drawing board and a T-square, such as used in simple technical drawing, he should be encouraged to use them. The making of geometric constructions can be taught only on the principle of "learn by doing." The pupils must be required actually to make the various constructions and scale drawings indicated in the exercises, so that they will acquire a reasonable skill in such work. Precise drawing, however, is not expected. Most pupils are interested in work of this kind, and, if supplied with proper instruments, they should have little difficulty with it.

Page 145

Aim: To review the kinds of angles and three ways of reading angles

Suggestions: The meaning of an angle is explained in ex. 1 on this page. In this work you should make clear that the size of an angle depends on the amount of rotation and not on how long the sides of the angle have been drawn. The notion of an angle may be illustrated before the class by spreading the legs of a pair of compasses or by opening and closing a pair of scissors. The terms *right angle, straight angle, acute angle,* and *obtuse angle* should be retaught. Three methods of reading angles are described in ex. 2.

Pages 146–147

Aim: To study the measurement of angles, including the use of the protractor

Suggestions: The measurement of angles by the use of a protractor is explained in ex. 4 on page 146. In this work it should be made clear to the pupils that the word *degree* refers to each of the 90 small angles formed at the vertex of a right angle and also to each of the 90 small arcs into which the arc of a right angle is divided. In other words, the degree is a unit for measuring arcs as well as angles.

The method of using the protractor to measure an angle, which is described in ex. 4 on page 146, should have very careful study. In this work caution the pupils to put the center of the protractor exactly over the vertex of the angle that is being measured and to see that one side of the angle passes through the 180° mark of the protractor. See the figure at the bottom of page 146.

The methods of using a protractor to draw an angle of a given size and to bisect an angle are described in ex. 3 and 5 on page 147. The pupil should acquire some skill in this work by constructing and bisecting angles as suggested in ex. 4 and 6 on page 147.

Ex. 2 on page 147 develops a very important theorem of geometry; namely, that *the sum of the angles of any triangle equals* 180°. To show experimentally that this is true, the pupil should draw with a ruler 6 or more triangles of different shapes and sizes and measure the three angles of each triangle by means of a protractor. In each triangle the sum of the three angles should equal approximately 180°. Results varying from 178° to 182° may be obtained in some cases, since measurements with the usual school protractor do not have a high degree of accuracy.

Workbook Reference: Arithmetic Workshop, Book 8, page 66

Pages 148–149

Aim: To show the use of angles in air navigation

Suggestions: In ex. 1 on page 148 the term *clockwise* means the direction in which the hands of a clock move. The direction opposite to the clockwise direction is called *counterclockwise.* In Fig. 1 in ex. 1 you think of side AN of the angle as pointing north and remaining fixed in this position while side AE rotates in the *clockwise* direction. In Fig. 1 it is seen that side AE has rotated through an angle of 90° and now points east. When a plane flies east, we say that it is flying on a *course* of 90°. In Fig. 2 side AN still remains fixed and points north while side AG has rotated *clockwise* through an angle of 135°. Side AG is pointing *southeast;* when a plane flies southeast, its course is 135°. Fig. 3 shows an angle of 270° with the side AW pointing west. Fig. 4 shows an angle of 315° with side AH pointing northwest. In Fig. 4 suppose that side AH rotates still more in the clockwise direction until it reaches side AN; in this event side AH will have rotated through 360° in all and will then point north just as side AN points north. It is thus seen that a direction of 360° is just the same as a direction of 0°.

Ex. 4 on page 149 gives the pupil practice in drawing the course of an airplane, and ex. 5 gives practice in measuring the course. Pupils should have considerable practice in this work.

Workbook Reference: Arithmetic Workshop, Book 8, page 67

Key: **2.** The amount of rotation is 135°; southeast; 315°; northeast, south, southwest, north. **3.** $360° - 45° = 315°$. **5.** 110°; 348°; 245°.

Page 150

Aims: To provide review problems, and to give practice in the addition of mixed numbers and of decimals representing dollars and cents

Key: **1.** (1) $\frac{1}{5} \times \$90 = \$18$; $\$90 + \$18 = \$108$; $\frac{1}{5} \times \$108 = \$21.60$; $\$108 - \$21.60 = \$86.40$; (2) no. **2.** 7 in.; $2\frac{1}{2}$ in. **3.** 175 ft.; 225 ft.; 350 ft.; 400 ft. **4.** (1) The bases will be $3\frac{1}{2}$ in. and $4\frac{1}{2}$ in., and the height will be $2\frac{1}{2}$ in.; (2) the bases will be $1\frac{3}{4}$ in. and $2\frac{1}{4}$ in., and the height will be $1\frac{1}{4}$ in. **5.** $13\frac{9}{16}$; $12\frac{1}{2}$; $6\frac{5}{6}$; $5\frac{5}{8}$; $15\frac{5}{8}$; $13\frac{1}{2}$; $6\frac{1}{10}$. **6.** $13\frac{1}{8}$; 19; $11\frac{3}{5}$; $14\frac{3}{4}$; 9; $8\frac{1}{12}$; $17\frac{7}{10}$. **7.** $16\frac{5}{6}$; $14\frac{1}{5}$; $19\frac{3}{8}$; $15\frac{1}{3}$; $17\frac{3}{10}$; $15\frac{1}{2}$; 14. **8.** \$1077.02; \$835.19; \$789.20; \$594.09; \$1087.43.

Page 151

Aim: To present another set of improvement tests in the subtraction of whole numbers

Suggestion: This set of improvement tests is to be administered in the same manner as preceding sets in subtraction. See page 375 of the text.

Key: **1.** \$145.36; \$648.22; \$589.92; \$374.01. **2.** \$140.78; \$456.87; \$109.54; \$47.64. **3.** \$131.28; \$589.57; \$416.29; \$303.89. **4.** \$580.76; \$863.94; \$268.13; \$359.07. **5.** \$5.89; \$153.68; \$278.45; \$667.29. **6.** \$125.78; \$265.89; \$862.94; \$487.28. **7.** \$92.58; \$753.89; \$77.72; \$165.67. **8.** \$239.09; \$307.91; \$356.76; \$498.49. **9.** \$550.75; \$236.89; \$399.86; \$164.18.

Pages 152–153

Aim: To reteach two ways of drawing perpendicular lines

Suggestions: The most rapid and accurate way to draw a line perpendicular to another is to use a draftsman's triangle, as described in ex. 3 on page 152. Hence, to succeed in this work, each pupil should be supplied with such a triangle, which is usually made of wood or plastic. If it is not practical for the school to provide such triangles, a substitute can be made from heavy cardboard. If such a triangle is made, one angle must be a right angle. To use a draftsman's triangle, the pupil slides the triangle along the edge of a ruler. Most rulers have a thin edge and a thicker edge. In most instances the results will be more satisfactory if the pupil slides the triangle along the thicker edge of the ruler. In classes in which technical drawing is taught, a T-square instead of a ruler is used, together with a draftsman's triangle, to draw perpendicular lines. A T-square and a draftsman's triangle are shown in the picture on page 153 of the text. The T-square is the long flat strip of wood lying on the drawing board under the man's wrists. The man is holding the draftsman's triangle close to the edge of the T-square.

When making the designs on page 153, the pupil should use his draftsman's triangle to draw the right angles at the corners of the outside squares.

Page 154

Aim: To show how to bisect an angle by using compasses

Suggestion: The pupil now has two methods of bisecting an angle. Emphasize the fact that the new method does not require measurement of the angle and is, therefore, independent of errors in measurement.

Page 155

Aim: To show how to draw a regular octagon using the method of bisecting an angle just learned

Suggestions: In drawing a regular octagon in a circle, as described in ex. 1, the work may be easier for some pupils if a radius of $1\frac{1}{2}$ in. or 2 in. is used in drawing the circle. Small figures may be harder for the pupils to draw than larger ones.

Directions for drawing design 7 at the bottom of the page can best be given by referring to the figure in ex. 1. First divide the circle into 8 equal parts, as shown in the figure in ex. 1. Next, with B as center and a radius equal to BF, draw an arc from F to H. Then, with H as center and the same radius, draw an arc from B to D. In like manner, with D as center and the same radius, draw an arc from H to G. Continue in this way around the circle. You will then have design 7, to which the color should be added.

Page 156

Aim: To teach how to use ruler and compasses to draw perpendicular lines and to bisect line segments

Suggestions: In ex. 3 on this page it should be noted that line segment CD not only bisects AB at O but is also perpendicular to AB. Another way to bisect a segment such as AB, without using compasses, is to measure the length of the segment with a ruler. Then locate the midpoint O so that it will be one half of this length from either endpoint of the segment. For example, if AB is $1\frac{1}{2}$ in. long, locate O by making a dot $\frac{3}{4}$ in. from A or from B. The method of bisecting a line segment given on this page does not, however, require the measurement of its length.

Pages 157–158

Aim: To review problem solving and computational skills

Suggestions: Although other topics are reviewed, these two pages carefully review all three types of percentage problems. You should go over with the class the meaning of the letters in the formula on page 140 for finding the rate of interest in installment buying.

Key: Page 157 **1.** $12 \times \$85 = \$1080$, total amount received as rent; $\$165.75 + \$194.25 = \$360.00$, total expenses; $\$1020 - \$360 = \$660$, net amount received from cottage; $\$660 \div \$6500 = .101$, or 10%. **2.** 480 mi. $\div$ 10 = 48 mi., usual speed; 3×48 mi. = 144 mi.; 480 mi. − 144 mi. = 336 mi., rest of distance; 10 hr. + $\frac{1}{2}$ hr. = $10\frac{1}{2}$ hr., time for whole trip; 3 hr. + $1\frac{1}{2}$ hr. = $4\frac{1}{2}$ hr., time used before trip was resumed; $10\frac{1}{2}$ hr. − $4\frac{1}{2}$ hr. = 6 hr., time left; 336 mi. $\div$ 6 = 56 mi. **3.** (1) 65% of S.P. = \$39; 1% of S.P. = \$39 $\div$ 65, or \$.60; 100% of S.P. = 100 $\times$ \$.60, or \$60; (2) \$64; (3) \$86. **4.** (1) $10 \times \$6.75 = \$67.50$; $\$67.50 + \$15 = \$82.50$, installment price; $\$82.50 - \$75 = \$7.50$, carrying charge; (2) $\$75 - \$15 = \$60$, unpaid balance; $C = \$7.50$, $P = \$60$, $n = 10$; $r = \dfrac{24 \times 7.50}{60(10+1)} = .2727$, or 27.3% (yearly rate). **5.** $\$1.45 - \$.71 = \$.74$; $\$.74 \div \$1.45 = .510$, or 51%. **6.** 18 oranges = $1\frac{1}{2}$ doz. oranges; $\$.74 \div 1\frac{1}{2} = \$.49\frac{1}{3}$, which is $\$.03\frac{2}{3}$ cheaper per dozen.

Page 158 **1.** \$6143.32; 2.960; $59\frac{2}{3}$; 100.0; 9 lb. 12 oz. **2.** 1.82; 20.6; 5.5; 590; 4600. **3.** 2400; 11.63; 7.49; .24; 49.04. **4.** $12\frac{1}{4}$; 4; $\frac{3}{8}$; $2\frac{1}{2}$. **5.** 108; $52\frac{1}{2}$; $\frac{3}{20}$; $\frac{1}{8}$. **6.** 6; $3\frac{3}{5}$; $5\frac{1}{4}$; $1\frac{1}{3}$. **7.** \$2.25; 35%; 160. **8.** \$6.02; 75%; 2200. **9.** \$.01; 40%; 152. **10.** \$12; \$76.50; \$39. **11.** \$14; \$33; \$14.70. **12.** \$.75; \$4.75; \$2.40. **13.** 142; 1951; 1875; 1929; 34; 913; 408; 985.

Pages 159–160

Aim: To reteach and extend the work on parallel lines first presented in Grade 7

Suggestions: In ex. 1 and 2 on page 159 parallel lines are defined as lines having the *same direction*, which means that each makes the same angle with a third line. This definition of parallel lines is applied in practice when you draw parallel lines by sliding a draftsman's triangle along the edge of a ruler, as is done in ex. 3 on page 159. Parallel lines may also be drawn by using a T-square, as shown in ex. 1 on page 160. The pupils should have practice in drawing pairs of parallel lines.

Page 161

Aim: To teach the names of different kinds of triangles and how to recognize each kind

Suggestions: In ex. 2 on page 147 the pupil learned that the sum of the three angles in any triangle is 180°. This fact may be used in answering the questions in ex. 4 on page 161. If a triangle had two right angles, these two angles alone would have a sum of 180°; hence, the third angle would be 0°, which is not possible. So a triangle cannot have two right angles. Likewise, it cannot have two obtuse angles, since their sum alone would be more than 180°, and a third angle would not be possible.

Key: **2.** Second; third; first. **3.** Third; second; first. **4.** No; no; yes. **5.** Right triangle. **6.** Scalene; isosceles; equilateral. **7.** Obtuse; acute; right.

Pages 162–163

Aim: To teach an important property of triangles

Suggestions: The work on page 162, which shows that a triangle is a rigid figure, is closely related to the work in ex. 2–4 on page 168.

Notice that a figure which is not rigid can sometimes be made rigid in more than one way. For a figure to be rigid it must be made up entirely of triangles. Thus, any plan which uses triangles is satisfactory.

Have the pupils bring to class pictures of structures, such as metal towers or bridges, in which triangles are used to give rigidity to the structures; see ex. 7 on page 163.

Page 164

Aim: To describe the parallel ruler

Suggestions: The parallel ruler gives the pupil another way of drawing parallel lines. It is also a good illustration of how geometric principles can be the basis of the design of a mechanical device. The pupil knows that the opposite sides of a parallelogram are parallel and equal. He should also be led to see that a four-sided figure that has equal opposite sides must be a parallelogram.

For purposes of demonstration, you can purchase a parallel ruler from a dealer in drawing instruments. Homemade parallel rulers are very useful in the classroom, and they can be made out of strips of wood or heavy cardboard and paper fasteners. You will find that pupils enjoy making them.

Page 165

Aim: To present another set of improvement tests in multiplication and also one in division

Suggestion: These improvement tests are to be administered in the same way as the previous sets in multiplication and division. See page 375 of the text.

Key: **1.** \$5051.61; \$26,963.75; \$16,353.06; \$22,066.44; \$16,456. **2.** \$15,420.24; \$31,184.45; \$29,123.52; \$61,063; \$44,524.80. **3.** \$75,646.99; \$7643.36; \$5715.84; \$36,232.70; \$36,072.40. **4.** 60.8; 52.3; 92.6. **5.** 84.9; 87.9; 89.3. **6.** 99.7; 82.9; 99.9. **7.** 98.2; 80.1; 84.8. **8.** 96.1; 86.6; 99.9. **9.** 99.7; 78.3; 67.3.

Pages 166–168

Aims: To show how to draw a triangle when the lengths of the three sides are given, and to teach experimentally some properties of triangles

Suggestions: On page 166 the pupils should actually draw the triangles and measure the angles as suggested in ex. 1–4. Ex. 2–5 on page 167 are applications of the fact that the sum of the three angles of any triangle equals 180°. The pupil should actually draw the triangles in ex. 6–10 on page 167, using compasses and ruler. On page 168, the pupil must be made familiar with the term *congruent*. This word is pronounced \\'käŋ-grə-wənt\\. Ex. 1–4 on this page show

experimentally that *two triangles are congruent if the three sides of one are respectively equal to the three sides of the other.*

Key: Page 166 **3.** The 3 angles are all 60°; the 3 angles of an equilateral triangle are equal. **4.** Scalene; 90°; right triangle.

Page 167 **2.** Subtract the sum of the base angles from 180° to get 44°. **3.** 45° and 90°. **4.** 180° − 40° = 140°; so each base angle is 70°. **5.** Acute; 90°; 60°. **11.** (9) 22°, 50°, 108°; 68°, 44°, 68°; 45°, 45°, 90°; (10) 41°, 56°, 83°; 34°, 73°, 73°; 60°, 60°, 60°.

Page 169

Aims: To summarize certain facts learned about triangles and to test the pupils' understanding of important geometric terms

Suggestions: The first half of this page requires careful study since it brings out the fact that every triangle has six parts; namely, three sides and three angles. To draw a triangle, however, it is not necessary to know the sizes of all six parts. The pupil has already seen on pages 166–168 that he can draw a triangle if he knows the lengths of its three sides. On pages 171 and 176 he will learn ways to draw a triangle when other parts are known.

Page 170

Aim: To give a mixed review of previous work

Key: **1.** \$12; \$8. **2.** \$150 + \$235.25 = \$385.25; \$385.25 − \$275.53 = \$109.72. **3.** $11\frac{1}{2} + 9\frac{2}{3} + 12\frac{1}{2} + 10\frac{5}{6} = 44\frac{1}{2}$; $44\frac{1}{2} \div 4 = 11\frac{1}{8}$. **4.** 900. **5.** $48\frac{5}{12}$. **6.** 8945. **7.** 65; $98\frac{2}{5}$; $10\frac{1}{2}$. **8.** $3\frac{1}{3}$; $12\frac{3}{4}$; $\frac{5}{6}$. **9.** (1) 1500 ft. ÷ 5280 ft. = .28 (mi.), or .3 mi.; (2) 3750 ft. ÷ 5280 ft. = .71 (mi.), or .7 mi.; (3) 4300 ft. ÷ 5280 ft. = .81 (mi.), or .8 mi.; (4) 4725 ft. ÷ 5280 ft. = .89 (mi.), or .9 mi. **10.** .025 × \$2500 = \$62.50; .0175 × \$3800 = \$66.50; so $1\frac{3}{4}$% of \$3800 is \$4 greater. **11.** 47 ÷ 50 = .94; 72 ÷ 75 = .96; so Team B had the better standing. **12.** 40° and 100°. **13.** Obtuse triangle. **14.** $\frac{1}{2}$ of 1% = .5%, .5% of \$250 = \$1.25; 50% of \$250 = \$125; no, the difference is \$123.75. **15.** $\frac{3}{4}$ of \$80 = \$60, $7\frac{1}{2}$% of \$800 = \$60, 750% of \$8000 = \$60,000, $\frac{3}{4}$% of \$8000 = \$60, 75% of \$80 = \$60; all but 750% of \$8000. **16.** 6 × \$13.50 = \$81; \$81 − \$75 = \$6, carrying charge; $C = \$6, P = \$75, n = 6$; $r = \dfrac{24 \times 6}{75(6+1)} = .274$, or 27%. **17.** \$381.5 million; \$94.6 million; 25%.

Page 171

Aim: To show how to draw a triangle when given two sides and the included angle

Suggestions: In ex. 1 and 2 make sure that the pupil understands the term *included angle.* In order fully to understand the work on this page, the pupil must actually draw the triangles indicated in ex. 1 and in ex. 3–7, using a protractor to draw the included angles. Ex. 7 on this page is very important and should be carefully done.

Key: **3.** 86°, $3\frac{1}{2}$ in., 34°; 42°, 3 in., 48°. **4.** 77°, $3\frac{3}{4}$ in., 44°; 40°, $6\frac{3}{8}$ in., 25°. **5.** 105°, 4 in., 35°; 89°, 4 in., 61°. **6.** 129°, 2 in., 33°; 40°, $4\frac{1}{4}$ in., 25°. **7.** 800 ft. = 4 in. on the scale; 450 ft. = $2\frac{1}{4}$ in. on the scale; the third side measures $3\frac{1}{2}$ in. on the drawing, or 700 ft.

Pages 172–173

Aim: To show how to measure distances indirectly by making use of triangles

Suggestions: These pages should be read slowly and carefully in order to understand what is meant by measuring a distance *indirectly*. In ex. 1 it is evident that you cannot measure the length of the pond directly because of the water, but you can find that length in a roundabout way, or *indirectly*. The explanation in ex. 1 must be carefully studied. The scale drawing which is mentioned in the explanation is very important since it is this scale drawing that finally provides the length of the pond. The pupils should make the scale drawings for ex. 2–5 and find the unknown distances.

The pupils will be interested in seeing a surveyor's transit, pictured on page 173, if one is available in your school.

Key: **2.** 500 ft. **3.** RT measures $3\frac{1}{4}$ in., or 325 ft. **4.** 275 ft. **5.** 550 ft.

Pages 174–175

Aim: To show how to make a field protractor and how to use it in measuring distances indirectly

Suggestions: The work on pages 174–175 will be found very interesting. The field protractor described on page 174 enables one to measure angles out of doors. You will find that the circular protractors described at the bottom of page 174 are inexpensive and very satisfactory for the construction of field protractors. You may wish to make a field protractor with the class or have each of several groups of pupils make one. If a large paper protractor cannot be purchased, one can be made as follows:

Using a radius of 4 in. or 5 in., draw a semicircle on paper and mark its center C. Place a small protractor on the paper so that its center is at point C, and its diameter lies along the diameter of the semicircle. Then place dots on the paper around the circular edge of the protractor at each 5-degree mark. Draw rays from C through each of these dots and extend these rays until they cut the circle. The circle is now marked in 5-degree intervals. By eye, divide each 5-degree space into 5 equal parts, and you will have a large protractor divided into 180°.

The method of using a field protractor is described on page 175. In the picture on page 175 the boy wants to find the distance AC; he has to find this distance *indirectly* since he cannot measure it directly because of the water. To measure AC indirectly, he uses the triangle which is shown by the dotted lines. It is necessary to measure the sides BA and BC of this triangle, and also the included angle at B. The picture shows him measuring angle B with the field

protractor. After the angle has been measured, he measures the distances BA and BC by pacing. He now has two sides and the included angle of the triangle. With this data, he can draw the triangle to scale, as explained in ex. 1 on page 172. After this triangle is drawn, he then measures side AC on the scale drawing to find the length of the pond.

The class will enjoy and profit from the experience of going outdoors to measure some unknown distances indirectly. Therefore, it is suggested that you arrange, if possible, for work of this kind. If your equipment budget permits, you will find a 100-foot steel tape useful for making the required direct measurements of the sides of a triangle.

Key: Page 175 **2.** 60°; 125°; 30°.

Page 176

Aim: To show how to draw a triangle when two angles and the included side are given

Suggestions: On this page, it is important for the pupils to draw each of the triangles as suggested in the exercises. In this work it must be made clear that the *included side* is the side between the two given angles. In the figure in ex. 1, the included side is AB; side BC is not the included side; likewise, side AC is not the included side.

Key: **2.** 3 in.; 4 in.; 90°. **4.** $2\frac{1}{2}$ in., $2\frac{1}{2}$ in., 60°; $3\frac{1}{2}$ in., 4 in., 106°; $7\frac{1}{2}$ in., 6 in., 37°. **5.** $7\frac{1}{2}$ in., $6\frac{1}{2}$ in., 30°; 6 in., 4 in., 56°; $4\frac{1}{2}$ in., $6\frac{1}{4}$ in., 59°. **6.** 3 in., 3 in., 90°; $2\frac{1}{2}$ in., 5 in., 52°; $3\frac{1}{2}$ in., $3\frac{1}{2}$ in., 60°. **7.** $4\frac{3}{8}$ in., $6\frac{5}{8}$ in., 49°; 3 in., $1\frac{3}{4}$ in., 40°; $2\frac{3}{4}$ in., $1\frac{1}{2}$ in., 64°. **8.** $3\frac{5}{8}$ in., $1\frac{3}{4}$ in., 89°; $2\frac{3}{4}$ in., $4\frac{1}{4}$ in., 117°; $3\frac{3}{4}$ in., $3\frac{1}{4}$ in., 50°. **9.** $2\frac{3}{8}$ in., 4 in., 48°; $1\frac{5}{8}$ in., $2\frac{3}{4}$ in., 83°; $2\frac{3}{4}$ in., $2\frac{1}{4}$ in., 59°.

Page 177

Aim: To provide a review of previous work

Suggestions: All the work required for the answers to the questions on this page should be done mentally.

Key: **1.** $62° + 75° + 43° = 180°$; since the sum of the three angles of a triangle should equal 180°, these 3 angles could be in one triangle. **2.** A decimal. **3.** More than the number. **4.** 485; 3.86. **5.** 1963. **6.** 80°. **7.** 3 out of 5; 9 out of 15. **8.** 16 out of 20. **9.** The numbers given may be represented by these 3 fractions: $\frac{26}{35}$, $\frac{26}{32}$, $\frac{26}{29}$. These fractions all have the same numerator. The value of each fraction depends upon the size of its denominator. As the denominator gets larger, the value of the fraction becomes less. Hence, $\frac{26}{35}$ has the smallest value and represents the smallest per cent; $\frac{26}{29}$ has the largest value and represents the largest per cent. **10.** This example is worked the same way as ex. 9; 37 out of 53 represents the smallest per cent. **11.** $\frac{3}{4}$ of 1% is less than 1%, while 75% is 75 times 1%. Therefore, 75% of 200 is much larger. **12.** On \$100 the interest at 6% is twice as much as the interest at 3%, but the interest for

30 days is half as much as the interest for 60 days. Hence, the interest at 6% for 30 days gives the same amount as the interest at 3% for 60 days. **13.** 60% + 33% = 93%; 100% − 93% = 7%. **14.** 0. **15.** 24. **16.** 100 games were played in all, and the team won 60 games; hence, its percentage is .600. **17.** Think of 19.6% as about 20%, or $\frac{1}{5}$; $\frac{1}{5} \times 275 = 55$; 53.9 is nearest to 55.

Page 178

Aim: To use the method of drawing a triangle given on page 176 in measuring distances indirectly

Suggestions: On page 178 the work on page 176 is applied to measure a distance indirectly. The picture at the top of page 178 shows how this is done. The steps in the work are as follows: Tom paces a distance of 300 ft. along the shore to make side AB of a triangle. Then he measures angles A and B. To measure angle A he places his field protractor at A and moves the pointer so that it points first at B and then at C. To measure angle B he places his field protractor at B and moves the pointer so that it points first at A and then at C. In this way he gets the measurements for two angles and the included side of a triangle. He then uses these measurements to draw this triangle to scale. From the scale drawing, he finds the distance AC indirectly, which is what he set out to find.

Key: **2.** 3 in.; $2\frac{1}{2}$ in.; 250 ft. **3.** 275 ft.; 325 ft. **4.** 225 ft.

Page 179

Aim: To give a true-false test on the concepts and facts of Chapter 5

Suggestion: You might ask pupils to tell why a statement is true or false.

Key: **1.** False; the sum of the angles is greater than 180°. **2.** True. **3.** True. **4.** False; lengths of sides do not affect size of an angle. **5.** True. **6.** False; a right angle is formed when one line is drawn perpendicular to another line. **7.** True. **8.** True; the two sides of the right angle can be equal, thus making an isosceles triangle. **9.** True. **10.** True. **11.** True. **12.** False; parallel lines never meet. **13.** True. **14.** True. **15.** False; the triangles *are* congruent. **16.** True. **17.** True.

Page 180

Aim: To provide a general review of problem solving

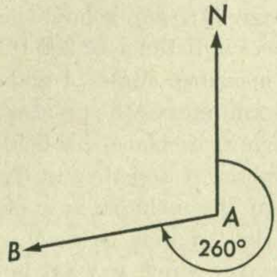
Key: **1.** .10 × \$48.75 = \$4.875, which is called \$4.88; .02 × \$48.75 = \$.975, which is called \$.98; \$48.75 + \$4.88 + \$.98 = \$54.61. **2.** 70.6 million ÷ \$133.6 million = .528, or 53%. **3.** (1) $\frac{\$150}{1} \times \frac{5}{100} \times \frac{1}{2} = \$3.75$, interest, \$177 + \$3.75 = \$180.75; (2) \$195 − \$180.75 = \$14.25. **4.** (1) 88 = 15 + 35 + 38; 15 kw-hr cost \$.95; 35 × \$.04 = \$1.40; 38 × \$.03 = \$1.14; \$.95 + \$1.40 + \$1.14 = \$3.49; (2) 143 = 15 + 35 + 50 + 43; 15 kw-hr cost \$.95; 35 × \$.04 = \$1.40; 50 × \$.03 = \$1.50; 43 × \$.02$\frac{1}{2}$ = \$1.07$\frac{1}{2}$, or \$1.08; \$.95 + \$1.40 + \$1.50 + \$1.08 = \$4.93; (3) 218 = 15

$+ 35 + 50 + 118$; 15 kw-hr cost \$.95; $35 \times \$.04 = \$1.40$; $50 \times \$.03 = \$1.50$; $118 \times \$.02\frac{1}{2} = \$2.95$; $\$.95 + \$1.40 + \$1.50 + \$2.95 = \$6.80$. **5.** $4 \times 12 = 48$ (sq. ft.); $5\frac{1}{2} \times 10\frac{1}{2} = 57\frac{3}{4}$ (sq. ft.); $6 \times 60 = 60$ (sq. ft.); $6\frac{1}{2} \times 9\frac{1}{2} = 61\frac{3}{4}$ (sq. ft.); $8 \times 8 = 64$ (sq. ft.); so the areas are not the same; $4' \times 12'$ is the smallest, and $8' \times 8'$ is the largest. **6.** $100 \div 25 = 4$, so 100 sheets are 4 times as many as 25 sheets; $4 \times \$.75 = \$3.00$; $\$3.00 - \$2.50 = \$.50$. **7.** 5% of S.P. = \$725; 1% of S.P. = \$725 ÷ 5, or \$145; 100% of S.P. = 100 × \$145, or \$14,500.

Page 181

Aim: To review the work of Chapter 5

Key: 3.



8. 80°. **9.** (1) $90° + 45° + 45° = 180°$, yes; (2) $90° + 60° + 30° = 180°$, yes; (3) $80° + 50° + 60° = 190°$, no; (4) $100° + 25° + 55° = 180°$, yes; (5) $34° + 62° + 89° = 185°$, no; (6) $90° + 15° + 90° = 195°$, no. **11.** 82°; 58°; $2\frac{1}{4}$ in. **12.** 98°; $1\frac{1}{4}$ in.; 2 in.

Page 182

Aim: To present Problem Test 5

Key: **1.** (1) $65 \times 100 = 6500$ (sq. ft.), $6500 \times \$3 = \$19,500$; (2) $\frac{1}{2} \times 96 \times 84 = 4032$ (sq. ft.), $4032 \times \$3 = \$12,096$; (3) $\frac{1}{2} \times 60 \times 200 = 6000$, $6000 \times \$3 = \$18,000$. **2.** $2\frac{1}{2}$ lb. + $1\frac{15}{16}$ lb. + $2\frac{1}{8}$ lb. + $1\frac{7}{8}$ lb. + $2\frac{1}{4}$ lb. + $2\frac{3}{16}$ lb. + $1\frac{1}{2}$ lb. + $1\frac{5}{8}$ lb. = 16 lb.; 16 lb. ÷ 8 = 2 lb. **3.** $12 \times \$43.33 = \$519.96$; $\$519.96 - \$500 = \$19.96$. **4.** $.20 \times \$50 = \$10$; $\$50 - \$10 = \$40$; 100% − 40% = 60%; 60% of S.P. = \$40; 1% of S.P. = \$40 ÷ 60, or \$.66667; 100% of S.P. = \$66.667, or \$66.67. **5.** \$15,580 + \$820 = \$16,400; \$820 ÷ \$16,400 = .05, or 5%. **6.** 26 da. (May) + 30 da. (June) + 7 da. (July) = 63 da.; $\frac{1}{2} \times \$.88 = \$.44$ (3 da.); \$8.80 (60 da.) + \$.44 (3 da.) = \$9.24 (63 da.). **7.** (1) \$880 − \$9.24 = \$870.76; (2) \$880. **8.** \$2.50 ÷ 500 = \$.005, or $\frac{1}{2}$¢; 30 × $\frac{1}{2}$¢ = 15¢. **9.** 25¢ − 21¢ = 4¢; 4¢ ÷ 21¢ = .190, or 19%. **10.** 72 ÷ 2 = 36, so 72 grapefruit are 36 times as many as 2 grapefruit; 36 × \$.25 = \$9.

Pages 183–184

Aim: To provide diagnostic tests with suggested pages for practice

Suggestions: Since the work of the first half year is now completed, two diagnostic tests are given on these pages in order to cover in detail several of the topics

studied during this period. These two tests are devoted primarily to the four fundamental operations with fractions, to division of whole numbers and decimals, and to the three types of percentage examples. Before proceeding to new work, make sure that the pupils have a satisfactory mastery of these topics. If necessary, re-explain basic procedures and assign practice.

Key: Page 183 **1.** $2603\frac{4}{9}$; 810. **2.** 8038; 2155. **3.** $14\frac{1}{8}$; 16; $10\frac{13}{16}$; $7\frac{1}{2}$; $10\frac{11}{12}$. **4.** $4\frac{2}{5}$; $6\frac{3}{4}$; $2\frac{4}{5}$; $\frac{5}{16}$; $2\frac{1}{6}$. **5.** 6; 1; $\frac{1}{6}$. **6.** $26\frac{2}{3}$; $\frac{5}{6}$; $\frac{21}{32}$. **7.** .5; .214; 5.58; .13. **8.** 90.33; .03; 12.01; 6800. **9.** 24%; $87\frac{1}{2}$%; 125%; $16\frac{1}{4}$%; 1045%. **10.** 7%; $8\frac{1}{2}$%; 364%; $7\frac{3}{4}$%; $412\frac{1}{2}$%. **11.** $81.00; $34.94. **12.** $175.50; $24.03.

Page 184 **1.** $2.31; $3.33. **2.** $77.66; $195.17. **3.** $1.88; $4.13. **4.** $2.58; $5.57. **5.** $816.00; $3452.00. **6.** $1023.00; $4260.00. **7.** 400; 50. **8.** 70; 12. **9.** 24%; 42%. **10.** 38.6%; 11.9%. **11.** $150; $165. **12.** $400; $160. **13.** $600; $160. **14.** $7.50; $46.25. **15.** $9.46; $1.45. **16.** $3.75; $9.

Chapter 6

Aims of Chapter 6. The major aims of Chapter 6 are to:

1. Show how a corporation is formed and operates, and describe the kinds of stock issued by a corporation.
2. Teach how stocks are bought and sold, and how brokers' commissions are computed.
3. Teach how to find the rate of income received from stocks that are owned.
4. Give information about bonds, including United States Savings Bonds.
5. Teach how bonds are bought and sold.
6. Teach how to find the rate of income on a bond.
7. Teach how money is borrowed from savings and loan associations and other lending agencies to build a house, and how it is paid back.
8. Explain the meaning of compound interest and teach how it is computed.

Page 185

Aim: To discuss ways of investing money

Suggestions: The great industrial activity which is so characteristic of the United States has been made possible by the growth of corporations. Very often these corporations may raise money for expanding their plants by the sale of shares of stock to the public. Stocks represent the means by which individuals may share in great enterprises. In other words, they are symbols of co-operative effort.

When a corporation needs to borrow money for expansion or other purposes, it may issue bonds which are sold to the public. These bonds are called *industrial bonds* to distinguish them from city, state, and United States Government bonds. These industrial bonds are another way in which one may invest his money.

When you have a class discussion of this page, ask the pupils to tell of other ways of investing savings.

Pages 186–187

Aim: To show in terms of a simple example how a corporation is formed and how it operates

Suggestions: An effective way to develop an understanding of our large corporations is to present a simple example, such as the Johnson Toy Company, that could be an experience of the pupils. If time permits, pupils can be asked to make up some additional problems about the Johnson Toy Company.

In connection with the *par value* of a stock, attention should be called to the fact that stock that has *no par value* is also issued. In such cases each share of stock represents a definite share in the business. After the initial issuance of common stock, par value has no great significance to the buyer. Actually, the

value of a stock is the price investors are willing to pay for it. A new issue of common stock today often has a par value lower than the price at which it is offered to the public.

Key: **1.** 300 shares. **2.** \$25; \$20; \$5; \$3. **3.** 10%. **4.** \$2.50, \$2.00, \$.50, \$.30; \$1.80; \$.90; \$3.50; \$4.00.

Page 188

Aim: To describe the kinds of stock issued by corporations

Suggestions: The great corporations of this country are owned by millions of people who have bought shares of stock in these companies. A few of the better known corporations are the General Motors Corporation, the United States Steel Corporation, the General Electric Company, the Union Pacific Railroad, the Standard Oil Company of New Jersey, the American Telephone and Telegraph Company, the Radio Corporation of America, and many others. Lists of the more important corporations in this country are given daily in the larger newspapers, together with the current prices at which the stocks of these corporations are selling.

Preferred stock, as stated in ex. 4, has a preference over common stock with respect to dividends. It also has a preference over common stock with respect to liquidation payments in the event a corporation is dissolved. On the other hand, preferred stockholders usually do not have the right to vote on matters related to the operation of the corporation.

Key: **1.** \$3.00; \$37.50.

Page 189

Aim: To give information about the New York Stock Exchange and the way in which stocks are bought and sold

Suggestions: The New York Stock Exchange is the largest and most important stock exchange in this country. It may, in fact, be considered both a national and an international market place for the sale of stocks and bonds. Orders to buy and sell on this exchange come in daily from practically all our larger cities, as well as from European cities. The brokers who buy and sell stocks on the New York Stock Exchange have to be members of the Exchange, the number of members being limited. These brokers have brokerage offices in the city of New York, and in many instances they also have branch offices in cities throughout the United States. Each branch office is connected by private wire with the broker's New York office. It is thus possible for a man in Seattle to give an order at his Seattle broker's office for the purchase of stocks on the New York Stock Exchange and to have a reply within a few minutes, stating that the order has been executed in New York.

On the New York Stock Exchange only those stocks are bought and sold which have been officially approved or "listed" by the officers of the Exchange.

Before approving the stock of any corporation, the authorities of the Stock Exchange require detailed information concerning the financial standing of the corporation in question. The fact that a stock is listed on the New York Stock Exchange is one evidence that it is a reputable stock. Fake stocks or those of doubtful value would not be accepted for listing on this exchange.

All transactions on the New York Stock Exchange are recorded regularly in the newspapers, some of the larger papers devoting three to eight pages daily to such news. None of the business of the Exchange is done in private or in secret. Through these newspaper quotations it is possible for anyone to find the present market value of any stock listed on the Exchange. This fact alone affords a great service and protection to the investing public.

In teaching stocks and bonds in the eighth grade, you can add much interest by having pupils bring to class for several days the financial pages of a newspaper and showing them how to read the stock quotations on these pages. If you wish detailed information about the financial pages of the newspaper, you can obtain from *The New York Times* for $1.95 a useful book entitled *How to Read and Understand Financial and Business News.*

Workbook Reference: Arithmetic Workshop, Book 8, page 68

Pages 190–191

Aim: To teach how stocks are bought and sold through brokers

Suggestions: The brokerage commissions for buying and selling stocks listed on the New York Stock Exchange are fixed by the Exchange. The commissions currently charged on 100-share lots are given on page 190. They vary from time to time because of changing economic conditions. Besides the commissions, certain federal taxes, known as *stock transfer taxes,* are charged on each stock transaction. These taxes are paid by the seller of the stock and not by the buyer. In addition to the federal tax, several states, including New York State, have a stock transfer tax. All selling prices for stocks sold on the New York Stock Exchange must include certain taxes. Since all these taxes change frequently, they are not included in the exercises in the textbook. The current practice with respect to these taxes can be learned by inquiry at a stockbroker's office.

A *round lot* of shares of stock is usually 100 shares, and an *odd lot* is anything less than 100 shares. Odd-lot trading is important, for it permits an investor to buy stock with limited funds and to diversify his investments. A regular broker turns over an odd-lot order to a special broker who deals in odd lots. Because of the extra work in handling an odd-lot order, the buyer must pay $12\frac{1}{2}$¢ more per share for stock selling under $40 per share, and 25¢ more per share for stock selling at or above $40 per share.

Supplement the work on these pages with some problems based on actual stock quotations from a newspaper.

Workbook Reference: Arithmetic Workshop, Book 8, page 69

Key: **3.** (1) $3 \times \$48.33 = \$144.99$; (2) $3 \times \$9325 = \$27{,}975$, $\$27{,}975 + \$144.99$

= \$28,119.99. **4.** (1) $\frac{1}{10} \times \$57.375 = \$5.7375$, or \$5.74; $\$39 + \$5.74 = \$44.74$, $2 \times \$44.74 = \$89.48$; (2) $200 \times \$57.375 = \$11{,}475.00$; $\$11{,}475.00 + \$89.48 = \$11{,}564.48$. **7.** $\frac{1}{10} \times \$56.25 = \$5.625$, or \$5.63; $\$39 + \$5.63 = \$44.63$, brokerage; $100 \times \$56.25 = \$5625$, cost of 100 shares; $\$5625 + \$44.63 = \$5669.63$. **8.** $\frac{1}{2} \times \$43.875 = \$21.9375$, or \$21.94; $\$19 + \$21.94 = \$40.94$, brokerage on 100 shares; $2 \times \$40.94 = \$81.88$, brokerage on 200 shares; $200 \times \$43.875 = \$8775$, cost of 200 shares; $\$8775 + \$81.88 = \$8856.88$. **9.** $\frac{1}{10} \times \$113 = \$11.30$; $\$39 + \$11.30 = \$50.30$, brokerage; $100 \times \$113 = \$11{,}300$, cost of 100 shares; $\$11{,}300 + \$50.30 = \$11{,}350.30$. **10.** $\$7 + \$17.38 = \$24.38$, brokerage on 100 shares; $3 \times \$24.38 = \$73.14$, brokerage on 300 shares; $300 \times \$17.375 = \$5212.50$, cost of 300 shares; $\$5212.50 + \$73.14 = \$5285.64$. **11.** $\frac{1}{10} \times \$84.75 = \$8.475$, or \$8.48, $\$8.48 + \$39 = \$47.48$, brokerage on 100 shares; $2 \times \$47.48 = \$94.96$, brokerage on 200 shares; $200 \times \$84.75 = \$16{,}950$; $\$16{,}950 + \$94.96 = \$17{,}044.96$.

Page 192

Aim: To show how profits and losses are determined when stocks are bought and sold

Suggestions: You will probably find it helpful to work out ex. 1 and 2 with the class. Stress the fact that to find the actual cost of stock you must add the brokerage, and to find the returns from a sale of stock you must subtract the brokerage.

The brokerage commissions are given for all exercises except ex. 1 and 2. For additional practice these commissions can be checked by using the rates given on page 190 if you wish.

Remind the pupils that the brokerage commissions that are given are the commissions for each 100 shares. If 200 shares are bought and sold, as in ex. 5, the brokerage commission is double the commission shown.

Workbook Reference: Arithmetic Workshop, Book 8, page 72

Key: **1.** (1) $\frac{1}{2} \times \$49.75 = \$24.875$, or \$24.88; $\$19 + \$24.88 = \$43.88$, brokerage when bought; (2) $\frac{1}{10} \times \$53.50 = \$5.35$; $\$39 + \$5.35 = \$44.35$, brokerage when sold; (3) $100 \times \$49.75 = \$4975$, cost of 100 shares; $\$4975 + \$43.88 = \$5018.88$, total cost; $100 \times \$53.50 = \$5350$, selling price; $\$5350 - \$44.35 = \$5305.65$, total sale return; $\$5305.65 - \$5018.88 = \$286.77$, profit. **2.** $100 \times \$6.50 = \$650$; $\$286.77 + \$650 = \$936.77$. **3.** (1) $100 \times \$14 = \$1400$, cost of 100 shares; $\$1400 + \$21 = \$1421$, total cost; $100 \times \$1 = \$100$, selling price; $\$100 - \$6 = \$94$, total sale return; $\$1421 - \$94 = \$1327$; (2) Mr. Bell was speculating. **4.** No. **5.** $200 \times \$49.50 = \$9900$; $2 \times \$43.75 = \$87.50$; $\$9900 + \$87.50 = \$9987.50$; $200 \times \$60.75 = \$12{,}150$; $2 \times \$45.08 = \$90.16$; $\$12{,}150 - \$90.16 = \$12{,}059.84$; $\$12{,}059.84 - \$9987.50 = \$2072.34$; $200 \times \$2.50 = \$500$; $\$2072.34 + \$500 = \$2572.34$, profit. **6.** $100 \times \$77 = \$7700$; $\$7700 + \$46.70 = \$7746.70$; $100 \times \$92.50 = \$9250$; $\$9250 - \$48.25 = \$9201.75$; $\$9201.75 - \$7746.70 = \$1455.05$; $100 \times \$4.50 = \$450$; $\$1455.05 + \$450 = \$1905.05$, profit. **7.** $200 \times \$38.25 = \$7650$; $2 \times \$38.13 = \$76.26$; $\$7650 + \$76.26 = \$7726.26$; $200 \times \$23.25$

= \$4650; 2 × \$30.25 = \$60.50; \$4650 − \$60.50 = \$4589.50; \$7726.26 − \$4589.50 = \$3136.76; 200 × \$1.50 = \$300; \$3136.76 − \$300 = \$2836.76, loss. **8.** 500 × \$18.875 = \$9437.50; 5 × \$25.88 = \$129.40; \$9437.50 + \$129.40 = \$9566.90; 500 × \$18 = \$9000; 5 × \$25 = \$125; \$9000 − \$125 = \$8875; \$9566.90 − \$8875 = \$691.90, loss. **9.** 100 × \$3.125 = \$312.50; \$312.50 + \$9.25 = \$321.75; 100 × \$7.75 = \$775; \$775 − \$14.75 = \$760.25; \$760.25 − \$321.75 = \$438.50; 100 × \$.60 = \$60; \$438.50 + \$60 = \$498.50, profit. **10.** 300 × \$252.50 = \$75,750; 3 × \$64.25 = \$192.75; \$75,750 + \$192.75 = \$75,942.75; 300 × \$260.125 = \$78,037.50; 3 × \$65.01 = \$195.03; \$78,037.50 − \$195.03 = \$77,842.47; \$77,842.47 − \$75,942.75 = \$1899.72; 300 × \$10 = \$3000; \$1899.72 + \$3000 = \$4899.72, profit. **11.** 100 × \$49.25 = \$4925; \$4925 + \$43.63 = \$4968.63; 100 × \$64 = \$6400; \$6400 − \$45.40 = \$6354.60; \$6354.60 − \$4968.63 = \$1385.97; 100 × \$3 = \$300; \$1385.97 + \$300 = \$1685.97, profit. **12.** 200 × \$83.625 = \$16,725; 2 × \$47.36 = \$94.72; \$16,725 + \$94.72 = \$16,819.72; 200 × \$78.875 = \$15,775; 2 × \$46.89 = \$93.78; \$15,775 − \$93.78 = \$15,681.22; \$16,819.72 − \$15,681.22 = \$1138.50, loss.

Page 193

Aim: To present another set of improvement tests in addition

Suggestion: This set of improvement tests is to be administered in the same manner as the previous sets in addition. Since these are addition tests, the answers are to be written on folded paper. See page 375 of the text.

Key: **1.** \$30.46; \$36.01; \$25.29; \$36.61; \$31.46; \$40.87. **2.** \$29.01; \$30.15; \$35.88; \$25.00; \$33.42; \$62.06. **3.** \$30.07; \$28.57; \$27.44; \$26.35; \$35.91; \$53.47.

Page 194

Aim: To show how to find the rate of income that is received from stocks that have been purchased

Suggestion: In ex. 2 on this page you find the rate of income by finding what per cent \$5 is of \$69.50. This is an application of the second type of percentage problem.

Workbook Reference: Arithmetic Workshop, Book 8, page 70

Key: **3.** 6.0%. **4.** 5.0%. **5.** 6.7%. **6.** 6.3%. **7.** 8.3%. **8.** 3.8%. **9.** 5.5%. **10.** 7.5%. **11.** 7.3%. **12.** 3.9%. **13.** 8.7%. **14.** 6.0%. **15.** 6.5%. **16.** 4.3%. **17.** 4.8%. **18.** 10.5%. **19.** 4.3%. **20.** 5.5%. **21.** 3.8%. **22.** 3.6%. **23.** 4.3%. **24.** 4.3%; 5.4%. The dividend on preferred stock is always limited to a stated amount regardless of how large the profits of the company may be. The dividend on common stock, however, is not limited. If the company is making good profits, a large dividend may be declared on the common stock; hence, the dividend on the common stock in some years may be larger than that on the preferred stock. Under such circumstances the common stock may sell at a higher price per share than the preferred stock.

Page 195

Aim: To give a general review

Key: **1.** (1) $.02 \times \$186 = \$3.72$; $\$186 - \$3.72 = \$182.28$; $\$182.28 + \$.90 = \$183.18$, $10 \times 12 = 120$ (pr.); $\$183.18 \div 120 = \$1.526$, or \$1.53; (2) $100\% - 40\% = 60\%$; 60% of S.P. = \$1.53; 1% of S.P. = $\$1.53 \div 60$, or \$.0255; 100% of S.P. = $100 \times \$.0255$, or \$2.55. **2.** $10 \times \$5.50 = \$55$; $\$55 + \$25 = \$80$; $.20 \times \$80 = \$16$. **3.** $\$75 - \$63 = \$12$; $\$12 \div \$75 = .16$, or 16%; $\$80 - \$66 = \$14$; $\$14 \div \$80 = .17\frac{1}{2}$, or $17\frac{1}{2}\%$; so \$80 reduced to \$66 represents the better rate of discount. **4.** $328 - 160 = 168$ (members); $168 \div 160 = 1.05$, or 105%. **5.** 4.55; 10.92; 1221.67; 13.21. **6.** .11; .05; 8.71; 354.29. **7.** $14\frac{7}{8}$; $13\frac{4}{5}$; $14\frac{1}{12}$; $16\frac{13}{24}$; 18; $17\frac{5}{12}$; $13\frac{11}{16}$. **8.** 60; 200; 12. **9.** 22; 180; 14.1. **10.** 11; 87.5; 14.

Pages 196–197

Aim: To describe bonds and to discuss the terms used in connection with bonds

Suggestions: When a corporation needs additional money for improvements or other purposes, it borrows money by issuing bonds. The corporation pays interest on these bonds, just as an individual pays interest on money that he borrows from a bank.

The bonds of our railroads, airlines, and other corporations, as well as many of the bonds of the United States Government, are listed on the New York Stock Exchange; hence, such bonds may be purchased quickly through a broker who is a member of the Exchange.

The owner of shares of the stock of a corporation usually has the power to vote, based on the number of shares owned, regarding the affairs of the corporation. The owner of one of its bonds, however, has no such power.

Bonds play a very important part in the investments of the world. The funds of estates, life insurance companies, savings banks, endowed colleges and universities, and so on may be invested in bonds, because this form of security is usually more stable and dependable than stocks.

In introducing bonds to eighth-grade pupils, you should endeavor to refer to local situations where bonds have been issued. Possibly the city in which the pupils live has raised money, through an issue of bonds, for a new school building or new waterworks. United States Savings Bonds should also be mentioned. See pages 201 and 202 of the text.

Key: **5.** \$25; \$50. **6.** Bond matures 22 yr. after the first of the 45 coupons is due.

Page 198

Aim: To show how much interest one gets from a bond

Suggestions: The bonds in ex. 5–8 are described in the form used in the financial pages of the newspaper. "American Oil 5s, 1974" means a bond of the American Oil Company which pays 5% interest and matures in 1974. In ex. 9 the interest paid on a bond each year is practically guaranteed as long as the company is

sound. If a company should fail, it may stop paying interest on its bonds and it may also be unable to redeem its bonds at maturity. In ex. 11, the most dependable income is that paid by a company on its first mortgage bonds because bonds come before preferred stock and common stock in their claims on the earnings of the company. If the earnings were not sufficient to pay dividends on the preferred and common stock, the interest on the bonds would still be paid.

Key: **1.** (1) $425,000 ÷ $1000 = 425 (bonds); (2) .05 × $425,000 = $21,250; ½ × $21,250 = $10,625. **2.** .03½ × $800,000 = $28,000. **3.** It is easier to pay off 20 matured bonds each year than it is to let all the bonds run for a long time and then pay them all off together. **4.** (1) $130,000,000 ÷ $1000 = 130,000 (bonds); (2) .0375 × $130,000,000 = $4,875,000; ½ × $4,875,000 = $2,437,500. **5.** (1) 5%, 1974, $25; (2) 3½%, 1985, $17.50. **6.** (1) 4½%, 1971, $22.50; (2) 6%, 1995, $30.00. **7.** (1) 4%, 2003, $20.00; (2) 2¾%, 1976, $13.75. **8.** (1) 3%, 1990, $15.00; (2) 3½%, 1981, $17.50. **9.** Yes; the rate is guaranteed. **10.** No; yes. **11.** Bonds.

Page 199

Aim: To show how bonds are bought and sold on a stock exchange through a broker

Suggestions: When a $1000 bond is first issued, it usually sells at, or close to, its par value, which is $1000. If the bond is a 4% bond, it pays $40 a year in interest. If the company that issued the bond becomes more and more prosperous, each year its bonds will be in demand and will usually sell at a higher price. The price of the bond may rise to 105 or more, in which case each bond will cost $1050 or more. The bond will continue, however, to pay only $40 interest a year regardless of the price at which the bond sells. On the other hand, if the company has several years of poor business, with decreasing profits, its bonds may drop to 95, or below, because the public has less confidence in them. In this event a $1000 bond can be bought for $950 or less. The interest on this bond, however, will continue to be $40 a year. Thus, you can see why the prices of bonds usually do not fluctuate so much as the prices of common stock. On the date of maturity a $1000 bond is paid off *at par*, which means that the company redeems the bond at $1000 regardless of the fluctuations in the price of the bond during its lifetime. If you originally bought the bond for $950, or for $1060, in either case you would get $1000 for it at maturity. This information will enable you to understand better ex. 6 on this page.

Key: **2.** $825.00; $717.50; $926.25; $1031.25; $1108.75; $872.50; $1003.75. **3.** (1) 2 × $883.75 = $1767.50; 2 × $5 = $10; $1767.50 + $10 = $1777.50; (2) 4 × $877.50 = $3510.00; 4 × $3 = $12; $3510 + $12 = $3522; (3) 7 × $1026.25 = $7183.75; 7 × $2.50 = $17.50; $7183.75 + $17.50 = $7201.25. **4.** (1) $1001.25 + $5 = $1006.25; (2) $22.50. **5.** 5 × $1036.25 = $5181.25; 5 × $2.50 = $12.50; $5181.25 + $12.50 = $5193.75. **6.** When a $1000 bond matures, the owner will receive only $1000.

Page 200

Aim: To show how to find the rate of income on a bond

Suggestions: The explanations in ex. 1 and 2 on this page should be carefully studied. From these explanations you see that a 4% bond doesn't necessarily pay 4% on the money that is invested in it. If you buy a 4% bond at 100, you do get 4% on your investment; but if you buy a 4% bond at 80, you get 5% on your investment. On the other hand, if you buy a 4% bond at 108, you get only 3.7% on your investment.

Supplement the work on pages 199 and 200 with some problems based on actual bond quotations from a newspaper.

Workbook Reference: Arithmetic Workshop, Book 8, page 71

Key: **3.** (1) $10 \times \$101 = \$1010$; (2) $.025 \times \$1000 = \$25$; (3) $\$25 \div \$1010 = .0247$, or 2.5%. **4.** (1) $.035 \times \$1000 = \$35$; (2) $10 \times \$82 = \$820$; $\$35 \div \$820 = .0426$, or 4.3%. **5.** $.03 \times \$1000 = \$30$; $10 \times \$89.25 = \$892.50$; $\$30 \div \$892.50 = .0336$, or 3.4%. **6.** 6.0%. **7.** 4.4%. **8.** 4.7%. **9.** 2.8%. **10.** 3.7%. **11.** 4.9%. **12.** 2.4%. **13.** 4.3%. **14.** 4.1%.

Pages 201–202

Aim: To give information about United States Savings Bonds of Series E

Suggestions: Series E United States Savings Bonds originally matured at the end of 10 years. In order to increase the interest rate and keep the purchase prices of these bonds the same, the Government has reduced the length of time to maturity. This length of time was first reduced to 9 yr. and 8 mo., and it is now 7 yr. and 9 mo. The redemption values have also, of course, been changed. The parents of the pupils may have purchased some of these earlier Savings Bonds, or the pupils may have received them as gifts. Ask the pupils to report, if possible, on former redemption values of Savings Bonds. These values can be compared with the redemption values given on page 202.

The maturity dates of the earlier Savings Bonds have been automatically extended 20 years. That is, the owner of one of these bonds may, if he wishes, continue to hold the bond for as long as 20 years after its maturity date and receive additional interest when the bond is redeemed. You can get detailed information regarding this optional extension at any bank. An extension for the newer Savings Bonds will be announced when their maturity date is approached.

Ex. 4 on page 202 brings out an important fact about United States Savings Bonds; namely, the longer you hold them, the higher the rate of interest you receive from them.

Key: Page 201 **3.** (1) $7\frac{3}{4} = 7.75$; $\$25 \div 7.75 = \$3.225$, or \$3.23; (2) $\$3.23 \div \$75 = .0430$, or 4.3%. **4.** (1) \$50; (2) \$12.50; (3) $3 \times \$37.50 = \$112.50$, $\$150 - \$112.50 = \$37.50$.

Page 202 **1.** \$11.00; \$2.75. **2.** \$89.60; \$14.60; \$2.92. **3.** $\$3.04\frac{2}{3}$; $\$3.15\frac{3}{7}$. **4.** Yes; yes. **5.** \$896.00; \$970.80.

Redemption Values of \$500 Bond That Cost \$375	
If Redeemed After	Its Value Is
1 yr.	\$383.80
2 yr.	398.00
3 yr.	413.20
4 yr.	430.00
5 yr.	448.00
6 yr.	466.40
7 yr.	485.40
7 yr. 9 mo.	500.00

Page 203

Aim: To present another set of improvement tests in the subtraction of whole numbers

Suggestions: This set of improvement tests should be administered in the same way as the preceding sets in subtraction. In this set the answers to each test are to be written on folded paper. See page 375 of the text.

Key: **1.** \$1796.69; \$4747.98; \$4526.69; \$7863.97. **2.** \$3986.59; \$4848.85; \$3573.35; \$2047.59. **3.** \$5296.85; \$5261.53; \$6574.26; \$3374.34. **4.** \$4964.87; \$1859.94; \$8386.12; \$2601.22. **5.** \$3315.65; \$7399.08; \$3104.36; \$1737.52. **6.** \$2857.93; \$1182.44; \$2616.46; \$4854.71. **7.** \$620.79; \$6652.85; \$2088.88; \$1309.57. **8.** \$5925.77; \$429.64; \$4499.62; \$1898.20. **9.** \$7963.10; \$767.54; \$8341.95; \$719.09.

Page 204

Aim: To review the four fundamental operations with fractions

Key: **1.** 150 ft.; 100 ft.; 225 ft.; 250 ft. **2.** (1) 350 ft. $\div 2\frac{1}{2}$ ft. = 140 (steps); (2) 2 ft. 4 in. = $2\frac{1}{3}$ ft.; 350 ft. $\div 2\frac{1}{3}$ ft. = 150 (steps); 150 − 140 = 10 (steps). **3.** 23 − 15 = 8 (words); $8 \times 6\frac{1}{2}$¢ = 52¢; \$1.45 + \$.52 = \$1.97; .10 × \$1.97 = \$.197, or \$.20; \$1.97 + \$.20 = \$2.17. **4.** $10\frac{1}{4}$; $6\frac{1}{4}$; $\frac{3}{8}$; 42. **5.** $2\frac{1}{4}$; $8\frac{3}{8}$; $1\frac{7}{12}$; $\frac{1}{16}$. **6.** 3; $2\frac{2}{3}$; $\frac{3}{10}$; $6\frac{7}{8}$. **7.** $7\frac{1}{4}$; $5\frac{1}{6}$; 27; $\frac{1}{3}$. **8.** $1\frac{4}{5}$; $\frac{3}{10}$; $\frac{1}{6}$; 5. **9.** $3\frac{3}{16}$; 1; $\frac{1}{2}$; $\frac{2}{3}$. **10.** $2\frac{1}{8}$; $1\frac{3}{4}$; $1\frac{17}{24}$; 12. **11.** $\frac{3}{10}$; $3\frac{11}{20}$; $\frac{3}{8}$; $\frac{1}{16}$. **12.** $2\frac{2}{5}$; $3\frac{1}{6}$; $\frac{2}{3}$; $3\frac{1}{4}$. **13.** $22\frac{1}{2}$; $1\frac{1}{2}$; 9; $\frac{1}{4}$. **14.** $31\frac{9}{16}$; $17\frac{1}{5}$. **15.** $21\frac{5}{6}$; $18\frac{3}{16}$.

Page 205

Aim: To discuss the services of savings and loan associations

Suggestions: The title "Savings and Loan Association" is today applied to an institution that formerly was, and still is in certain sections of the country, called a "Building and Loan Association." These associations furnish one of the princi-

pal sources for borrowing money for the purchase or construction of a home, and they are also one of the most popular types of savings institutions.

Formerly, all savings and loan associations were chartered by the various states. In 1933 Congress enacted legislation providing for the federal chartering of this type of association. In the years following the enactment of this law, many of the organizations gave up their state charters for federal charters. At the present time, therefore, both state and federal associations are operating in the field.

In the United States today there are 6358 savings and loan associations which are serving millions of people. Some of these people have borrowed money from the associations to build homes, while the rest are investing their savings in these institutions.

Key: **2.** \$8000. **4.** (1) 12 × \$20 = \$240; 5 × \$240 = \$1200; \$1325.54 − \$1200 = \$125.54. **5.** 10 × \$240 = \$2400; \$2942.96 − \$2400 = \$542.96. **6.** (1) 32,431,000 ÷ 6358 = $5100\frac{2600}{3179}$, or about 5101 savings accounts; (2) \$70,838,000,000 ÷ 32,431,000 = \$2184.268, or about \$2184.27.

Pages 206–207

Aim: To show how money is borrowed from savings and loan associations to build a house and how it is paid back

Suggestions: Ex. 2–4 on pages 206 and 207 should be most carefully studied. It is very important to show how each item in the table in ex. 2 is computed. In these exercises it must be understood that Mr. Hill pays \$86 each month to the savings and loan association and that each \$86 covers interest as well as a payment to reduce the debt. It is also helpful to note, as stated in ex. 3, that the interest becomes *smaller* each month while payment on the debt becomes *larger*.

It will be of interest to study Column *A* in the table in ex. 2 on page 206. Each month the amount shown in Column *A* gets smaller, meaning that Mr. Hill's debt grows smaller each month. This column shows also that the risk which the savings and loan association is carrying on this loan is becoming smaller and smaller each month. This fact contributes to the safety and stability of the savings and loan association.

If there is a savings and loan association in your locality, it will be worthwhile to get pamphlets describing its arrangements for making loans on houses.

Key: **5.**

	A Unpaid Balance	B Total Monthly Payment	C Interest on Unpaid Balance	D Amount Used to Reduce Debt
6th month	\$11,868.70	\$86	\$59.34	\$26.66
7th month	11,842.04	86	59.21	26.79
8th month	11,815.25	86	59.08	26.92

Pages 208–209

Aim: To show how the monthly payment on a loan depends upon the size of the loan, the rate of interest, and the time that is taken to repay the loan

Suggestions: Supplement the work on these pages with additional problems that can be solved by using the table in ex. 2. For prices of houses, use those that are common in your locality.

The regulations affecting FHA loans vary from time to time, depending on economic conditions. It will add interest to this work if you or the pupils obtain from a bank or another lending agency in your community information about current regulations for these loans.

Workbook Reference: Arithmetic Workshop, Book 8, pages 74 and 75

Key: **2.** \$6.60; \$5.85; \$10.86; \$8.18; \$6.88. **3.** \$39.55; \$33.00; \$34.08; \$26.16; \$20.28. **4.** (1) 9 × \$6.60 = \$59.40; (2) 12 × \$59.40 = \$712.80; 20 × \$712.80 = \$14,256; (3) \$14,256 − \$9000 = \$5256. **5.** (1) 9 × \$7.91 = \$71.19; (2) 12 × \$71.19 = \$854.28; 15 × \$854.28 = \$12,814.20; \$12,814.20 − \$9000 = \$3814.20. **6.** $\frac{2}{3}$ × \$16,500 = \$11,000; 11 × \$7.91 = \$87.01. **7.** (1) $\frac{4}{5}$ × \$15,000 = \$12,000; 12 × \$6.88 = \$82.56; (2) \$15,000 − \$12,000 = \$3000. **8.** .85 × \$20,000 = \$17,000; 17 × \$6.60 = \$112.20.

Pages 210–211

Aim: To explain the meaning of compound interest and show how it is computed

Suggestions: Compound interest is a topic of basic importance in the business and economic world because successful investment depends on the reinvestment of all interest earned, thus applying the principle of compound interest. When money is placed in a savings bank, it earns compound interest if each installment of interest is left in the bank when it becomes due. Life insurance companies and trust companies are also examples of organizations whose funds are so invested as to earn compound interest. In transactions involving interest, compound interest is usually used if the time is greater than one year.

The important thing is to get the pupil to understand what compound interest means. At first he can best get this understanding by doing some simple exercises in computing compound interest, such as those on pages 210 and 211 of the text. It will be helpful if you introduce this work by doing the computations in ex. 2 on the board so that the pupils may see how each step is obtained.

Key: **3.** \$6.00. **4.** (1) From the table the amount at age 13 is \$337.82.

.02 × \$337 = \$6.74	\$337.82 + \$6.74 = \$344.56, amt. age 13½
.02 × \$344 = \$6.88	\$344.56 + \$6.88 = \$351.44, amt. age 14
.02 × \$351 = \$7.02	\$351.44 + \$7.02 = \$358.46, amt. age 14½
.02 × \$358 = \$7.16	\$358.46 + \$7.16 = \$365.62, amt. age 15
.02 × \$365 = \$7.30	\$365.62 + \$7.30 = \$372.92, amt. age 15½
.02 × \$372 = \$7.44	\$372.92 + \$7.44 = \$380.36, amt. age 16

(2) $\$380.36 - \$300 = \$80.36$; (3) $\frac{\$300}{1} \times \frac{4}{100} \times 6 = \$72$; (4) $\$80.36 - \$72 = \$8.36$.

5. $.015 \times \$400 = \$6.00$	$\$400 + \$6.00 = \$406$, amt. end of $\frac{1}{2}$ yr.
$.015 \times \$406 = \$6.09$	$\$406 + \$6.09 = \$412.09$, amt. end of 1 yr.
$.015 \times \$412 = \$6.18$	$\$412.09 + \$6.18 = \$418.27$, amt. end of $1\frac{1}{2}$ yr.
$.015 \times \$418 = \$6.27$	$\$418.27 + \$6.27 = \$424.54$, amt. end of 2 yr.
$.015 \times \$424 = \$6.36$	$\$424.54 + \$6.36 = \$430.90$, amt. end of $2\frac{1}{2}$ yr.
$.015 \times \$430 = \$6.45$	$\$430.90 + \$6.45 = \$437.35$, amt. end of 3 yr.
$.015 \times \$437 = \$6.56$	$\$437.35 + \$6.56 = \$443.91$, amt. end of $3\frac{1}{2}$ yr.
$.015 \times \$443 = \$6.65$	$\$443.91 + \$6.65 = \$450.56$, amt. end of 4 yr.

Pages 212–213

Aim: To teach the use of a table in computing compound interest

Suggestions: Compound interest tables are widely used because of the work they eliminate in the computation of compound interest. It is, therefore, important for the pupils to understand how to use such a table. Ex. 1–3 on page 212 show how to use the compound interest table in various situations where the interest is compounded annually. The table may be used also when interest is compounded semiannually.

Ex. 1 on page 213 should be carefully studied. Make sure that the pupils understand this exercise before they attempt the other work on this page.

Workbook Reference: Arithmetic Workshop, Book 8, page 76

Key: Page 212 **2.** \$1.08; \$1.08; \$2.43; \$3.26. **4.** (1) $\$411.84 - \$300 = \$111.84$; (2) $\frac{\$300}{1} \times \frac{2}{100} \times \frac{16}{1} = \$96$; (3) $\$111.84 - \$96 = \$15.84$. **5.** \$594.38; \$873.55; \$1250.46; \$3049.19; \$3600.77.

Page 213 **2.** 10 periods, 16 periods, 9 periods, 40 periods; 2%, $1\frac{1}{2}$%, 1%, $\frac{3}{4}$%, 3%. **3.** $300 \times \$1.21899 = \$365.697$, or \$365.70. **4.** $700 \times \$1.37279 = \$960.953$, or \$960.95. **5.** \$673.70. **6.** \$1337.36. **7.** \$1435.23. **8.** \$522.74. **9.** \$3974.47. **10.** \$2264.20. **11.** \$4160.66. **12.** \$348.16; \$888.29; \$647.73. **13.** \$1625.50; \$2120.33; \$660.66. **14.** (1) $1500 \times \$1.81136 = \$2717.04$; (2) $\$2717.04 - \$1500 = \$1217.04$.

Page 214

Aim: To provide a general review of problem solving and computational skills

Key: **1.** 64.5; 15.9; 349.7. **2.** 37.0; 5.1; 290.9. **3.** 254.6; 34.4; 87.8. **4.** \$.48; \$6.67; \$.48. **5.** \$.10; \$10.08; \$.19. **6.** \$.26; \$8.16; \$.71. **7.** $42\frac{1}{4}$; 42.25; yes. **8.** $A = \frac{1}{2}h(a + b)$; $A = \frac{1}{2} \times 8 \times 19\frac{1}{2}$, or 78 (sq. ft.). **9.** 15 da. (June) + 9 da. (July) = 24 da.; $4 \times \$.45$ (int. for 6 da.) = \$1.80. **10.** $.01 \times \$4000 = \$40$, $\$4000 + \$40 = \$4040$, at end of $\frac{1}{2}$ yr.; $.01 \times \$4040 = \$40.40$, $\$4040 + \$40.40 = \$4080.40$, at end of 1 yr. **11.** $\$2.50 \div \$47.50 = .0526$, or 5.3%. **12.** $35 + 19 = 54$ (games played), $35 \div 54 = .648$; $37 + 21 = 58$ (games played), $37 \div 58$

= .638; so the first team has the better score. **13.** \$38 − \$29 = \$9; \$9 ÷ \$38 = .236, or 24%. **14.** (1) 1000 × \$2.19112 = \$2191.12; (2) 1000 × \$2.20804 = \$2208.04; (3) \$2208.04 − \$2191.12 = \$16.92.

Page 215

Aims: To present another set of improvement tests in multiplication and to review important terms used in this chapter

Suggestion: The improvement tests in multiplication should be administered in the same manner as previous sets in multiplication. See page 375 of the text.

Key: **1.** 43,743,014; 19,639,596; 43,220,310; 19,376,578. **2.** 10,993,608; 15,074,917; 38,032,460; 7,921,628. **3.** 89,494,848; 47,392,380; 70,287,840; 8,987,808.

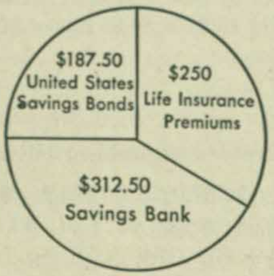
Page 216

Aim: To present plans for savings

Suggestions: The per cent of income that a person can be expected to save depends upon his responsibilities as well as upon his income. However, unless a person has a goal toward which to work, he is unlikely to save as much as he should. The per cents given in ex. 1 should be presented as goals, under average conditions, for savings of persons having the corresponding incomes in the table. For some persons these per cents are too large, and for others they could be increased. Point out to the pupils that an appropriate per cent for a specific person might be determined by making a careful study of his necessary expenditures over a certain period of time. Notice that the given incomes represent incomes *after* the deduction of income taxes.

Key: **1.** \$240; \$360; \$504; \$720; \$810. **2.** (1) 52 × \$85 = \$4420; (2) .12 × \$4420 = \$530.40, or about \$530. **3.** 12 × \$25 = \$300; \$530 − \$300 = \$230. **4.** \$230 − \$115 = \$115; \$115 ÷ \$37.50 = 3 (bonds), \$2.50 over. **5.** \$750. **6.** (1) $\frac{1}{4}$ × \$750 = \$187.50 (bonds); (2) $\frac{1}{3}$ × \$750 = \$250 (life insurance); (3) \$187.50 + \$250 = \$437.50; \$750 − \$437.50 = \$312.50 (savings bank).

7.



8. The amount used to reduce the debt.

Page 217

Aim: To provide practice in solving problems which involve previous work

Key: **1.** \$46.28 = brokerage on 100 shares; 3 × \$46.28 = \$138.84, brokerage on 300 shares; 300 × \$72.75 = \$21,825; \$21,825 − \$138.84 = \$21,686.16. **2.** \$4.75 ÷ \$77 = .0616, or 6.2%. **3.** \$16,800 − \$15,500 = \$1300; \$1300 ÷ \$15,500 = .0838, or 8.4%. **4.** 1500 × \$1.34686 = \$2020.29. **5.** (1) 60% of S.P. = \$33; 1% of S.P. = \$33 ÷ 60, or \$.55; 100% of S.P. = 100 × \$.55, or \$55, regular S.P.; .30 × \$55 = \$16.50; \$55 − \$16.50 = \$38.50, reduced S.P. of last coat; \$38.50 − \$33 = \$5.50; so he sold the coat for \$5.50 more than it cost; (2) .32 × \$55 = \$17.60; \$17.60 − \$5.50 = \$12.10; so he lost \$12.10 on the coat sold at the reduced price. **6.** 30 words − 15 words = 15 words; 15 × 7.5¢ = 112½¢, or \$1.13; \$1.60 + \$1.13 = \$2.73; .10 × \$2.73 = \$.273, or \$.27; \$2.73 + \$.27 = \$3.00. **7.** 8 × \$50 = \$400; 10 × \$20 = \$200; 15 × \$10 = \$150; \$400 + \$200 + \$150 = \$750; .01 × \$750 = \$7.50; \$750 + \$7.50 = \$757.50. **8.** \$40.75 = brokerage on 100 shares; 2 × \$40.75 = \$81.50; 200 × \$43.50 = \$8700; \$8700 + \$81.50 = \$8781.50.

Page 218

Aim: To provide a review of Chapter 6

Key: **1.** A large company requires a large amount of capital to establish it, and the amount is more than a single individual cares to risk; hence, this large airplane company is owned by a corporation rather than by an individual. **2.** The profits of a company must first be used to pay the dividends on preferred stock; the remainder of the profits may then be applied to pay dividends on the common stock. If these remaining profits are small, the dividend on the common stock may be very small or it may not be paid at all. The preferred stock always has the same rate of dividend. Sometimes this dividend is paid even though it is not earned, and it is always paid if it is earned. Also see the answer in the Key for ex. 24 on page 194. **3.** Series E Savings Bonds are a good investment because they are backed by the United States Government and pay a higher rate of interest than is paid by many savings banks. In order to get this higher rate of interest, the Series E bonds must be held until maturity. **4.** The safer investment would be a bond of a first-class corporation paying 4%, provided the corporation was making good profits. **5.** A bond is usually a safer investment than the common stock of the same company because a bond carries a definite promise to pay a certain amount of interest each year. Another advantage of a bond is that it is often protected by a mortgage on the company's property. **8.** \$100.

10. .03 × \$500 = \$15	\$500 + \$15 = \$515, amt. end of ½ yr.
.03 × \$515 = \$15.45	\$515 + \$15.45 = \$530.45, amt. end of 1 yr.
.03 × \$530 = \$15.90	\$530.45 + \$15.90 = \$546.35, amt. end of 1½ yr.
.03 × \$546 = \$16.38	\$546.35 + \$16.38 = \$562.73, amt. end of 2 yr.
.03 × \$562 = \$16.86	\$562.73 + \$16.86 = \$579.59, amt. end of 2½ yr.
.03 × \$579 = \$17.37	\$579.59 + \$17.37 = \$596.96, amt. end of 3 yr.

11. 100 × \$92.125 = \$9212.50; \$9212.50 + \$48.21 = \$9260.71. **12.** The interest on a bond is usually paid twice a year. The dividends on stock are usually paid four times a year, though some corporations pay dividends only twice a year.

Page 219

Aim: To present Problem Test 6

Workbook Reference: Arithmetic Workshop, Book 8, page 77

Key: **1.** (1) \$37.50 ÷ 15 = \$2.50; (2) \$2.75 + \$2.50 + \$2.50 + \$2.50 + \$2.95 = \$13.20, \$13.20 ÷ 5 = \$2.64; yes; (3) \$37.50 − \$13.20 = \$24.30, \$24.30 ÷ 10 = \$2.43. **2.** 21 da. (Apr.) + 31 da. (May) + 30 da. (June) + 31 da. (July) + 1 da. (Aug.) = 114 da.; $\frac{\$90}{1} \times \frac{6}{100} \times \frac{114}{360} = \$1.71$. **3.** 62 × \$.45 = \$27.90; 42 × \$.48 = \$20.16; 46 × \$.51 = \$23.46; \$27.90 + \$20.16 + \$23.46 = \$71.52; \$71.52 ÷ 150 = \$.476, which is called \$.48. **4.** 4296 − 3580 = 716; 716 ÷ 3580 = .20, or 20%. **5.** 45 min. = $\frac{3}{4}$ hr.; in $\frac{3}{4}$ hr. the plane travels 285 mi.; in $\frac{1}{4}$ hr. the plane travels $\frac{1}{3}$ × 285 mi., or 95 mi.; in 1 hr. ($\frac{4}{4}$ hr.) the plane travels 4 × 95 mi., or 380 mi. **6.** 9 × \$5 = \$45; \$45 + \$19 = \$64; \$64 − \$56.95 = \$7.05; \$7.05 ÷ \$56.95 = .123, which is called 12%. **7.** 792,194,000 rounds off to 792 million; 187,769,000 rounds off to 188 million; 188 ÷ 792 = .237, which is called 24%. **8.** 42 × 7.3 lb. = 306.6 lb. **9.** 6 × 12 oranges = 72 oranges; 72 ÷ 8 = 9, so 72 oranges are 9 times as many as 8 oranges; 9 × \$.45 = \$4.05. **10.** Bill and Joe arrived between 6:43 and 7:43, so they must wait for the 7:43 bus; from 6:55 to 7:43 is 48 min.

Page 220

Aim: To provide a diagnostic test with page references for practice

Suggestion: If you wish to review the computation of brokerage, have the pupils show how the brokerage for 100 shares can be found in ex. 1–4 by using the procedure presented on page 190.

Key: **1.** 100 × \$13.50 = \$1350; \$1350 + \$20.50 = \$1370.50. **2.** 3 × \$40.50 = \$121.50, brokerage on 300 shares; 300 × \$43 = \$12,900; \$12,900 + \$121.50 = \$13,021.50. **3.** 5 × \$44.63 = \$223.15, brokerage on 500 shares; 500 × \$56.25 = \$28,125; \$28,125 + \$223.15 = \$28,348.15. **4.** 4 × \$49.80 = \$199.20, brokerage on 400 shares; 400 × \$108 = \$43,200; \$43,200 + \$199.20 = \$43,399.20. **5.** (1) \$5 ÷ \$80 = .0625, or 6.3%; (2) \$2.75 ÷ \$44 = .0625, or 6.3%. **6.** 5.7%; 8.2%. **7.** 7.8%; 6.5%. **8.** 13 × \$6.88 = \$89.44. **9.** 19.5 × \$6.45 = \$125.775, or \$125.78. **10.** (1) 500 × \$1.16054 = \$580.27; (2) 600 × \$1.60471 = \$962.826, or \$962.83. **11.** \$902.36; \$1447.60. **12.** \$549.12; \$1010.15.

Aims of Chapter 7. The major aims of Chapter 7 are to:

1. Review how to find the volume of a rectangular prism.
2. Describe other prisms and teach how to find the volume of any prism.
3. Teach how to find the volume of a cylinder.
4. Teach how to find the area of the curved surface of a cylinder.
5. Describe the various kinds of pyramids, and teach how to find the volume of a pyramid.
6. Describe Egyptian numerals.
7. Teach how to find the volume of a cone.
8. Teach how to find the area of the surface of a sphere.
9. Teach how to find the volume of a sphere.
10. Teach how to measure lumber and how to find its cost.

Page 221

Aim: To discuss different kinds of geometric figures having three dimensions

Suggestions: You will find that models of cylinders, cones, spheres, pyramids, and prisms are very helpful in the work of this chapter. Such models can be made of heavy paper, wood, or plastic. The pupils will enjoy and profit from making some models out of heavy paper. Perhaps the Industrial Arts Department of your school will help you make some of them out of wood and plastic. Excellent models can also be purchased, if your equipment budget permits, from dealers in educational supplies. These can be used for demonstration purposes and for exhibits.

Pages 222–224

Aim: To review finding the volume of a rectangular prism, including the use of the formula for the volume

Suggestions: The work on finding the volume of a rectangular solid was first taught in Grade 6 and was retaught in Grade 7. This work, therefore, will be a review for many pupils. The pupils must be thoroughly familiar with the terms *rectangular prism, cube, height, depth, width, length,* and *dimensions*. The concept of volume is a difficult one for pupils; hence, you should try to make this work as real as possible. In reviewing the volume of a rectangular prism, as is done on page 222, it will be helpful to use an actual box, such as a shoe box or a chalk box, using inch cubes to fill the box if such cubes are available. Possibly you can make a number of such cubes out of wood or have an Industrial Arts teacher make them for you. In all this work encourage the pupils to measure the dimensions of boxes of various sizes and to compute their volumes. The more real this topic can be made, the better it will be understood by the pupils. In using the formula $V = lwh$, the pupils should remember that lwh means $l \times w \times h$.

In teaching the finding of volumes, emphasize the fact that the three dimensions must all be given in, or changed to, the same unit of measure, as indicated on page 223. The table of cubic measure given in ex. 3 on page 224 should be memorized.

Workbook Reference: Arithmetic Workshop, Book 8, page 79

Key: Page 222 **5.** $\frac{1}{2}$ ft. = 6 in.; 6 in. × 5 in. × 6 in. = 180 cu. in. **6.** 1 ft. = 12 in.; 12 in. × 9 in. × 1 in. = 108 cu. in. **7.** 4 in. = $\frac{1}{3}$ ft.; 6 ft. × 2 ft. × $\frac{1}{3}$ ft. = 4 cu. ft. **8.** 1 ft. 2 in. = 14 in.; 9 in. × 6 in. × 14 in. = 756 cu. in. **9.** 3 ft. 6 in. = $3\frac{1}{2}$ ft.; 15 ft. × 12 ft. × $3\frac{1}{2}$ ft. = 630 cu. ft. **10.** 9 ft. 3 in. = $9\frac{1}{4}$ ft.; 12 ft. × 18 ft. × $9\frac{1}{4}$ ft. = 1998 cu. ft.

Page 223 **1.** $13\frac{1}{2}$ cu. ft. **2.** 9 in. = $\frac{3}{4}$ ft.; 6 in. = $\frac{1}{2}$ ft.; 3 in. = $\frac{1}{4}$ ft.; $\frac{3}{4}$ ft. × $\frac{1}{2}$ ft. × $\frac{1}{4}$ ft. = $\frac{3}{32}$ cu. ft. **3.** $13\frac{1}{2}$ cu. ft. ÷ $\frac{3}{32}$ cu. ft. = 144 (boxes). **4.** 16 boxes; 9 layers; 144 boxes; yes. **5.** (1) 36 in. × 24 in. × 27 in. = 23,328 cu. in.; (2) 9 in. × 6 in. × 3 in. = 162 cu. in.; (3) 23,328 cu. in. ÷ 162 cu. in. = 144 (boxes); yes. **6.** 1800 cu. ft.; 2160 cu. ft. **7.** 576 cu. in. or $\frac{1}{3}$ cu. ft.; 1584 cu. in. or $\frac{11}{12}$ cu. ft.; 23,328 cu. in. or $13\frac{1}{2}$ cu. ft. **8.** 720 cu. in. or $\frac{5}{12}$ cu. ft.; 1050 cu. in. or $\frac{175}{288}$ cu. ft.; 5040 cu. in. or $2\frac{11}{12}$ cu. ft. **9.** 3456 cu. in. or 2 cu. ft.; 4320 cu. in. or $2\frac{1}{2}$ cu. ft.; 4800 cu. in. or $2\frac{7}{9}$ cu. ft.

Page 224 **4.** 108 cu. ft.; 18 cu. ft. **5.** 3456 cu. in. **6.** 27 ft. = 9 yd.; 18 ft. = 6 yd.; 6 ft. = 2 yd.; 9 × 6 × 2 = 108 (cu. yd.). **7.** (1) 12 cu. ft.; (2) $2\frac{1}{4}$ ft. × $2\frac{1}{4}$ ft. × $2\frac{1}{4}$ ft. = $11\frac{25}{64}$ cu. ft.; so the volume of the chest is more. **8.** 8 × 4 × 4 = 128 (cu. ft.); 128 × 60 lb. = 7680 lb. **9.** (1) The cookie boxes can be laid in 4 $2\frac{1}{2}$-inch rows with 3 boxes 8 in. long in each row, or in dimensions 10 in. wide and 24 in. long; they can be laid in 6 $2\frac{1}{2}$-inch rows with 2 boxes 8 in. long in each row, or in dimensions 15 in. wide and 16 in. long; they can be laid in 3 $2\frac{1}{2}$-inch rows with 4 boxes 8 in. long in each row, or in dimensions of $7\frac{1}{2}$ in. wide and 32 in. long; (2) 6 × 2 in. = 12 in. **10.** 60 × 50 × 14 = 42,000 (cu. ft.); 42,000 ÷ 200 = 210 (pupils).

Pages 225–226

Aim: To give a project and some everyday problems that apply the work on finding the volumes of rectangular prisms

Suggestions: Certain exercises on page 226 are devoted to the measurement of liquids. When you wish to find the number of gallons of water that a small rectangular prism, such as a milk carton, will hold, you usually use the fact that 231 cu. in. = 1 gal. This equivalent is used in ex. 3, 4, and 6 on page 226. When the rectangular prism is larger, such as a ditch, a tank, or a swimming pool, you frequently use the fact that 1 cu. ft. = about $7\frac{1}{2}$ gal. The latter value is an approximate equivalent which is sufficiently accurate for many practical purposes; this equivalent is used in ex. 5 on page 226. It is an easy matter to show the pupils that 1 cu. ft. = about $7\frac{1}{2}$ gal. There are 1728 cu. in. in 1 cu. ft. If you divide 1728 by 231, you get $7\frac{111}{231}$, which is *about* $7\frac{1}{2}$. Hence, there are *about* $7\frac{1}{2}$ gal. in 1 cu. ft.

Workbook Reference: Arithmetic Workshop, Book 8, page 80

Key: Page 225 **1.** 6912 cu. in. **2.** (1) 2016 cu. in.; (2) 1080 cu. in.; (3) 3120 cu. in.; (4) 960 cu. in.; (5) 7176 cu. in.; (6) 7176 cu. in. − 6912 cu. in. = 264 cu. in.; so the suitcases contain 264 cu. in. more. **3.** 4 cu. ft.

Page 226 **1.** (1) $7 \times 10 \times 3 = 210$ (cu. ft.), $210 \div 35 = 6$ (T.); (2) $7 \times 10 \times 4 = 280$ (cu. ft.), $280 \div 35 = 8$ (T). **2.** 10 tons of coal occupy 10×35 cu. ft., or 350 cu. ft.; $350 \div 70 = 5$ (depth in feet). **3.** $2\frac{3}{4} \times 2\frac{3}{4} \times 7\frac{3}{4} = 58\frac{39}{64}$ (cu. in.); $\frac{1}{4} \times 231$ cu. in. $= 57\frac{3}{4}$ cu. in.; so the container holds 1 qt. of milk. **4.** $2\frac{1}{8} \times 2\frac{1}{8} \times 6\frac{1}{2} = 29\frac{45}{128}$ (cu. in.); 8 pt. = 1 gal., $\frac{1}{8} \times 231$ cu. in. $= 28\frac{7}{8}$ cu. in.; so the container holds 1 pt. of milk. **5.** (1) $25 \times 20 \times 3\frac{1}{2} = 1750$ (cu. ft.), $1750 \times 7\frac{1}{2}$ gal. = 13,125 gal.; (2) $1750 \div 100 = 17.5$ (hundreds), $17.5 \times \$.20 = \$3.50$. **6.** $18 \times 14 \times 11 = 2772$ (cu. in.); $2772 \div 231 = 12$ (gal.). **7.** 18 in. $= 1\frac{1}{2}$ ft.; 14 in. $= 1\frac{1}{6}$ ft.; 12 in. = 1 ft.; $1\frac{1}{2} \times 1\frac{1}{6} \times 1 = 1\frac{3}{4}$ (cu. ft.); $1\frac{3}{4} \times 62\frac{1}{2}$ lb. $= 109\frac{3}{8}$ lb.

Page 227

Aim: To present another set of improvement tests in division and a review of column addition

Suggestion: The improvement tests on this page should be given in the same manner as previous improvement tests in division. See page 375.

Key: **1.** 99.93; 95.19; 41.04. **2.** 83.92; 54.54; 86.83. **3.** 73.14; 43.11; 41.78. **4.** 61.62; 45.15; 22.08. **5.** 55.14; 37.88; 66.48. **6.** 77.81; 27.64; 38.33. **7.** 3007; 3397; 3591; 3380; 4569; 4178; 2933. **8.** 533. **9.** 554. **10.** 402.

Page 228

Aim: To give another project that applies the work on finding the volume of a rectangular prism

Suggestions: In ex. 1 the pupils should consider the amount of water in the swimming pool to be equal to the volume of a rectangular prism which has the same length and width as the pool and whose height is equal to the average depth of the water. The volume of a swimming pool will be studied in more detail and with more care on page 230. This project should motivate the study of that page. Use on page 228 the fact that there are about $7\frac{1}{2}$ gal. in 1 cu. ft., as was done on page 226.

Key: **1.** $60 \times 50 \times 5\frac{1}{2} = 16{,}500$ (cu. ft.). **2.** $16{,}500 \div 100 = 165$ (hundreds); $165 \times \$.12 = \$19.80$. **3.** 5 ft. 9 in. $= 5\frac{3}{4}$ ft.; $60 \times 50 \times 5\frac{3}{4} = 17{,}250$ (cu. ft.); $17{,}250 \div 100 = 172.5$ (hundreds); $172.5 \times \$.12 = \$20.70$; $\$20.70 - \$19.80 = \$.90$. **4.** 2 in. $= \frac{1}{6}$ ft.; $60 \times 50 \times \frac{1}{6} = 500$ (cu. ft.); $500 \div 100 = 5$ (hundreds); $5 \times \$.12 = \$.60$. **5.** $16{,}500 \times 7\frac{1}{2}$ gal. = 123,750 gal. **6.** $123{,}750 \div 150 = 825$ (min.), or $13\frac{3}{4}$ hr.

Pages 229–230

Aims: To describe prisms and to give and use the formula for finding the volume of any prism

Suggestions: The following rule, given in ex. 3 on page 229, should be carefully explained, since it is of basic importance:

Volume of prism = area of base × height

The value of this rule lies in the fact that it applies to any prism, regardless of the shape of its base. In applying this rule, remember that it is not always necessary to regard the side on which a prism lies or stands as its base. For example, in ex. 2, 3, and 6 on page 230, you see that the part of the prism which is used as the base is one of the sides of the pool which has the shape of a trapezoid.

The prisms discussed in this text are all *right* prisms. In a right prism the sides are perpendicular to the bases and are rectangles. Other prisms are called *oblique*. In an oblique prism the sides are *not* perpendicular to the bases and are parallelograms. The formula given in ex. 3 on page 229 is, however, true for *all* prisms. It is not necessary at this grade level to introduce oblique prisms. The pupil will study them in the senior high school.

Key: Page 229 **4.** 140 cu. in. **5.** 540 cu. ft.

Page 230 **1.** 10 ft. + 4 ft. = 14 ft.; 14 ft. ÷ 2 = 7 ft. **2.** (1) Bases are 4′ and 10′, height is 60′; $A = \frac{1}{2} \times 60 \times 14 = 420$ (sq. ft.); (2) 420 × 25 = 10,000 (cu. ft.). **3.** Bases are 3′ and 9′, height is 60′; $\frac{1}{2} \times 60 \times 12 = 360$ (sq. ft.); 360 × 25 = 9000 (cu. ft.). **4.** 10 ft. − 1 ft. = 9 ft.; 4 ft. − 1 ft. = 3 ft.; 9 ft. + 3 ft. = 12 ft.; 12 ft. ÷ 2 = 6 ft. **5.** 9000 × $7\frac{1}{2}$ gal. = 67,500 gal. **6.** (1) Bases are 8 ft. and 2 ft., height is 120 ft.; $A = \frac{1}{2} \times 120 \times 10 = 600$ (sq. ft.); 600 × 80 = 48,000 (cu. ft.); (2) 48,000 × $7\frac{1}{2}$ gal. = 360,000 gal. **7.** (1) 10 in. = $\frac{5}{6}$ ft.; $80 \times 60 \times \frac{5}{6} = 4000$ (cu. ft.); (2) 4000 × $57\frac{1}{2}$ lb. = 230,000 lb.; (3) 230,000 ÷ 2000 = 115 (T.). **8.** 120 × 100 × $\frac{5}{6}$ = 10,000 (cu. ft.); 10,000 × $57\frac{1}{2}$ lb. = 575,000 lb.; 575,000 ÷ 2000 = $287\frac{1}{2}$ (T.).

Pages 231–233

Aims: To develop the formula for the volume of a cylinder and to apply it in problem solving

Suggestions: On page 231 the pupils should become familiar with the formula for finding the volume of the cylinder and learn how to apply it in the various exercises. They should become accustomed to using both $3\frac{1}{7}$ and 3.14 as the value of π. Give the pupils practice in rounding off answers to a given precision.

In the work on these pages make use of models of cylinders, as previously suggested. In addition it would be interesting to have an exhibit of everyday articles which take the form of a cylinder. Pictures of tank cars, milk trucks, and other things which make use of cylinders could also be included.

Workbook Reference: Arithmetic Workshop, Book 8, page 81

Key: Page 231 **4.** (1) $V = \pi r^2 h$; $V = 3\frac{1}{7} \times 1\frac{1}{2} \times 1\frac{1}{2} \times 4 = 28\frac{2}{7}$ (cu. in.); (2) 231 ÷ $28\frac{2}{7} = 8\frac{1}{6}$; so the jar holds about 1 pt. **5.** 3.14 × 8 × 8 × 60 = 12,057.6 (cu. in.); 12,057.6 ÷ 231 = 52.1, or about 52 gal. **6.** 27 cu. in. **7.** 140 cu. in. **8.** 60 cu. ft. **9.** 42 cu. ft. **10.** 120 cu. ft. **11.** 301.44 cu. in. **12.** 113.04 cu. ft. **13.** 37.68 cu. in. **14.** 846.23 cu. ft. **15.** 2198 cu. in.

Page 232 **2.** 6776 cu. ft.; 4620 cu. ft.; 3080 cu. ft.; 1540 cu. ft. **3.** 242 da. **4.** (1) $3\frac{1}{7} \times 3\frac{1}{2} \times 3\frac{1}{2} \times 7 = 847$ (cu. ft.); no; (2) $847 \div 6776 = \frac{1}{8}$.

Page 233 **1.** $\frac{3}{4} \times 40$ ft. = 30 ft.; $3\frac{1}{7} \times 1\frac{1}{2} \times 1\frac{1}{2} \times 30 = 212\frac{1}{7}$ (cu. ft.); $212\frac{1}{7} \times 7\frac{1}{2}$ gal. = $1591\frac{1}{14}$ gal., or about 1591 gal. **2.** $3\frac{1}{7} \times 4 \times 4 \times 14 = 704$ (cu. ft.); $704 \times 7\frac{1}{2}$ gal. = 5280 gal. **3.** (1) $3\frac{1}{7} \times 5\frac{1}{4} \times 5\frac{1}{4} \times 4 = 346\frac{1}{2}$ (cu. in.); $346\frac{1}{2} \div 231 = 1\frac{1}{2}$ (gal.); (2) 6 qt. **4.** (1) 2 ft. 4 in. = 28 in., $3\frac{1}{7} \times 10\frac{1}{2} \times 10\frac{1}{2} \times 28 = 9702$ (cu. in.), $9702 \div 231 = 42$ (gal.); (2) 4 ft. = 48 in., $3\frac{1}{7} \times 10\frac{1}{2} \times 10\frac{1}{2} \times 48 = 16{,}632$ (cu. in.), $16{,}632 \div 231 = 72$ (gal.). **5.** $3\frac{1}{7} \times 2 \times 2 \times 10 = 125\frac{5}{7}$ (cu. ft.); $125\frac{5}{7} \times 7\frac{1}{2}$ gal. = $942\frac{6}{7}$ gal., or about 943 gal. **6.** $3\frac{1}{7} \times 2\frac{1}{2} \times 2\frac{1}{2} \times 30 = 589\frac{2}{7}$ (cu. ft.); $589\frac{2}{7} \times 7\frac{1}{2}$ gal. = $4419\frac{9}{14}$ gal., or about 4420 gal.

Page 234

Aims: To develop the formula for finding the area of the curved surface of a cylinder and to give problems in finding such areas

Suggestions: The experiment described in ex. 1 on this page should be performed in class. Instead of a tomato can, you may use any other convenient cylinder and wrap a sheet of paper around its curved surface. Make sure that the pupils understand the difference between the *area of the curved surface* of a cylinder and the *total area* of the surface of a cylinder.

Workbook Reference: Arithmetic Workshop, Book 8, page 81

Key: **2.** $2 \times 3\frac{1}{7} \times 2 \times 5 = 62\frac{6}{7}$ (sq. in.). **3.** $A = \pi r^2$; $A = 3\frac{1}{7} \times 2 \times 2 = 12\frac{4}{7}$ (sq. in.); $62\frac{6}{7} + 12\frac{4}{7} + 12\frac{4}{7} = 88$ (sq. in.). **4.** $2 \times 3\frac{1}{7} \times 7 \times 44 = 1936$ (sq. ft.); $1936 \div 400 = 4\frac{21}{25}$, or about 5 gal. **5.** $5 \times \$2.88 = \$14.40$. **6.** 84 sq. ft. **7.** 2190 sq. in. **8.** $3\frac{1}{7} \times 35 \times 9 = 990$ (sq. in.). **9.** $2 \times 3\frac{1}{7} \times 4 \times 25 = 628\frac{4}{7}$ (sq. in.). **10.** $3\frac{1}{7} \times 8 \times 20 = 502\frac{6}{7}$ (sq. ft.). **11.** $2 \times 3\frac{1}{7} \times 7 \times 31 = 1364$ (sq. in.).

Page 235

Aim: To provide practice on (1) the four fundamental operations with whole numbers, fractions, and decimals and (2) the first two cases of percentage

Suggestions: It will be helpful if you review concepts and procedures related to these examples. Some of these are: how to find the least common denominator, how to use a trial divisor in long division, how to place the decimal point in a quotient.

Key: **1.** 237,252; 230,374; $116\frac{5}{16}$; $50\frac{7}{10}$; 31.625. **2.** 28,946; 27,242; $13\frac{11}{12}$; $5\frac{2}{5}$; \$16.25. **3.** $67\frac{1}{2}$; $\frac{1}{4}$; $3\frac{1}{3}$; $9\frac{5}{8}$. **4.** $5\frac{1}{3}$; 6; $6\frac{1}{4}$; $5\frac{1}{4}$. **5.** $4\frac{3}{8}$; $13\frac{1}{2}$; 8; $\frac{5}{16}$. **6.** 81.5; 821.3; 21.8. **7.** .7; 246.4; 56.4. **8.** \$2.74; \$612.75; \$.63. **9.** \$4.95; \$8.78; \$2.15. **10.** 13.8%; 4.2%; .8%; 68.9%. **11.** 30.1%; 10.8%; .5%; 64.8%. **12.** 81.8%; 28.8%; 2.5%; 40.4%.

Pages 236–237

Aims: To define the various kinds of pyramids, to develop the formula for the volume of a pyramid, and to give problems using this formula

Suggestions: In the work on pyramids, emphasis should be placed on the fact that the sides of every pyramid are triangles and that the *height* of a pyramid is the perpendicular distance from the vertex of the pyramid to its base. The height is *not* the length of one of the slanting edges of the pyramid. See the figure in ex. 3 on page 236, which shows that the *height* of the pyramid is the length of AB, which runs from the vertex to the base. AB is perpendicular to the base.

It will add interest to the work on these pages if you ask the pupils to look up more information about the Egyptian pyramids and report to the class.

On page 237 the experiment in ex. 1 can easily be carried out in class by making a pyramid and a prism of cardboard or heavy paper and cellophane tape. The bases of these solids should be squares of the same size, and the heights should also be the same. In order to make the heights equal, the pyramid should be made first, after which the prism can easily be made the same height.

In connection with the work of this chapter, considerable interest is added if the pupils are encouraged to look for prisms, cylinders, pyramids, and cones on churches and other buildings. Church steeples are usually pyramids or cones. Cylinders are frequently found, a few examples being drinking glasses, water tanks, oil tanks, silos, towers, trunks of trees, water pipes, and so on. The closer you can relate this work in geometry to familiar objects, the more real it will be to the pupils.

Workbook Reference: Arithmetic Workshop, Book 8, page 83

Key: Page 236 **3.** Shorter.

Page 237 **2.** $V = \frac{1}{3} \times 24 \times 8$, or 64 (cu. in.). **3.** (1) $\frac{1}{3} \times 746 \times 746 \times 461 = 85{,}517{,}958\frac{2}{3}$ (cu. ft.); (2) 27 cu. ft. = 1 cu. yd.; $85{,}517{,}958\frac{2}{3} \div 27 = 3{,}167{,}331\frac{65}{81}$ (cu. yd.). **4.** $\frac{1}{3} \times 30 \times 30 \times 15 = 4500$ (cu. ft.). **5.** $\frac{1}{3} \times 5 \times 5 \times 4 = 33\frac{1}{3}$ (cu. ft.); $33\frac{1}{3}$ cu. ft. $\div 1\frac{1}{4}$ cu. ft. $= 26\frac{2}{3}$ (bu.). **6.** 1232 cu. in. **7.** $A = \frac{1}{2}bh$; $A = \frac{1}{2} \times 8 \times 14 = 56$ (sq. in.); $\frac{1}{3} \times 56 \times 24 = 448$ (cu. in.). **8.** 1056 cu. in.

Page 238

Aim: To acquaint pupils with another number system which does not have place value

Suggestions: Since the pupils have just studied pyramids, it is appropriate for them to see how the ancient Egyptians wrote numbers. A study of these numerals should add to the pupils' appreciation and understanding of our number system. Bring out the convenience of place-value systems when compared to Egyptian numerals. See Key on G-105.

Page 239

Aim: To present another set of improvement tests in addition

Suggestions: This set of improvement tests in addition should be administered in the same manner as previous sets in addition. In each test the answers are to be written on folded paper. See page 375. See Key on G-105.

Key: **1.** XLIX; XCIX; CDXCIX; CMXCIX.
Page 238 **3.** 𓍢𓍢𓎆𓎆𓎆𓎆𓏺𓏺𓏺; 𓍢𓍢𓎆𓏺𓏺𓏺; 𓆼𓍢𓍢𓍢𓍢𓎆𓎆𓏺; 𓆼𓆼𓏺𓏺𓏺; 𓂭𓍢𓏺; 𓆼𓍢𓍢𓍢𓍢𓍢𓍢; 𓆼𓍢𓏺𓏺𓏺𓏺𓏺𓏺; 𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓎆𓏺𓏺𓏺 ($123_8 = 83$). **4.** 𓆼𓍢𓍢𓏺𓏺𓏺; 𓍢𓍢𓏺𓏺; 𓂭𓂭𓆼𓆼𓆼𓆼𓆼𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓍢𓎆𓏺𓏺𓏺𓏺. **5.** 27 symbols (𓍢𓍢𓍢𓍢𓍢𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺𓏺𓏺 𓍢𓍢𓍢𓍢𓎆𓎆𓎆𓎆𓏺𓏺𓏺𓏺).

Key: Page 239 **1.** 48,091; 41,176; 41,704; 42,636; 30,223. **2.** 41,040; 44,340; 50,841; 50,212; 45,373. **3.** 53,300; 54,950; 41,461; 50,521; 52,564.

Page 240

Aims: To give the formula for the volume of a cone and to use it in problem solving

Suggestions: The pupils' attention should be called to the fact that the rules for finding the volumes of cones and pyramids are alike, each volume being $\frac{1}{3} \times$ area of base $\times$ height. The base of a cone is a circle, while the base of a pyramid is a square, a rectangle, a triangle, or any other figure having straight lines for sides.

You can repeat, if you wish, the experiment in ex. 1 on page 237 with a cone and a cylinder each having the same base and height. These figures can be made of heavy paper and cellophane tape. When you have emptied the sand from the cone into the cylinder, the cylinder will be just $\frac{1}{3}$ full.

Workbook Reference: Arithmetic Workshop, Book 8, page 84

Key: **4.** $91\frac{2}{3}$ cu. in. **5.** $\frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 4\frac{2}{3} = 99$ (cu. ft.); 27 cu. ft. = 1 cu. yd.; $99 \div 27 = 3\frac{2}{3}$ (cu. yd.). **6.** (1) $\frac{1}{2} \times 2\frac{1}{2}$ in. = $\frac{5}{4}$ in.; $\frac{1}{3} \times \frac{22}{7} \times \frac{5}{4} \times \frac{5}{4} \times 3 = 4\frac{51}{56}$ (cu. in.), or about 5 cu. in.; (2) $231 \div 5 = 46\frac{1}{5}$ (scoops), or 46 scoops.

Page 241

Aim: To provide a general review of problem solving

Key: **1.** $.935 \times 76{,}036{,}400 = 71{,}094{,}034$ (dial telephones). **2.** $200 \times \$32.625 = \$6525$; $\frac{1}{2} \times \$32.625 = \$16.3125$, which is called \$16.31; $\$19 + \$16.31 = \$35.31$, brokerage on 100 shares; $2 \times \$35.31 = \$70.62$, brokerage on 200 shares; $\$6525 - \$70.62 = \$6454.38$.

3. Savings and loan associations: $.44 \times 360° = 158.4°$
Commercial banks: $.16 \times 360° = 57.6°$
Individuals: $.12 \times 360° = 43.2°$
Insurance companies: $.04 \times 360° = 14.4°$
Savings banks: $.05 \times 360° = 18.0°$
Others: $.19 \times 360° = 68.4°$

4. $870 \times .6$ mi. = 552 mi. **5.** $1500 \times \$1.34392 = \$2015.88$. **6.** $38{,}309 - 37{,}998 = 311$ (deaths); $311 \div 37{,}998 = .0081$, or $.8\%$. **7.** $76.5\% + 16.5\% + 3.6\% = 96.6\%$; $100\% - 96.6\% = 3.4\%$; $.765 \times \$575{,}000 = \$439{,}875$, costs and operating expenses; $.165 \times \$575{,}000 = \$94{,}875$, payroll; $.036 \times \$575{,}000 = \$20{,}700$, taxes; $.034 \times \$575{,}000 = \$19{,}550$, net earnings. **8.** $10.4 + 19.9 + 33.6 + 31.7 + 13.2 = 108.8$ (inches); 108.8 in. $\div 6 = 21.76$ in. **9.** $250 \times \frac{1}{2}$ lemon = 125 lemons; $125 \div 12 = 10\frac{5}{12}$ (doz.); for $10\frac{5}{12}$ doz., $10\frac{1}{2}$ doz. must be bought; $10\frac{1}{2} \times \$.50 = \$5.25$.

Pages 242–243

Aim: To present the formula for the area of the surface of a sphere and to use it in solving problems

Suggestions: To use the formula $A = 4\pi r^2$ it is necessary to know the length of the radius of the sphere. When the diameter of the sphere is given, you can use the formula $A = \pi d^2$ given in ex. 4, or you can first take one half of the diameter to find the radius and then use the formula $A = 4\pi r^2$. Thus you may teach one or two formulas for the area of a sphere, as you prefer.

Ex. 12 on page 243 brings out an important fact: if the radius is doubled, the area of the surface of the sphere becomes 4 times as large.

Workbook Reference: Arithmetic Workshop, Book 8, page 86

Key: **4.** You know that $d = 2r$; hence, $d^2 = 4r^2$. This result shows that $4r^2$ can be replaced by d^2. In the formula $A = 4\pi r^2$ you can rearrange the order of the letters and write the formula as $A = \pi 4r^2$. You can then replace $4r^2$ by d^2 to get $A = \pi d^2$. Then $A = 4\pi r^2$ is equivalent to $A = \pi d^2$. **5.** 1256 sq. in.; 125,600 sq. ft.; 4069.44 sq. in. **6.** 31,400 sq. ft.; 200.96 sq. in.; 314 sq. ft. **7.** 17,194.64 sq. ft.; 3.14 sq. in.; 379.94 sq. ft. **8.** 616 sq. ft.; 2464 sq. in.; 61,600 sq. ft.

9. 15,400 sq. in.; 9856 sq. yd.; 554,400 sq. in. **10.** $\frac{22}{7} \times \frac{21}{1} \times \frac{21}{1} = 1386$ (sq. ft.); 1386 sq. ft. $\div$ 2 = 693 sq. ft. **11.** (1) $\frac{22}{7} \times \frac{42}{1} \times \frac{42}{1} = 5544$ (sq. ft.); (2) 5544 $\div$ 600 = $9\frac{6}{25}$ (gal.), which is called 10 gal., since you cannot buy part of a gallon; (3) 10 $\times$ \$4.37 = \$43.70. **12.** (1) $\frac{22}{7} \times \frac{21}{1} \times \frac{21}{1} = 1386$ (sq. ft.); (2) 5544 sq. ft. is area of surface of larger tank (see ex. 11); Fred takes $\frac{1}{2}$ of 5544 sq. ft., which would be 2772 sq. ft., as the area of the smaller tank; so Fred is wrong because the area of the smaller tank is 1386 sq. ft.; (3) 5544 $\div$ 1386 = 4; hence, the area of the larger tank is 4 times as much as that of the smaller tank, or the area of the smaller tank is $\frac{1}{4}$ that of the larger tank. **13.** $2 \times 3.14 \times \frac{75}{2} \times \frac{75}{2} = 8831.25$ (sq. ft.).

Page 244

Aim: To present the formula for the volume of a sphere and to use it in problems

Suggestions: To use the formula $V = \frac{4}{3}\pi r^3$, it is necessary to know the length of r, which is the radius. In some of the problems on page 244, the length of the diameter instead of the radius is given; remind the pupils in these cases that they must first take one half of the diameter to get the radius before using the formula. Also remind the pupils to give the units in which the volumes are expressed.

Workbook Reference: Arithmetic Workshop, Book 8, page 86

Key: **4.** $\frac{4}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times \frac{9}{2} = 381\frac{6}{7}$ (cu. in.). **5.** $523\frac{17}{21}$ cu. in. **6.** (1) $\frac{4}{3} \times \frac{22}{7} \times \frac{21}{1} \times \frac{21}{1} \times \frac{21}{1} = 38{,}808$ (cu. ft.); (2) 38,808 $\times$ $7\frac{1}{2}$ gal. = 291,060 gal. **7.** $\frac{4}{3} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1} \times \frac{7}{1} = 1437\frac{1}{3}$ (cu. in.); $1437\frac{1}{3}$ cu. in. $\div$ 2 = $718\frac{2}{3}$ cu. in.; 231 cu. in. = 1 gal.; $718\frac{2}{3} \div 231 = 3\frac{1}{9}$ (gal.), or about 3 gal. **8.** (1) $\frac{4}{3} \times \frac{22}{7} \times 1 \times 1 \times 1 = 4\frac{4}{21}$ (cu. ft.); $4\frac{4}{21} \times 42$ lb. = 176 lb., weight of Joe's snowball; (2) $\frac{4}{3} \times \frac{22}{7} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = 1\frac{43}{56}$ (cu. ft.); $1\frac{43}{56} \times 42$ lb. = $74\frac{1}{4}$ lb. **9.** $11{,}498\frac{2}{3}$ cu. ft. **10.** $1437\frac{1}{3}$ cu. in. **11.** $14{,}142\frac{6}{7}$ cu. ft. **12.** $4190\frac{10}{21}$ cu. in. **13.** $2145\frac{11}{21}$ cu. ft. **14.** 310.464 cu. ft. **15.** 38.808 cu. in.

Page 245

Aim: To present problems involving spheres, cones, and cylinders

Suggestions: Ex. 4 brings out the important fact that if the radius or the diameter of a sphere is doubled, its volume becomes 8 times as large. Compare this with what happens to the area of a circle when its radius or diameter is doubled.

For supplementary work solve ex. 1 and 2 with prices different from those given.

Workbook Reference: Arithmetic Workshop, Book 8, page 88

Key: **1.** (1) $V = \frac{1}{3}\pi r^2 h$; $V = \frac{1}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 4 = 9\frac{3}{7}$ (cu. in.), volume of smaller cup; $V = \frac{1}{3} \times \frac{22}{7} \times \frac{15}{8} \times \frac{15}{8} \times 5 = 18\frac{93}{224}$ (cu. in.), volume of larger cup; so the larger cup is almost twice the size of the smaller cup; (2) the larger one gives more for the money. **2.** $V = \pi r^2 h$; $V = \frac{22}{7} \times \frac{11}{8} \times \frac{11}{8} \times 5 = 29\frac{159}{224}$ (cu. in.); $29\frac{159}{224}$ cu. in. is about 3 times the size of the smaller cup and about $1\frac{1}{2}$ times the

size of the larger cup; so the glass is the best value. **3.** $A = \pi d^2$; $A = \frac{22}{7} \times \frac{8000}{1} \times \frac{8000}{1} = 201{,}142{,}857\frac{1}{7}$ (sq. mi.), or about 200,000,000 sq. mi. **4.** (1) $V = \frac{4}{3}\pi r^3$; $V = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = 179\frac{2}{3}$ (cu. ft.); (2) $\frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 1437\frac{1}{3}$ (cu. ft.); (3) $1437\frac{1}{3} \div 179\frac{2}{3} = 8$; so the volume of a sphere is multiplied 8 times when the diameter is doubled. **5.** (1) $V = \frac{4}{3}\pi r^3$; $V = \frac{4}{3} \times \frac{22}{7} \times 1 \times 1 \times 1 = 4\frac{4}{21}$ (cu. ft.); (2) $4\frac{4}{21} \times 170$ lb. $= 712\frac{8}{21}$ lb.

Pages 246–248

Aim: To teach how to measure lumber and how to find its cost

Suggestions: The definition of a *board foot*, given in ex. 1 on page 246, must be clearly explained. Notice that a board foot measures 1 ft. long, 1 ft. wide, and 1 in. thick. If a piece of board is less than 1 in. thick, but still 1 ft. long and 1 ft. wide, it also counts as a *board foot*. Any thickness less than 1 in. counts the same as 1 in. in measuring lumber. On the other hand, any thickness greater than 1 in. is counted in full. For example, a piece of lumber 1 ft. long, 1 ft. wide, and 2 in. thick counts as 2 board feet; if the thickness were $1\frac{1}{2}$ in. instead of 2 in., it would count as $1\frac{1}{2}$ board feet. You will find it useful in the development of the meaning of this unit of measure to have in the classroom several pieces of lumber 1 ft. long, 1 ft. wide, and 1 in. thick.

The rule for finding the number of board feet in a piece of lumber is as follows: *Multiply the thickness in inches by the width in feet by the length in feet.* Notice that the *length* and the *width* must both be expressed in *feet*, while the thickness must be expressed in *inches*. In using this rule, any thickness less than 1 in. is to be used as 1 in. If the width is given in inches, it must first be changed to feet, as shown in ex. 3 on page 246.

The term *lineal foot*, used in ex. 8 on page 248, must not be confused with *board foot*. When wooden molding is sold by the *lineal foot*, its width or thickness is not considered; it is only the *length* that counts. The *lineal foot* is sometimes called the *running foot*. Selling molding by the lineal foot corresponds to selling ribbon by the yard.

Key: Pages 246–247 **3.** 4 bd. ft. **4.** 40 bd. ft.; 30 bd. ft.; 45 bd. ft. **6.** 10 in. $= \frac{5}{6}$ ft.; $3 \times \frac{5}{6} \times 15 = 37\frac{1}{2}$ (bd. ft.); $30 \times 37\frac{1}{2}$ bd. ft. $= 1125$ bd. ft. **7.** (1) $7180 \div 36{,}742 = .195$, or about 20%; (2) 36,742 million − 7180 million = 29,562 million (bd. ft.); (3) $29{,}562 \div 36{,}742 = .804$, or about 80%.

Page 248 **1.** $8'' = \frac{2}{3}'$; $1\frac{1}{2} \times \frac{2}{3} \times 10 = 10$ (bd. ft.); 7×10 bd. ft. $= 70$ bd. ft. **2.** $9'' = \frac{3}{4}'$; $1 \times \frac{3}{4} \times 10 = 7\frac{1}{2}$ (bd. ft.); $6 \times 7\frac{1}{2}$ bd. ft. $= 45$ bd. ft. **3.** $10'' = \frac{5}{6}'$; $2\frac{1}{2} \times \frac{5}{6} \times 10 = 20\frac{5}{6}$ (bd. ft.); $42 \times 20\frac{5}{6}$ bd. ft. $= 875$ bd. ft. **4.** (1) $8'' = \frac{2}{3}'$, $1 \times \frac{2}{3} \times 12 = 8$ (bd. ft.), 60×8 bd. ft. $= 480$ bd. ft.; (2) $10'' = \frac{5}{6}'$, $1 \times \frac{5}{6} \times 12 = 10$ (bd. ft.), 144×10 bd. ft. $= 1440$ bd. ft.; (3) $6'' = \frac{1}{2}'$, $2 \times \frac{1}{2} \times 16 = 16$ (bd. ft.), 48×16 bd. ft. $= 768$ bd. ft.; (4) $8'' = \frac{2}{3}'$, $4 \times \frac{2}{3} \times 16 = 42\frac{2}{3}$ (bd. ft.), $30 \times 42\frac{2}{3}$ bd. ft. $= 1280$ bd. ft.; (5) 480 bd. ft. + 1440 bd. ft. + 768 bd. ft. + 1280 bd. ft. = 3968 bd. ft. **5.** \$33.60. **6.** $9'' = \frac{3}{4}'$; $1 \times \frac{3}{4} \times 16 = 12$ (bd. ft.); 6×12 bd. ft. $= 72$ bd. ft.; $\frac{72}{1000} = .072$; $.072 \times \$180 = \$12.96$. **7.** (1) $\frac{250}{1000} = .25$, $.25 \times \$170 = \$42.50$;

(2) $\frac{1300}{1000} = 1.3$, $1.3 \times \$170 = \$221$; (3) $\frac{2785}{1000} = 2.785$, $2.785 \times \$170 = \$473.45$; (4) $\frac{860}{1000} = .86$, $.86 \times \$170 = \$146.20$. **8.** \$6.94. **9.** (1) $\frac{150}{100} = 1.5$, $1.5 \times \$12.75 = \$19.125$, or \$19.13; (2) $\frac{80}{100} = .8$, $.8 \times \$12.75 = \$10.20$; (3) $\frac{240}{100} = 2.4$, $2.4 \times \$12.75 = \$30.60$; (4) $\frac{125}{100} = 1.25$, $1.25 \times \$12.75 = \$15.9375$, or \$15.94.

Page 249

Aim: To present another set of improvement tests in the subtraction of whole numbers

Suggestion: These improvement tests in subtraction should be administered in the same way as previous sets in subtraction. See page 375.

Key: **1.** 477,563; 293,499; 61,759; 154,159. **2.** 714,517; 71,967; 624,838; 308,135. **3.** 783,795; 122,956; 413,796; 869,961. **4.** 388,425; 750,287; 577,257; 689,389. **5.** 125,803; 915,925; 684,443; 236,499. **6.** 538,098; 216,530; 170,424; 804,393. **7.** 246,068; 222,970; 136,193; 102,970. **8.** 206,228; 461,363; 331,075; 815,859. **9.** 494,257; 513,738; 57,784; 769,945.

Page 250

Aim: To review the work of Chapter 7

Key: **1.** Suggestions: (1) Shoe box; (2) tomato can; (3) spherical gas tank; (4) ice cream cone. **2.** Suggestions: (1) Brick; (2) Egyptian pyramid; (3) stone column; (4) wooden ball. **3.** Cube, rectangular prism, triangular prism. **4.** (1) Square, triangle, hexagon; (2) only one — circle. **5.** (1) $V = \frac{1}{3}\pi r^2 h$; (2) $V = \pi r^2 h$; (3) $V = \frac{1}{3}Bh$; (4) $V = \frac{4}{3}\pi r^3$; (5) $A = 4\pi r^2$. **6.** $V = \frac{1}{3}Bh$, $V = \frac{1}{3} \times 6 \times 5 \times 12 = 120$ (cu. ft.). **7.** (1) $V = \frac{4}{3}\pi r^3$, $V = \frac{4}{3} \times 3.14 \times 30 \times 30 \times 30 = 113{,}040$ (cu. in.); (2) $A = 4\pi r^2$, $A = 4 \times 3.14 \times 30 \times 30 = 11{,}304$ (sq. in.). **8.** $S = 2\pi rh$, $S = 2 \times \frac{22}{7} \times 2 \times 7 = 88$ (sq. in.), area of curved surface; $A = \pi r^2$, $A = \frac{22}{7} \times 2 \times 2 = 12\frac{4}{7}$ (sq. in.), area of one end, $2 \times 12\frac{4}{7}$ sq. in. $= 25\frac{1}{7}$ sq. in., area of two ends; 88 sq. in. + $25\frac{1}{7}$ sq. in. = $113\frac{1}{7}$ sq. in. **9.** (1) $V = \pi r^2 h$, $V = \frac{22}{7} \times 2 \times 2 \times 7 = 88$ (cu. in.); (2) $88 \div 58 = 1.51$ (qt.), or about $1\frac{1}{2}$ qt. **10.** (1) 1 ft. = 12 in., 12 in. ÷ 4 in. = 3 (boxes), number in length; 10 in. ÷ $2\frac{1}{2}$ in. = 4 (boxes), number in width; 9 in. ÷ $1\frac{1}{2}$ in. = 6 (boxes), number in depth; $3 \times 4 \times 6 = 72$ (boxes), number in all; (2) $3 \times 4 = 12$, number of boxes in each layer; (3) 6 layers. **11.** $V = lwh$, $V = 90 \times 60 \times 4\frac{1}{2} = 24{,}300$ (cu. ft.); $24{,}300 \times 7\frac{1}{2}$ gal. = 182,250 gal.

Page 251

Aim: To present Problem Test 7

Workbook Reference: Arithmetic Workshop, Book 8, page 23

Key: **1.** 1 mi. = 5280 ft.; 3 min. = 180 sec.; $2\frac{3}{4} \times 5280$ ft. = 14,520 ft.; 14,520 ft. ÷ 180 = $80\frac{2}{3}$ ft. **2.** (1) $\frac{1}{10}$ of the actual price = tax; $\frac{10}{10}$ of the actual price = actual price; $\frac{11}{10}$ of the actual price = actual price + tax; $\frac{11}{10}$ of the actual price = \$13.20; $\frac{1}{11}$ of the actual price = $\frac{1}{11} \times \$13.20$, or \$1.20, tax; (2) $\frac{10}{10}$ of the actual price = $10 \times \$1.20$, or \$12.00, actual price. **3.** $579 \div 827 = .7001$, or 70.0%.

4. Compound interest: $\frac{1}{2}$ of $4\% = 2\%$, rate of interest every 6 mo.

$.02 \times \$8000 = \$160$	$\$8000 + \$160 = \$8160$,	amt. end of $\frac{1}{2}$ yr.
$.02 \times \$8160 = \$163.20$	$\$8160 + \$163.20 = \$8323.20$,	amt. end of 1 yr.
$.02 \times \$8323 = \$166.46$	$\$8323.20 + \$166.46 = \$8489.66$,	amt. end of $1\frac{1}{2}$ yr.
$.02 \times \$8489 = \$169.78$	$\$8489.66 + \$169.78 = \$8659.44$,	amt. end of 2 yr.

Simple interest: $.04 \times \$8000 = \$320$; $2 \times \$320 = \$640$. $\$8659.44 - \$8000 = \$659.44$, compound interest for 2 yr.; $\$659.44 - \$640 = \$19.44$.

5. $V = \frac{4}{3}\pi r^3$; $V = \frac{4}{3} \times \frac{22}{7} \times 20 \times 20 \times 20 = 33{,}523\frac{17}{21}$ (cu. ft.), which is called 33,524 cu. ft. **6.** $V = \frac{1}{3}\pi r^2 h$; $V = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{19}{4} = 15\frac{23}{96}$, or 15.2 (cu. in.); $\frac{1}{2} \times 29$ cu. in. $= 14.5$ cu. in., so the cup will hold more than $\frac{1}{2}$ pt. **7.** $2\frac{1}{2} \times \$.55 = \$1.37\frac{1}{2}$, which is called \$1.38; $1\frac{1}{2} \times \$.19 = \$.28\frac{1}{2}$, which is called \$.29; $5\frac{1}{4} \times \$.65 = \$3.4125$, which is called \$3.42; 9 lemons is 3 times 3 lemons, $3 \times \$.11 = \$.33$; $\$1.38 + \$.29 + \$3.42 + \$.33 = \$5.42$. **8.** (1) $32\% + 8\% = 40\%$, $100\% - 40\% = 60\%$, cost; 60% of S.P. $= \$33$; 1% of S.P. $= \$33 \div 60$, or \$.55; 100% of S.P. $= 100 \times \$.55$, or \$55; (2) $.08 \times \$55 = \$4.40$, expected profit; $\$55 - \$48 = \$7$, discount; $\$7 - \$4.40 = \$2.60$, loss. **9.** $\$75 \div \$1000 = .075$, or $7\frac{1}{2}\%$. **10.** 13 mi. $\div 3.5 = 3.71$ mi., or 3.7 mi.

Page 252

Aim: To provide a diagnostic test with references for practice pages

Key: **1.** $V = Bh$; (1) $V = 7 \times 4\frac{1}{2} = 31\frac{1}{2}$ (cu. ft.); (2) $3\frac{1}{2}$ ft. $= 42$ in., $V = 84 \times 42 = 3528$ (cu. in.). **2.** (1) $V = 8 \times 9\frac{1}{4} = 74$ (cu. in.); (2) 66 in. $= 5\frac{1}{2}$ ft., $V = 4\frac{1}{2} \times 5\frac{1}{2} = 24\frac{3}{4}$ (cu. ft.). **3.** $V = lwh$; (1) $V = 8 \times 7 \times 3 = 168$ (cu. in.); (2) $V = 1\frac{1}{2} \times 1\frac{1}{2} \times 1\frac{1}{2} = 3\frac{3}{8}$ (cu. in.). **4.** (1) $V = 6 \times 3 \times 1 = 18$ (cu. ft.); (2) $V = 8\frac{1}{4} \times 2\frac{2}{3} \times 3\frac{1}{2} = 77$ (cu. in.). **5.** $V = \pi r^2 h$; (1) $V = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 8 = 308$ (cu. in.); (2) $V = \frac{22}{7} \times 5 \times 5 \times 21 = 1650$ (cu. in.). **6.** (1) $V = \frac{22}{7} \times 14 \times 14 \times 9 = 5544$ (cu. ft.); (2) $V = \frac{22}{7} \times 21 \times 21 \times 25 \times 34{,}650$ (cu. ft.). **7.** $V = \frac{1}{3}\pi r^2 h$; (1) $V = \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 14 \times 528$ (cu. in.); (2) $V = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 16 = 205\frac{1}{3}$ (cu. ft.). **8.** $A = 4\pi r^2$; (1) $A = 4 \times \frac{22}{7} \times 3 \times 3 = 113\frac{1}{7}$ (sq. ft.); (2) $A = 4 \times \frac{22}{7} \times 14 \times 14 = 2464$ (sq. in.); (3) $A = 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 1386$ (sq. ft.); (4) $A = 4 \times \frac{22}{7} \times 35 \times 35 = 15{,}400$ (sq. ft.). **9.** $V = \frac{4}{3}\pi r^3$; (1) $V = \frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3 = 113\frac{1}{7}$ (cu. in.); (2) $V = \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 1437\frac{1}{3}$ (cu. in.); (3) $V = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = 179\frac{2}{3}$ (cu. ft.); (4) $V = \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21 = 38{,}808$ (cu. ft.). **10.** $S = 2\pi rh$; (1) $S = 2 \times \frac{22}{7} \times 7 \times 12 = 528$ (sq. in.); (2) $S = 2 \times \frac{22}{7} \times \frac{21}{2} \times 30 = 1980$ (sq. ft.).

Aims of Chapter 8. The major aims of Chapter 8 are to:

1. Develop an understanding of the different kinds of life insurance policies.
2. Present typical premiums for \$1000 life insurance policies, and show how to find the premiums for policies of other sizes.
3. Develop an understanding of cash and loan values of life insurance policies.
4. Give information about some of the benefits of workers insured under Social Security.
5. Teach how to compute the premium for a fire insurance policy if the rate is given.
6. Give information and problems about the provisions and premiums of automobile insurance.
7. Describe accident and health insurance.
8. Teach how the real estate tax rate is determined and computed.
9. Give a brief treatment of the computation of income taxes.
10. Give information about gasoline taxes.
11. Discuss prime and composite numbers, and show how to find the largest common factor and least common multiple of two or more numbers.

Page 253

Aim: To show the importance of life insurance

Suggestions: The total amount of life insurance recently in force in the United States was over \$629 billion. This insurance includes regular life insurance, group life insurance, industrial insurance, and credit life insurance, these being the types sold by private companies. (Credit life insurance protects the lender in case a borrower dies before repaying a loan.) It does not include United States Government life insurance issued to members of the armed forces of the United States. Figures regarding life insurance change from year to year just as figures concerning our population are constantly changing.

It should be understood that the life insurance that is being discussed in this chapter is the so-called *legal reserve* life insurance, which is sold by life insurance companies. The law requires that in all legal reserve life insurance a *reserve fund* be set aside for each policy, a good portion of each annual premium being used to build up this fund. In order that this fund may grow as rapidly as possible, it is invested at compound interest. This reserve fund makes life insurance safe and dependable. Life insurance should be an important part of every family's financial plan.

Key: 1. $1444 - 444 = 1000$ companies; $1000 \div 444 = 2.252$, which is called 225%. **2.** (1) $40 \div 103 = .388$, or about 39%; (2) $33 \div 103 = .320$, or about 32%; (3) $30 \div 103 = .291$, or about 29%. **3.** \$629,493,000,000 is about \$629 billion; \$253,140,000,000 is about \$253 billion; \$629 billion − \$253 billion = \$376 billion;

\$376 billion ÷ \$253 billion = 1.486, or about 149%. **4.** \$48,427,000,000 ÷ 24,145,000 = about \$2000.

Pages 254–255

Aim: To describe four kinds of life insurance policies

Suggestions: The pupil should be made thoroughly familiar with the meaning of the terms *face of policy, insured, beneficiary, premium, in force,* and *renewable.* These terms are frequently used throughout the work in life insurance.

No one kind of insurance policy can be said to be the best for all persons. The kind of insurance a person should buy depends upon his income, the needs of his dependents, and his financial obligations, such as mortgage payments and debts. It is very important for the pupil to understand each of the four types of life insurance policies so that later when he buys insurance he can make an intelligent choice of policy. The major purpose of life insurance is to provide funds for a person's dependents in the event of his death. Life insurance can be used also to provide at death for the payment of debts. It has an important savings feature which will be discussed in connection with cash values of policies.

The *ordinary life* policy, described in ex. 1 on page 254, is the type of policy most frequently purchased. It is cheaper than any other form of policy that insures one for his entire lifetime. The premium remains the same each year, and the insured pays it as long as he lives.

The *limited payment* policy also insures a person throughout his life. Since the premium is paid for only a limited time, such as 20 yr., it is higher than the premium for the ordinary life policy. If a person has a relatively high income when he is young, this type of policy might be more suitable for him.

The *endowment* policy insures a person for a limited time, such as 20 yr. At the end of this time the insured, if living, receives a payment equal to the face of the policy. Thus a large part of the premium can be considered to be savings. Endowment insurance is expensive.

Term insurance is the cheapest form of insurance. It insures a person for limited periods, such as 5, 10, or 20 yr. Because of its lower cost, a person with a low income may be able to buy the amount of insurance that he needs if he selects this type of policy. A person can use term insurance for extra protection during periods when he has heavier responsibilities. Term insurance is usually not available after a person reaches age 60 or 65. Some term policies are *renewable.* That is, they may be renewed for additional periods without a medical examination. Each time that a renewable term policy is renewed the premium increases because of the increased age of the insured. Term policies are often *convertible.* That is, they may be changed without medical examination to other types of policies if the change is made within a stated period of time.

Pages 256–257

Aims: To teach how to find the premium on any amount of life insurance and to give problems in figuring premiums

Suggestions: Ex. 9 on page 257 illustrates a frequent modern business practice, which is to insure the lives of important officers of the company upon whom the success of the company depends.

Supplement the work of these pages by having the pupils give a situation in which each type of insurance policy would be suitable.

Workbook Reference: Arithmetic Workshop, Book 8, pages 90 and 91

Key: Page 256 **2.** 5 × \$23.36 = \$116.80. **3.** (1) 15 × \$4.51 = \$67.65; (2) 15 × \$5.05 = \$75.75; (3) 15 × \$8.15 = \$122.25. **4.** 10 × \$29.49 = \$294.90; 10 × \$18.76 = \$187.60; \$294.90 − \$187.60 = \$107.30. **5.** Payments on a 20-payment life policy end in 20 yrs., whereas payments on an ordinary life policy will go on longer during a normal life span. **6.** (1) 3 × \$43.20 = \$129.60; (2) 3 × \$43.33 = \$129.99; (3) 3 × \$44.27 = \$132.81; (4) 3 × \$45.24 = \$135.72. Page 257 **1.** 20 × \$18.76 = \$375.20. **2.** 30 × \$8.15 = \$244.50. **3.** 6 × \$26.15 = \$156.90. **4.** 2 × \$43.20 = \$86.40. **5.** 200 × \$33.90 = \$6780. **6.** (a) 10-year term; (b) 20-year endowment; (c) 20-payment life. **7.** \$7000; \$4000; \$2000. **8.** If a person applies for life insurance when he is young, he is more likely to be able to pass the medical examination. An older man may not be able to pass such an examination. When a person is young, the premiums are lower than they would be at an older age. **9.** \$2278; \$815; the company.

Pages 258–259

Aims: To explain the reserve fund of a life insurance policy and to show its relation to the cash value and the loan value of the policy

Suggestions: In ex. 1 on page 258 the reserve fund of a life insurance policy is explained. Each year when the premium on the policy is paid, a part of this premium is set aside to build up this reserve fund, which grows larger each year. The *actual cost* each year, to the insurance company, of insuring a person's life increases as his age increases. This increase is caused by the increased rate of death as age increases. Because of the reserve on each policy, it is possible for the insurance company to continue to insure a policyholder at advanced ages without increasing the premium. You may wish to show the pupils some of the mortality tables used by insurance companies when premiums are computed. They can be found in books of mathematical tables.

The table in ex. 1 on page 258 shows that the reserve fund for this particular policy amounts to \$147 at the end of 10 years, to \$244 at the end of 15 years, and to \$345 at the end of 20 years. It is the reserve fund that makes it possible for the company to return something to the policyholder in case he discontinues his insurance. In this event the amount paid back, which is called the *cash value* of the policy, is equal approximately to the total reserve fund accumulated on the policy up to the time it is discontinued. If an insurer does not drop his insurance, it is possible for him to borrow money from the company; the amount loaned, which is called the *loan value* of the policy, is limited to the total reserve

fund then available. It is very important for the pupils to be fully acquainted with the terms *reserve fund, cash value,* and *loan value*.

A situation in which one borrows money on a life insurance policy is described in ex. 2 on page 259. When a policyholder borrows money from a life insurance company, no security is required. The policyholder is obliged to sign a note, and the amount borrowed is endorsed on the policy. If the policyholder dies before the loan is paid, the amount of the loan is deducted from the face of the policy, and only the balance is paid to the beneficiary. It is thus seen that a life insurance loan is equivalent to a mortgage on one's insurance; hence, it is desirable to repay such loans as soon as possible. When a man borrows on a life insurance policy, he is expected, of course, to keep the policy in force by paying the premiums as they become due. If he does not, the cash value of the policy protects the company from losing the amount lent to the policyholder.

Workbook Reference: Arithmetic Workshop, Book 8, page 93

Key: Page 258 **1.** \$147; \$244; \$345; \$529. **3.** \$168; \$732; \$1587; \$2440.

Page 259 **1.** \$1220. **2.** (1) $7 \times \$345 = \$2415$; yes, since the value of the policy is more than the \$2000; (2) $.05 \times \$2000 = \$100$, yearly interest. **3.** (1) $\frac{1}{2} \times \$100 = \$50$, interest for 6 mo.; $\$2000 + \$50 = \$2050$; $\$7000 - \$2050 = \$4950$. **4.** \$336. **5.** \$1176. **6.** \$3703. **7.** \$3450. **8.** \$280. **9.** \$3660. **10.** \$3675.

Page 260

Aims: To explain life insurance dividends and to show how they may be used

Suggestions: Some insurance companies are *stock* companies, and others are *mutual* companies. Stock companies are operated like other business corporations, and earnings are paid to their stockholders as dividends. In a mutual company a surplus of funds remaining at the end of the year after paying the costs of operation is divided among the policyholders. These payments are also called *dividends*. To find the actual cost for a given year of an insurance policy issued by a mutual company, you must subtract your dividend from the premium. Initial premiums of mutual companies are usually higher than corresponding premiums of stock companies. The premiums given on page 256 are typical of a stock company. It will add interest to this work on insurance if you obtain sample rates for insurance policies from the local agents of both kinds of companies. You can also find out the dividends that a mutual company has paid in the past. Ex. 2, 3, and 4 on this page show how dividends may be used each year to reduce the amount of the annual premium. You may, if you wish, leave your dividends with the company and let them accumulate at compound interest as in a savings bank fund. This accumulated fund will be paid to the policyholder at any time upon request or, in the event of his death, it will be paid to his beneficiary in addition to the face of the policy. When dividends are left to accumulate, the policyholder pays the full premium each year on his policy.

Workbook Reference: Arithmetic Workshop, Book 8, page 92

Key: **2.** \$30.37. **3.** (1) \$3.49 + \$3.83 + \$4.18 + \$4.53 + \$4.87 = \$20.90, \$20.90 ÷ 5 = \$4.18; (2) \$22.47 − \$4.18 = \$18.29. **4.** (1) \$54.29 + \$74.00 = \$128.29; (2) 10 × \$19.62 = \$196.20; (3) \$196.20 − \$128.29 = \$67.91; (4) \$67.91 ÷ 10 = \$6.791, or \$6.79.

Page 261

Aim: To present two more sets of improvement tests in multiplication and division

Suggestion: These improvement tests are to be administered in the same way as the previous sets in multiplication and division. See page 375 of the text.

Key: **1.** \$65,517.12; \$61,498.85; \$5314.95; \$51,242.78; \$38,614.16. **2.** \$26,500.24; \$40,888.35; \$78,657.60; \$38,406.34; \$18,455.82. **3.** \$32,986.01; \$46,755.14; \$15,401.90; \$23,152.38; \$7637.50. **4.** 66.4; 35.6. **5.** 94.6; 47.4. **6.** 80.8; 41.2. **7.** 69.0; 44.4. **8.** 25.0; 80.3. **9.** 48.3; 46.9.

Page 262

Aim: To describe industrial insurance, group life insurance, and United States Government insurance

Suggestions: Group life insurance, which is described in ex. 2 on this page, is available only to those persons who are members of a group of 25 or more employees, all working in the same business or industry. The amount of group insurance that an employee can get usually depends on his salary, or on the number of years he has worked for his employer, or on a combination of these. Group life insurance is the cheapest form of life insurance that one can get since it costs the insurance company much less to handle group insurance than other forms of insurance. The premiums on such insurance are usually paid in a lump sum by the employer for the entire group of 25 or more people. There is also no expense for medical examination since such examinations are not required in group insurance.

The disadvantage of group insurance is that it is often not available to employees after they have retired or have become unable to work, at which time death is more likely to occur. One should not, therefore, depend exclusively upon group insurance to meet his insurance needs. Such insurance should be regarded as a supplement to the forms of insurance described on pages 254 and 255 of the text, rather than as a substitute for them.

Key: **1.** \$5.20. **2.** \$192,000,000,000 ÷ 46,000,000 = \$4173.913, or \$4173.91.

Page 263

Aim: To give a review of important technical terms used in arithmetic

Key: **1.** Promissory note; date of maturity. **2.** Interest. **3.** Commission. **4.** Installment plan. **5.** Lending. **6.** 24. **7.** Dividends. **8.** Cost, expenses, profit. **9.** Compound interest. **10.** Less. **11.** Formula. **12.** Parallel lines. **13.** Height or altitude. **14.** Brokerage. **15.** Congruent.

Pages 264–265

Aim: To describe some of the benefits available to workers under the Social Security Act

Suggestions: The Social Security Act is changed or amended frequently by Act of Congress of the United States. These changes usually affect the amount of the contributions that the worker and the employer make to the government, the amount of monthly benefit that the worker or his survivors receive, and the different groups of persons who are eligible for these benefits. The latest information on these matters can be obtained by writing to the Government Printing Office, Washington 25, D. C., which publishes pamphlets on Social Security. The latest information on the Social Security Act is available also in *The World Almanac,* which is published annually. The contributions made by the worker and by the employer have been raised by Congress to $3\frac{5}{8}\%$ of the worker's wages, beginning in January of 1963. This amount is payable both by the worker and by the employer, making $7\frac{1}{4}\%$ in all. This rate is scheduled to rise to $4\frac{5}{8}\%$ each for worker and employer in 1968. Remind the pupils before they solve the problems on page 265 that the Social Security tax is paid on only the *first* \$4800 of a worker's wages each year.

The regulations regarding the requirements for a worker to be "fully insured" or "currently insured" and thus eligible for Social Security benefits are complicated and contain many exceptions and variations. Hence, no attempt was made to go into that aspect of Social Security on these pages.

Key: **3.** \$95.00. **4.** \$84.00; \$126.00. **5.** \$60.00 − \$40.00 = \$20.00; \$20.00 ÷ \$40.00 = .50, or 50%; for any of the average monthly earnings, the percentage will come out to about 50%. **6.** \$202.40. **8.** (1) .03625 × \$4000 = \$145; (2) \$145. **9.** .03625 × \$4800 = \$174.

Pages 266–269

Aims: To give information on fire insurance, including the premium rates charged for such insurance, and to give problems in calculating premiums

Suggestions: The latest figures on fire losses in the United States can be found in *The World Almanac.* The class will profit by a discussion of the reasons why fire insurance rates vary according to the type of building and the locality in which it is situated. See ex. 1 on page 267. In this connection the pupils should be asked to find the rates charged in their locality on buildings of various kinds and to explain the reasons for the variation in these rates. Such information may easily be obtained from the local agent of a fire insurance company.

The method of reducing the cost of insurance by buying 3-year and 5-year policies, as stated on page 268, should also be made clear. Incidentally, this furnishes a valuable illustration of how savings can often be made.

Workbook Reference: Arithmetic Workshop, Book 8, pages 96 and 97

Key: Page 266 **2.** 482,000 ÷ 1,024,000 = .470, or 47%.

Page 267 **4.** $\$6000 \div \$100 = 60$ (hundreds); $60 \times \$.34 = \$20.40$. **5.** \$8.40. **6.** \$7.70. **7.** \$15.12. **8.** \$14.80. **9.** \$11.40. **10.** \$23.60. **11.** \$21.28. **12.** \$35.84. **13.** \$128. **14.** \$23.70. **15.** \$45. **16.** \$123.20. **17.** \$552. **18.** \$252. **19.** \$18.56. **20.** \$76.

Page 268 **1.** (1) $3 \times \$20 = \$60$; $\$60 - \$54 = \$6$; (2) $5 \times \$18 = \$90$; $4.4 \times \$18 = \$79.20$; $\$90 - \$79.20 = \$10.80$. **2.** (1) $\$13{,}000 \div \$100 = 130$ (hundreds), $130 \times \$.32 = \$41.60$; (2) $2.7 \times \$41.60 = \$112.32$; (3) $4.4 \times \$41.60 = \$183.04$. **3.** (1) $\$29{,}000 \div \$100 = 290$ (hundreds), $290 \times \$.40 = \$116$, premium for 1 yr.; $15 \times \$116 = \$1740$, total premiums for 15 1-yr. policies; $2.7 \times \$116 = \$313.20$, premium for 1 3-yr. policy, $5 \times \$313.20 = \$1566$, premiums for 5 3-yr. policies; $\$1740 - \$1566 = \$174$; (2) $4.4 \times \$116 = \$510.40$, premium for 1 5-yr. policy, $3 \times \$510.40 = \$1531.20$, premiums for 3 5-yr. policies; $\$1740 - \$1531.20 = \$208.80$. **4.** \$6. **5.** \$98.66. **6.** \$76.95. **7.** \$336.60. **8.** \$39.29. **9.** \$64.26. **10.** \$24.36. **11.** \$9.45. **12.** \$121.50. **13.** \$192.51. **14.** \$291.06. **15.** \$1927.20. **16.** \$149.23. **17.** \$112. **18.** \$105.30. **19.** \$1703.46. **20.** $\$15{,}000 \div \$100 = 150$ (hundreds); $150 \times \$.26 = \$39$; $3 \times \$39 = \$117$; $\$117 - \$105.30 = \$11.70$.

Page 269 **1.** \$1500; \$5000. **2.** \$11,200. **3.** (1) $.80 \times \$45{,}000 = \$36{,}000$, $\$36{,}000 \div \$100 = 360$ (hundreds), $360 \times \$.24 = \$86.40$, $2.7 \times \$86.40 = \$233.28$; (2) $4.4 \times \$86.40 = \$380.16$. **4.** $.90 \times \$12{,}000 = \$10{,}800$; $\$10{,}800 \div \$100 = 108$ (hundreds); $108 \times \$.20 = \$21.60$; $2.7 \times \$21.60 = \$58.32$. **5.** (1) $\$7000 \div \$100 = 70$ (hundreds), $70 \times \$.32 = \$22.40$, $.60 \times \$22.40 = \$13.44$; (2) if you plan to sell your house in 6 months, there is no need to insure it for a longer period of time.

Page 270

Aim: To discuss automobile insurance and the reasons for carrying it

Suggestions: The National Safety Council, whose main office is in Chicago, is an organization which devotes its energies to educational campaigns to reduce or prevent automobile and other types of accidents. Each year the Council publishes a book entitled "Accident Facts," which gives statistical data on the accidents of the preceding year. Copies of this book may be had from the Council at a small cost per copy.

Page 271

Aims: To present another set of improvement tests in addition and to review arithmetical terms

Suggestion: This set of improvement tests should be administered in the same manner as previous sets in addition. See page 375 of the text.

Key: **1.** \$3923.27; \$3112.11; \$2582.37; \$2209.31; \$2221.25. **2.** \$2141.41; \$2696.35; \$2549.43; \$2088.28; \$3198.20. **3.** \$2692.25; \$3404.38; \$3650.95; \$2924.63; \$3262.97.

Pages 272–275

Aim: To give information on the more important kinds of automobile insurance, the premium rates for such insurance, and automobile insurance problems

Suggestions: In teaching the topic of automobile insurance, you can add interest by having the pupils inquire about the insurance that their parents carry on their cars and bring this information to class. In recent years auto accidents have greatly increased in number, with the result that the rates for automobile insurance have been raised considerably. Ask the pupils to inquire from their parents concerning the most recent rates in your locality.

The premiums vary widely throughout the country. They depend on whether the car is used for business, driven certain distances to work, or used only for pleasure driving. They depend also on the ages of the male drivers of the car. If a male driver under age 25 has taken an approved driver education course, the premium is reduced. If your high school provides such a course, the pupils will be interested in this regulation.

In ex. 7 on page 275 notice that the insurance company's liability for Mr. Hill's damages is limited to \$10,000. In liability insurance the first limit applies, in one accident, to *each* individual and the second limit applies to the *total* damages that the company will pay. Thus the company will pay \$10,000 to Mr. Hill and \$5000 to Mrs. Hill.

Key: Pages 272–273 **1.** (1) $1.32 \times \$43 = \$56.76$; (2) Mr. Baker was wise to carry the additional protection for the small extra cost, since in case of accident, damages of large amounts are often awarded; (3) there is less chance of accident if young people do not drive, and there is less use of the car if it is not used for business purposes; (4) the company will be risking greater amounts of money. **2.** $\$56.76 + \$18 + \$12 + \$46 = \$132.76$. **3.** $1.50 \times \$46 = \$64.50$. **4.** (1) $\$89.78 - \$50 = \$39.78$; (2) \$50. **5.** (1) (a) $.81 \times \$23 = \$18.63$, (b) $.44 \times \$23 = \$10.12$; (2) (a) $.81 \times \$43.40 = \$35.154$, or \$35.15, (b) $.44 \times \$43.40 = \$19.096$, or \$19.10; (3) (a) $.81 \times \$74.12 = \$60.0372$, or \$60.04, (b) $.44 \times \$74.12 = \$32.6128$, or \$32.61.
Pages 274–275 **2.** \$29, \$33.35, \$38.28, \$39.73, \$43.50, \$46.98; \$34, \$39.10, \$44.88, \$46.58, \$51, \$55.08; \$41, \$47.15, \$54.12, \$56.17, \$61.50, \$66.42; \$56, \$64.40, \$73.92, \$76.72, \$84, \$90.72; \$69, \$79.35, \$91.08, \$94.53, \$103.50, \$111.78; \$84, \$96.60, \$110.88, \$115.08, \$126, \$136.08. **3.** \$15, \$16.50, \$17.25, \$17.70, \$18.15, \$18.45, \$18.75, \$19.50; \$18, \$19.80, \$20.70, \$21.24, \$21.78, \$22.14, \$22.50, \$23.40; \$22, \$24.20, \$25.30, \$25.96, \$26.62, \$27.06, \$27.50, \$28.60; \$25, \$27.50, \$28.75, \$29.50, \$30.25, \$30.75, \$31.25, \$32.50; \$30, \$33, \$34.50, \$35.40, \$36.30, \$36.90, \$37.50, \$39; \$36, \$39.60, \$41.40, \$42.48, \$43.56, \$44.28, \$45, \$46.80. **6.** The limits of Mr. Smith's policy were \$10,000 and \$20,000, which means that the insurance company will pay damages up to \$10,000 to an injured person. **7.** $\$10,000 + \$5000 = \$15,000$. **8.** $\$6500 - \$5000 = \$1500$.

Pages 276–278

Aim: To give information on accident and health insurance and hospitalization plans

Suggestions: A typical accident and health policy, such as is sold by insurance companies, is described on page 276.

A typical hospitalization plan is described on page 278. Hospitalization plans have been established in many towns and cities throughout the United States. Many of these plans are nonprofit organizations; their purpose is to provide hospital services to members. In a recent year these plans had a total combined membership of about 60 million people, or about one third of all the people in the United States. The rates and benefits of such plans vary from community to community. Have the pupils inquire about hospitalization plans that have been established in your community and bring to class information concerning the services offered and the rates charged.

A newer kind of health insurance is becoming important. It is called *catastrophe* or *major medical expense* insurance. The insurance company pays, above a certain amount such as \$500, *all* expenses of a major illness. This includes hospital bills, nurses' fees, physicians' fees, medicines, and so on. You may wish to supplement the work on these pages with information about this kind of insurance. Insurance agents in your community will be glad to give you the provisions and the premiums of typical policies.

Key: Page 277 **2.** 5 wk.; \$250. **3.** (1) 7 × \$93 = \$651, loss of pay; (2) \$651 − \$250 = \$401. **4.** Many illnesses make persons inactive for less than 2 weeks. If the company had to pay for these short periods of illness, it would have to charge a higher premium. By excluding these short illnesses, the company can give a lower premium. On the other hand, if these short illnesses are included, a higher premium must be charged. **5.** \$5000. **6.** (1) 200 × \$50 = \$10,000; (2) 100 × \$50 = \$5000. **7.** 9 × \$50 = \$450; \$450 + \$785 = \$1235. **8.** Some occupations are more dangerous than others.

Page 278 **3.** (1) \$28 − \$15 = \$13, 17 × \$13 = \$221; (2) 17 × \$15 = \$255, \$255 + \$32 + \$37 = \$324, \$324 − \$44.40 = \$279.60. **4.** (1) 20 × 7 da. = 140 da., 140 da. = 120 da. + 20 da., \$21 − \$15 = \$6, 120 × \$6 = \$720 (cost of room for 120 da.), 21 × \$20 = \$420 (cost of room for 20 da.); \$720 + \$420 = \$1140; (2) 140 × \$21 = \$2940, \$2940 + \$140 + \$80 + \$57 = \$3217, \$3217 − \$44.40 = \$3172.60; \$3172.60 − \$1140 = \$2032.60.

Page 279

Aim: To provide a review of work previously presented

Key: **1.** .289. **2.** \$6.56. **3.** 22 ft. **4.** 75%; 111%. **5.** $3\frac{1}{2}\%$ of \$50 is \$.10 greater. **6.** 26 da. (Sept.) + 31 da. (Oct.) + 30 da. (Nov.) + 31 da. (Dec.) + 8 da. (Jan.) = 126 da. **7.** 900 × \$1.81402 = \$1632.618, or \$1632.62. **8.** $V = lwh$; the product of the length and the height is $4 \times 3\frac{3}{4}$, or 15; the product of 15 and the width is 35, so you can find the width by dividing 35 by 15, which gives $2\frac{1}{3}$ (ft.). **9.** 4 yd. 2 ft. = 168 in.; 3 ft. 6 in. = 42 in.; $\frac{42}{168} = \frac{1}{4}$. **10.** $16\frac{1}{2} + 18\frac{3}{4} + 15\frac{1}{2} + 19\frac{1}{4} = 70$; $70 \div 4 = 17\frac{1}{2}$. **11.** .20 × \$1000 = \$200, \$1000 − \$200 = \$800, .10 × \$800 = \$80, \$800 − \$80 = \$720, \$1000 − \$720 = \$280; .10 × \$1000 = \$100,

$\$1000 - \$100 = \$900$, $.20 \times \$900 = \$180$, $\$900 - \$180 = \$720$, $\$1000 - \$720 = \$280$; no more. **12.** (1) $A = \frac{1}{2}h(a + b)$; (2) $A = \frac{1}{2} \times 38 \times 95 = 1805$ (sq. ft.). **13.** 3291.67. **14.** (1) \$8 (60 da.) + \$.80 (6 da.) + \$.80 (6 da.) = \$9.60; (2) 4% is $\frac{2}{3}$ of 6%, $\frac{2}{3} \times \$6 = \$4$, int. at 4% for 60 da., \$2 (30 da.) + \$.20 (3 da.) = \$2.20. **15.** $r = \frac{24C}{P(n+1)}$; $r = \frac{24 \times 10.25}{100(9+1)}$; $r = \frac{246}{1000}$, or $\frac{24.6}{100}$, or 24.6%. **16.** $\frac{192}{360}$; .533; .526.

Pages 280–281

Aim: To discuss prime and composite numbers, and to show how to find the largest common factor and least common multiple of two or more numbers

Suggestions: The pupil's knowledge of number relationships and properties should develop continuously as he studies mathematics. The presentation of prime numbers and related topics on pages 280–281 will assist with this development and will be of interest to pupils.

Have some pupils make a chart with the first 100 integers. Discuss with the class how to strike out all of the composite numbers (multiples of 2, 3, 5, except 2, 3, 5, and so on), thus leaving only the prime numbers between 1 and 100. Point out different ways of finding all of the prime factors of a number.

Emphasize how to find multiples of one number. As is done in ex. 7, select two small composite numbers and build multiples of each. Then note which of these multiples are common multiples of the two numbers. Select the least common multiple of the pair of numbers and express it as a product of prime factors. Compare these prime factors with the prime factors of the two original numbers. Do this for several pairs of small composite numbers and lead pupils to discover how to find the least common multiple of two numbers from their prime factors.

Key: **1.** Prime: 2, 3, 5, 7, 11, 13, 17, 19, 23; composite: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25. **2.** $2 \times 2 \times 2$; 3×3; $2 \times 2 \times 3$; 2×7; 3×5; $2 \times 3 \times 3$; 3×7; 5×5; $2 \times 2 \times 7$; $2 \times 2 \times 2 \times 2 \times 2$; $2 \times 2 \times 2 \times 5$. **3.** Answers will vary. **4.** 15; 10; 2; 15; 3; 7; 15. **5.** and **6.** Answers will vary. **8.** 48; 45; 36; 120; 120. **9.** 24; 30; 144.

Pages 282–283

Aim: To show how the local tax rate is determined and how taxes on property are computed

Suggestions: The reasons for local taxation and the method of computing a property tax are given on these pages. It will make the topic of taxation more real to the pupils if you can get a copy of the budget of your city in order to see for what purposes the city needs money. Such a study gives the pupils a clear idea of the method of determining the tax rate.

By inquiry at home, the pupils will probably be able to find the assessed valuation of certain pieces of property in the city; this will make clear to them the meaning of the term *assessed valuation*. It should be pointed out, as is done

in ex. 13–15 on page 283, that the assessed valuation of a piece of property is usually less than its actual valuation and that the rate of taxation depends on the assessed valuation. For example, if property in a given town is assessed for 60% of its actual value, this assessment will require a higher tax rate than if the property is assessed for 80% of its actual value. In either case the *amount of tax* would be the same, but in the one case the *tax rate* is higher than in the other.

Workbook Reference: Arithmetic Workshop, Book 8, pages 98 and 99

Key: Page 282 **3.** \$2.84; \$3.05; \$2.60; \$2.87; \$3.09; \$4.15. **4.** \$221,250 ÷ \$6,250,000 = .0354, or 3.54%, or \$3.54 per \$100. **5.** \$4.15. **6.** \$2.92. **7.** \$2.98. **8.** \$3.32. **9.** \$4.45. **10.** \$3.65.

Page 283 **1.** \$256.50. **2.** \$9200 ÷ \$100 = 92 (hundreds); 92 × \$2.95 = \$271.40. **3.** \$323.40. **4.** \$435.75. **5.** \$1107.36. **6.** \$3718.30. **7.** \$3348.55. **8.** \$7100. **9.** \$4228.20. **10.** \$16,521.68. **11.** \$8782.32. **12.** \$23,179.50. **13.** (1) $\frac{3}{4}$ × \$65,299,200 = \$48,974,400, \$2,081,412 ÷ \$48,974,400 = .0425, or \$4.25 per \$100; (2) \$8500 ÷ \$100 = 85 (hundreds), 85 × \$4.25 = \$361.25. **14.** .70 × \$20,000 = \$14,000; \$14,000 ÷ \$100 = 140 (hundreds); 140 × \$3.55 = \$497.00. **15.** .60 × \$16,000 = \$9600, \$9600 ÷ \$100 = 96 (hundreds), 96 × \$4.75 = \$456; .80 × \$16,000 = \$12,800, \$12,800 ÷ 100 = 128 (hundreds), 128 × \$3.30 = \$422.40; \$456 − \$422.40 = \$33.60 more paid by Mr. Johnson.

Page 284

Aim: To teach how to use a tax table

Suggestions: For the purpose of computing taxes, many towns and cities use a tax table similar to the one illustrated on this page. Mathematical tables, such as tax tables, interest tables, and wage tables, play an important part in life today; hence, it is desirable for the pupils to become familiar with using such tables.

Key: **2.** \$274.77; \$319.50; \$93.72; \$149.10; \$185.31; \$232.17; \$302.46; \$257.73. **3.** \$2.13; \$193.83, \$195.96, \$198.09, \$200.22, \$202.35, \$204.48, \$206.61, \$208.74, \$210.87; \$236.43, \$238.56, \$240.69, \$242.82, \$244.95, \$247.08, \$249.21, \$251.34, \$253.47. **4.** By doubling the amount for \$3500; \$2.13; \$340.80, \$342.93, \$345.06, \$347.19, \$349.32, \$351.45, \$353.58, \$355.71, \$357.84, \$359.97, \$362.10; \$383.40, \$385.53, \$387.66, \$389.79, \$391.92, \$394.05, \$396.18, \$398.31, \$400.44, \$402.57, \$404.70; \$426.00; \$428.13, \$430.26, \$432.39, \$434.52, \$436.65, \$438.78, \$440.91, \$443.04, \$445.17, \$447.30.

Page 285

Aim: To discuss kinds of federal and state taxes

Suggestions: Federal income taxes of wage earners are usually collected weekly or monthly on the pay-as-you-go principle, these taxes being deducted from the weekly or monthly pay of the wage earner. If the total amount of taxes collected in this way from a wage earner turns out at the end of the year to be more than

he ought to pay, the excess amount is returned by the federal government. On the other hand, if the wage earner owes more federal tax than has been collected on the pay-as-you-go plan, he must pay the additional tax. Such additional taxes for a given year have to be paid on or before April 15 of the following year. Many of the states of the United States have state income taxes which the individual has to pay in addition to the federal income tax. Some states do not have such state taxes. Ask the pupils if they can find out what states do have such taxes.

Page 286

Aims: To give a very brief discussion of federal income tax returns, and to give an illustration of how income taxes are computed

Suggestions: The tax rate schedules provided by the tax collector, one of which is given in ex. 3, vary in accordance with new tax rates passed by Congress. It will be of interest to bring to class one of the latest federal tax forms and to have the pupils use the tax rate schedules printed in it in computing income taxes. You should emphasize to the pupils that taxes are computed on *taxable* income. A person's taxable income is found by subtracting from his actual income his exemptions and allowable deductions.

You may wish also to show the pupils how the tax tables given in the federal forms are used when a taxpayer finds the amount of his taxes this way.

Workbook Reference: Arithmetic Workshop, Book 8, page 102

Key: **4.** $3150 − $2000 = $1150; .22 × $1150 = $253; $400 + $253 = $653. **5.** (1) .20 × $1750 = $350; (2) $2800 − $2000 = $800, .22 × $800 = $176, $400 + $176 = $576; (3) $4275 − $4000 = $275, .26 × $275 = $71.50, $840 + $71.50 = $911.50; (4) $5600 − $4000 = $1600, .26 × $1600 = $416, $840 + $416 = $1256; (5) $7500 − $6000 = $1500, .30 × $1500 = $450, $1360 + $450 = $1810; (6) $8430 − $8000 = $430, .34 × $430 = $146.20, $1960 + $146.20 = $2106.20; (7) $9250 − $8000 = $1250, .34 × $1250 = $425, $1960 + $425 = $2385.

Page 287

Aim: To review circle graphs by means of a project showing how a city gets and spends it money

Key: **1.** $\frac{1}{2}$ × $38,500,000 = $19,250,000. **2.** $19,250,000 ÷ $550,000,000 = .035, or $3.50 per $100. **3.** (1) .10 × $38,500,000 = $3,850,000; (2) .20 × $38,500,000 = $7,700,000; (3) .20 × $38,500,000 = $7,700,000. **4.** $3,850,000 ÷ .02 = $192,500,000. **5.** (1) $\frac{1}{4}$ × $38,500,000 = $9,625,000; (2) .18 × $38,500,000 = $6,930,000; (3) $\frac{1}{4}$ × $38,500,000 = $9,625,000. **6.** (1) .15 × $38,500,000 = $5,775,000; (2) .17 × $38,500,000 = $6,545,000.

Page 288

Aim: To give information and problems on gasoline taxes

Suggestion: Gasoline taxes vary from time to time and from state to state. Ask the pupils to find out the current federal tax per gallon on gasoline and also their state tax.

Key: **2.** \$72,984,000 ÷ \$.06 = 1,216,400,000 (gal.). **3.** 1,216,400,000 × \$.04 = \$48,656,000. **4.** \$3,625,755,000 is about \$3626 million, \$360,532,000 is about \$361 million; \$361 ÷ \$3626 = .0995, or 10.0%. **5.** \$3,625,755,000 − \$3,470,882,000 = \$154,873,000; \$154,873,000 ÷ \$3,470,882,000 = .0446, or 4.5%. **6.** (1) 7500 mi. ÷ 15 mi. = 500 (gal.); (2) $5\frac{1}{2}$¢ + 4¢ = $9\frac{1}{2}$¢, tax per gallon; 500 × \$.09$\frac{1}{2}$ = \$47.50.

Page 289

Aim: To give information and problems on customs duties and internal revenue

Suggestions: Duties (also called *customs* or *tariff*) are taxes imposed by law on goods brought into the United States from foreign countries. Only certain imports are subject to duty. Many articles of merchandise may be brought into the country duty-free; that is, without any customs charge. The term *tariff* usually applies to the schedule or list of all duties charged on imports. From year to year Congress decides which kinds of goods are to be subject to duty and which are not.

The term *internal revenue* applies to federal taxes collected *within* the United States. These internal taxes are made up of *federal income taxes* and *excise taxes*. Income taxes are levied against the incomes and profits of individuals, business houses, and corporations and are the chief source of internal revenue. Excise taxes are levied on services and articles made and sold in this country and include taxes on jewelry, luggage, furs, electrical appliances, automobiles, gasoline, tobacco, alcoholic liquors, air travel tickets, telephone and telegraph services, and so on. These excise taxes are frequently changed by Congress. For lists of current excise taxes, see the latest issue of *The World Almanac*.

Workbook Reference: Arithmetic Workshop, Book 8, pages 103 and 104

Key: **3.** \$2.19; \$3.52; \$6.50; \$127.50; \$185. **4.** \$.17, \$.12, \$.22, \$.33; \$1.87, \$1.32, \$2.42, \$3.63. **5.** .05 × \$161.20 = \$8.06, \$161.20 + \$8.06 = \$169.26, cost of 1 full-fare ticket; 2 × \$169.26 = \$338.52, cost of 2 full-fare tickets; $\frac{1}{2}$ × \$169.26 = \$84.63, cost of 1 half-fare ticket; 3 × \$84.63 = \$253.89, cost of 3 half-fare tickets; \$338.52 + \$253.89 = \$592.41. **6.** .10 × \$7.50 = \$.75.

Page 290

Aim: To review the work of Chapter 8

Key: **1.** An ordinary life insurance policy requires premium payments during the entire lifetime of the insured, while a 20-payment life insurance policy requires premium payments for only 20 years, after which no further payments are required. The premium on an ordinary life insurance policy is less than the premium on a 20-payment life insurance policy if the person insured is the same

age in each case. **2.** (a) If a life insurance policy runs for a number of years and is then dropped, the company returns a certain amount of money to the insured; this amount is called the *cash value* of the policy. (b) A life insurance premium is the amount the insured pays each year to have his life insured. (c) Usually a life insurance company finds at the end of each year that it has not spent as much money as it expected to spend; hence, it has accumulated a small amount in savings. These savings are divided among the policyholders in the form of dividends. **4.** If a building were used to store gasoline, explosives, or other inflammable goods, the risk of fire would be greater; hence, there would be a high rate for fire insurance. **5.** $5000 ÷ $100 = 50 (hundreds); 50 × $.34 = $17, premium for 1 year; 4.4 × $17 = $74.80, premium on 1 5-year policy; 3 × $74.80 = $224.40, premiums on 3 5-year policies; 2.7 × $17 = $45.90, premium on 1 3-year policy; 5 × $45.90 = $229.50, premiums on 5 3-year policies; $229.50 − $224.40 = $5.10. **6.** Because accidents were increasing in number. **10.** $4500 = $1000 + $2000 + $1500; .02 × $1000 = $20, .04 × $2000 = $80, .06 × $1500 = $90; $20 + $80 + $90 = $190. **11.** 57,877,826,000 × $.04 = $2,315,113,040.

Page 291

Aim: To present Problem Test 8

Workbook Reference: Arithmetic Workshop, Book 8, page 107

Key: **1.** $20,000 ÷ $1000 = 20 (thousands); 20 × $21.30 = $426.00. **2.** $5000 ÷ $1000 = 5 (thousands); 5 × $18.21 = $91.05; 10 × $91.05 = $910.50; $910.50 − $430 = $480.50. **3.** $8000 ÷ $100 = 80 (hundreds); 80 × $.20 = $16, premium for 1 yr.; 3 × $16 = $48, cost of premiums for 3 1-yr. policies; 2.7 × $16 = $43.20, cost of premium for 1 3-yr. policy; $48 − $43.20 = $4.80. **4.** $1.60 ÷ $40 = $\frac{4}{100}$, or 4%. **5.** $78,372 ÷ $2,488,000 = .0315, or $3.15 per $100. **6.** 89¢ − 55¢ = 34¢; 34¢ ÷ 55¢ = .618, or 62%. **7.** 11 × $4 = $44; $44 + $10 = $54; $54 − $50 = $4. **8.** 24 min. = $\frac{2}{5}$ hr.; in $\frac{2}{5}$ hr. he can row 2 mi.; in $\frac{1}{5}$ hr. he can row $\frac{1}{2}$ of 2 mi., or 1 mi.; in $\frac{5}{5}$ hr. (1 hr.) he can row 5 × 1 mi., or 5 mi. **9.** (1) 300 ÷ 25 = 12, so 300 sheets are 12 times as many as 25 sheets; 12 × $.09 = $1.08; (2) 175 ÷ 25 = 7, so 175 sheets are 7 times as many as 25 sheets; 7 × 9¢ = 63¢. **10.** If $\frac{3}{4}$ in. represents 15 mi., $\frac{1}{4}$ in. represents 5 mi.; then 1 in. represents 20 mi.; $2\frac{1}{4}$ × 20 mi. = 45 mi.

Page 292

Aim: To provide a diagnostic test with page references for practice

Key: **1.** $16.77; $118.74. **2.** $3.19; $2.75. **3.** $1792; $319. **4.** $12; $12.50. **5.** $.75; $5.25. **6.** $5.04; $3.20. **7.** $1.50; $9.75. **8.** $730; $14. **9.** $600; $4.50. **10.** $400; $8.75. **11.** 175; 220. **12.** 190; 116. **13.** 28; 69. **14.** 79.8; 55.2. **15.** 35.4; 81.0.

Aims of Chapter 9. The major aims of Chapter 9 are to:

1. Discuss mathematical sentences, equations, and formulas.
2. Teach the use of equations to find unknown numbers.
3. Show how to represent an unknown number by a mathematical phrase.
4. Apply four principles to the solution of equations.
5. Teach how to solve problems by using equations.
6. Develop an understanding of ratios and proportions.
7. Teach how to solve problems by using proportions.
8. Discuss inequalities, and show how inequalities are used to solve problems.

Page 293

Aim: To discuss mathematical sentences, equations, and formulas

Suggestions: The equation is one of the most useful instruments in mathematics. It is one of the most important methods of expressing relationships and of solving problems that has ever been developed. The equation is used all over the world to solve important problems in science, in engineering, in statistics, and in business.

Equations have already been used extensively in *American Arithmetics* to express relationships and general principles. The equation will now be used to solve certain types of arithmetic problems and to introduce basic algebraic methods.

Pages 294–295

Aim: To show how to use letters to represent unknown numbers, how to write number sentences in brief form, and how to find unknown numbers mentally

Suggestions: The equation is presented in very simple form on these pages. The exercises on these pages are to be worked by a simple process of reasoning, the pupil finding the answers by inspection. In ex. 17 on page 295 make clear to the pupils that *to solve* an equation means to find a value for the letter that makes the equation a true sentence. In equations of the type given here, there is a definite value for the unknown number, and only that value substituted for the unknown number will make the equation a true sentence.

Key: Page 294 **3.** 11 yr. **4.** 9 yr. **6.** $8n$; $\frac{1}{4}n$; $100n$; $n-6$; $n+50$. **7.** $\frac{1}{2}a = 7$; 14 yr. **8.** $\frac{1}{3}a = 25$; 75 yr.
Page 295 **1.** $4 + x = 12$, 8; $6x = 18$, 3. **2.** $x + 5 = 13$; 8. **3.** $\frac{1}{2}x = 6$, 12; $10 - x = 3$, 7. **4.** $x + 12 = 20$; 8. **5.** $x - 8 = 14$; 22. **6.** $\frac{1}{3}x = 5$; 15. **7.** $10 - x = 7$; 3. **8.** 2. **9.** 6. **10.** 1. **11.** 7. **12.** 17. **13.** 19. **14.** 4. **15.** 4. **16.** 28. **18.** 6. **19.** 13. **20.** 5. **21.** 1. **22.** 6. **23.** 3. **24.** 21. **25.** 7. **26.** 18. **27.** 8. **28.** 5. **29.** 3. **30.** 3. **31.** 80. **32.** 24.

Pages 296–298

Aims: To explain more about equations, and to show how to solve an equation by using the subtraction principle

Suggestions: On these pages the equation is compared to a seesaw in perfect balance and to a balance scale. In these illustrations it should be made clear that when both sides are equal in value, the balance is perfect. Thus, to solve an equation, one must find a value of the unknown number that will make the balance perfect. On page 298 the principle of subtracting the same number from both sides of an equation is introduced, all the work being confined to this principle.

Key: Pages 296–297 **5.** $n + 6$. **6.** $\frac{1}{3}n$. **7.** $n - 7$. **8.** $5n$. **9.** $3n + 7$. **10.** $4n - 5$. **11.** $12 - 8n$. **12.** 7. **13.** 24. **14.** 8. **15.** 26. **16.** 9. **17.** 8. **18.** 12. **19.** 14. **20.** 9. **21.** 7. **22.** 70. **23.** 8. **24.** $7\frac{1}{2}$. **25.** 15. **26.** 40. Page 298 **3.** 32. **4.** 125. **5.** 37.

Page 299

Aim: To present another set of improvement tests in subtraction

Suggestion: This set of improvement tests is to be administered in the same way as preceding sets in subtraction. See page 375 of the text.

Key: **1.** \$5814.99; \$1339.85; \$4218.76; \$2055.85. **2.** \$3664.96; \$321.16; \$5314.85; \$5794.87. **3.** \$9486.78; \$3520.30; \$7809.59; \$1357.86. **4.** \$6354.87; \$1495.94; \$3930.49; \$3764.97. **5.** \$2777.58; \$283.29; \$8048.25; \$586.10. **6.** \$1096.47; \$8204.31; \$2181.37; \$6188.09. **7.** \$3415.82; \$5072.69; \$5639.27; \$1192.76. **8.** \$1903.76; \$756.92; \$8584.60; \$7932.48. **9.** \$6096.53; \$3369.60; \$5927.96; \$6878.34.

Page 300

Aim: To teach the use of the addition principle in solving equations

Suggestions: The addition principle is stated in red at the end of ex. 1. Ex. 2 on this page should be very carefully studied since it shows how to use the addition principle to solve the equation $x - 5 = 8$. In ex. 2 it should be noted that the original equation $x - 5 = 8$ is changed to $x = 13$ by the use of the addition principle; that is, by adding 5 to both sides of $x - 5 = 8$. In this work, and also in the work on page 298, it should be noted that the word *transpose*, which is often used in high-school algebra, has not been used. The process of transposing a term from one side of an equation to the other by changing its sign from $-$ to $+$, or from $+$ to $-$, is not recommended for use in Grade 8 because at this time it might become a mechanical procedure about which the pupil would have little understanding. Instead, it is far better for the pupil to understand what he is doing by using the addition principle or the subtraction principle in order to make the necessary transformations in the original equation.

Key: **3.** Add 7. **4.** Add 3; to keep both sides equal. **5.** $x - 4 = 9$; $x - 4 + 4 = 9 + 4$; $x = 13$. *Check:* $13 - 4 = 9$. **6.** 6. **7.** 11. **8.** 8. **9.** 50. **10.** 147. **11.** 27. **12.** 80. **13.** $a - 400 = 531$; $a - 400 + 400 = 531 + 400$; $a = 931$. *Check:* $931 - 400 = 531$. **14.** 823. **15.** 426. **16.** 452.

Page 301

Aim: To summarize the subtraction and addition principles, and to provide practice in using them to solve equations

Key: **4.** $x + 4 = 25$; $x + 4 - 4 = 25 - 4$; $x = 21$. *Check:* $21 + 4 = 25$. **5.** 60. **6.** 33. **7.** $9 + x = 16$; $9 + x - 9 = 16 - 9$; $x = 7$. *Check:* $9 + 7 = 16$. **8.** 39. **9.** 13. **10.** 27. **11.** 59. **12.** 22. **13.** 62. **14.** 557. **15.** 539. **16.** 360. **17.** 405. **18.** 225. **19.** $n + 14 = 36$; $n + 14 - 14 = 36 - 14$; $n = 22$. *Check:* $22 + 14 = 36$. **20.** $n - 125 = 346$; $n - 125 + 125 = 346 + 125$; $n = 471$. *Check:* $471 - 125 = 346$. **21.** $45 + n = 93$; $45 + n - 45 = 93 - 45$; $n = 48$. *Check:* $45 + 48 = 93$.

Pages 302–303

Aim: To teach the use of the multiplication and division principles in solving equations

Suggestions: Ex. 2 on page 302 should be carefully explained to the pupils so that they will understand how the multiplication principle is used to solve the equation $\frac{1}{2}n = 7$. The same is true of ex. 2 on page 303, which explains the use of the division principle. Attention is called to the fact that all the exercises on these two pages, as well as those on pages 300 and 301, are to be checked by substituting in the original equation the value found for the letter.

Key: Page 302 **3.** $\frac{x}{3}$; $\frac{c}{4}$. **4.** 3. **5.** $\frac{1}{2}x = 8$; $2 \times \frac{1}{2}x = 2 \times 8$; $x = 16$. *Check:* $\frac{1}{2} \times 16 = 8$. **6.** 27. **7.** 15. **8.** 12. **9.** 21. **10.** $\frac{b}{5} = 2$; $5 \times \frac{b}{5} = 5 \times 2$; $b = 10$. *Check:* $\frac{10}{5} = 2$. **11.** 30. **12.** 10. **13.** 42. **14.** 12. **15.** 24. **16.** 96. **17.** 64. **18.** 36. **19.** 80. **20.** 75. **21.** 26. **22.** 54.

Page 303 **3.** 8. **4.** $5x = 30$; $\frac{5x}{5} = \frac{30}{5}$; $x = 6$. *Check:* $5 \times 6 = 30$. **5.** 5. **6.** 15. **7.** 4. **8.** 1. **9.** $\frac{1}{3}$. **10.** $\frac{1}{2}$. **11.** 2. **12.** $2\frac{1}{2}$. **13.** 4. **14.** 250. **15.** 50. **16.** 50. **17.** 25. **18.** 30. **19.** $8 = 8$; $\frac{8}{4} = \frac{8}{4}$; $2 = 2$.

Page 304

Aim: To summarize the four principles of equations and to use them in solving equations

Suggestions: The four principles summarized in ex. 1 are often called *axioms* in classes in high-school algebra. In Grade 8, however, it is much better to call

them *principles* than to use a new and strange word. Today some texts in algebra also refer to these four principles as *principles* rather than *axioms*.

It should be noted that the unknown number is not always on the left side of the equation; sometimes it is on the right side, as is seen in some of the exercises on this page and on previous pages.

Workbook Reference: Arithmetic Workshop, Book 8, page 108

Key: **3.** Subtract 7; add 12; divide by 11; multiply by 4; subtract 11. **4.** Add 4; subtract 14; divide by 90; multiply by 5; subtract 8. **5.** 8. **6.** 9. **7.** 1. **8.** 75. **9.** 18. **10.** $x - 18 = 27$; $x - 18 + 18 = 27 + 18$; $x = 45$. *Check:* $45 - 18 = 27$. **11.** $10 + d = 48$; $10 + d - 10 = 48 - 10$; $d = 38$. *Check:* $10 + 38 = 48$. **12.** 14. **13.** 26. **14.** 36. **15.** $\frac{1}{5}x = 11$; $5 \times \frac{1}{5}x = 5 \times 11$; $x = 55$. *Check:* $\frac{1}{5} \times 55 = 11$. **16.** $4m = 32$; $\frac{4m}{4} = \frac{32}{4}$; $m = 8$. *Check:* $4 \times 8 = 32$. **17.** 8. **18.** 3. **19.** 150. **20.** 23. **21.** 6. **22.** 12. **23.** 2. **24.** 56. **25.** 1. **26.** 55. **27.** 1. **28.** $\frac{1}{2}$.

Page 305

Aim: To teach how to understand and to write the language of algebra

Suggestion: The pupils will need help in seeing that they are to answer the questions in ex. 1–9 by giving number phrases which involve a letter as well as numbers.

Workbook Reference: Arithmetic Workshop, Book 8, page 110

Key: **3.** $2x$ tickets; $\frac{1}{2}x$ tickets. **4.** $(s + 8)$ dollars; $(s - 3)$ dollars; $2s$ dollars; $\frac{1}{2}s$ dollars. **5.** $(d - 1)$ dollars. **6.** $(x + 1)$ miles. **7.** $2x$ years; $(x + 5)$ years; $(x - 3)$ years. **8.** $(x - 19)$ cookies. **9.** $(x + 123)$ cones. **10.** 14; 5; 3. **11.** 14; 21; 25.

Pages 306–307

Aim: To teach the use of the equation in the solution of problems

Suggestions: Page 306 is one of the most important pages in this chapter since it teaches the procedure that must be followed in solving a problem by the aid of an equation. This procedure is outlined in ex. 4 on page 306; the most important step in this procedure is Step (2), which states that *by studying the problem, you must find two different number phrases that represent the same quantity.* After finding these two phrases, you write the symbol = between them and thus form an equation. Step (2) of ex. 4 will become clear to the pupil after he has carefully studied ex. 1–3 on page 306. Before assigning the exercises on page 307, you should go over some of them in class with the pupils.

Be sure to make clear to the pupils the differences between a *phrase* involving a letter to answer the questions on page 305 and using an *equation* to solve the problems on page 307.

Workbook Reference: Arithmetic Workshop, Book 8, page 111

Key: Page 306 **5.** Let x = number of cents Ann has now; $x + 15 = 87$; $x = 72$ (number of cents Ann has now).

Page 307 **1.** Let x = number of dollars they have now; $x + \$2.75 = \$15.00$; $x + \$2.75 - \$2.75 = \$15.00 - \$2.75$; $x = \$12.25$. *Check:* $\$12.25 + \$2.75 = \$15.00$. **2.** Let x = number of tickets at first; $x - 1477 = 273$; $x - 1477 + 1477 = 273 + 1477$; $x = 1750$ (tickets at first). *Check:* $1750 - 1477 = 273$. *Note:* In ex. 3–9 the necessary equation is given, but the steps required to solve the equation are omitted. **3.** Let x = number of pupils enrolled; $x - 140 = 35$; $x = 175$ (pupils enrolled). **4.** Let x = number in large box; $\frac{1}{4}x$ = number in small box; $\frac{1}{4}x = 18$; $x = 72$ (chocolates in large box). **5.** Let x = number of boys entered; $\frac{1}{5}x = 9$; $x = 45$ (boys entered). **6.** Let x = number of dollars still needed; $x + \$9.50 = \$43.50$; $x = \$34.00$. **7.** Let x = number of weeks; $\$3.50x = \$28.00$; $x = 8$ (weeks). **8.** Let x = number of hours; $35x = 210$; $x = 6$ (hours). **9.** Let x = number of dollars saved this week; $3x = \$6.75$; $x = \$2.25$. **10.** Let x = number of miles per hour; $2x = 6\frac{1}{2}$; $x = 3\frac{1}{4}$ (mi.).

Pages 308–309

Aim: To teach the solution of equations requiring the use of two principles

Suggestions: At this stage of the pupil's study of equations he should use the principles of addition and subtraction before he uses the principles of multiplication and division. Later on in his study of algebra he will learn that these principles can be applied to an equation in *any* order, and that the order which works out best will vary from equation to equation.

On page 309, in ex. 3, 5, and 6, make sure that the pupils understand and know how to write algebraically such phrases as "1 quart more than twice as many," "\$1 more than three times as much," "1 more than half the number," and so on.

Workbook Reference: Arithmetic Workshop, Book 8, page 109

Key: Page 308 **2.** $3x + x = 16$; $4x = 16$; $x = 4$. **3.** $6n + n = 56$; $7n = 56$; $n = 8$. **4.** $5r + 3r = 24$; $8r = 24$; $r = 3$. **5.** $7x + x = 40$; $8x = 40$; $x = 5$. **6.** $7x - 2x = 5$; $5x = 5$; $x = 1$. **7.** $3x - 2x = 9$; $x = 9$. **8.** $4n - 2n = 8$; $2n = 8$; $n = 4$. **9.** $3a - a = 2$; $2a = 2$; $a = 1$. **10.** 3. **11.** 4. **12.** 5. **13.** $2\frac{1}{2}$. **16.** 7. **17.** 3. **18.** 5. **19.** $3x + 8 = 20$; $3x + 8 - 8 = 20 - 8$; $3x = 12$; $x = 4$. **20.** 4. **21.** $\frac{1}{2}x + 5 = 11$; $\frac{1}{2}x + 5 - 5 = 11 - 5$; $\frac{1}{2}x = 6$; $2 \times \frac{1}{2}x = 2 \times 6$; $x = 12$. **22.** 28. **23.** 10. **24.** 15. **25.** 35. **26.** $\frac{3}{4}x = 6$; $\frac{1}{4}x = 2$; $x = 8$. **27.** 9.

Page 309 **1.** Let x = the number; $7x - 5 = 37$; $7x = 42$; $x = 6$. **2.** Let x = the number; $5x + x = 18$; $6x = 18$; $x = 3$. **3.** Let x = number of quarts he picked last week; $2x + 1 = 25$; $2x = 24$; $x = 12$ (qt. picked last week). **4.** Let x = Fred's age in years; $3x + 30 - x = 56$; $3x - x = 26$; $2x = 26$; $x = 13$ (Fred's age in years). **5.** Let x = dollars he put in bank in May; $3x + 1 = 7$; $3x = 6$; $x = 2$ (dollars put in bank in May). **6.** Let x = number of boys in the class; $\frac{1}{2}x + 1 = 11$; $\frac{1}{2}x = 10$; $x = 20$ (boys in class). **7.** Let x = number of dollars he saved a year ago; $3x + 5 = 50$; $3x = 45$; $x = 15$ (dollars saved a year ago).

8. Let x = number of cents he used to pay; $2x - 5 = 55$; $2x = 60$; $x = 30$ (number of cents he used to pay). **9.** Let x = number of dollars the house cost; $2x - 200 = 15{,}800$; $2x = 16{,}000$; $x = 8000$ (cost of house in dollars). **10.** Let x = number of miles to Newton; $\frac{3}{5}x = 12$; $\frac{1}{5}x = 4$; $x = 20$ (number of miles to Newton). **11.** Let x = number of dollars earned last week; $3x = 15$; $x = 5$ (number of dollars earned last week).

Page 310

Aim: To show how to find two unknown numbers by using an equation

Suggestions: In finding two unknown numbers, the pupil first has to decide how *each* of the two numbers will be represented. In ex. 1, he lets one number be represented by x and the other by $3x$. In this case the second number, $3x$, is said to be expressed "in terms of x." After deciding how to represent the numbers, he then forms the equation and solves it as shown in ex. 1. The pupils will need careful instruction in the procedure presented on this page.

Key: **2.** $2x$ hours; $3x$ hours. **3.** $3x = 6$; $x = 2$ (hours on Friday); $2x = 4$ (hours on Saturday). **4.** Let x = cost of apples in cents; $x + 30$ = cost of candy in cents; $x + x + 30 = 80$; $2x + 30 = 80$; $2x = 50$; $x = 25$ (cost of apples in cents); $x + 30 = 55$ (cost of candy in cents). **5.** Let x = number of points Walter scored; $x + 50$ = number of points Jack scored; $x + x + 50 = 500$; $2x + 50 = 500$; $2x = 450$; $x = 225$ (Walter's points); $x + 50 = 275$ (Jack's points). **6.** Let x = number of quarts Bill picked; $2x$ = number of quarts Mary picked; $3x$ = number of quarts Joe picked; $x + 2x + 3x = 12$; $6x = 12$; $x = 2$ (quarts Bill picked); $2x = 4$ (quarts Mary picked); $3x = 6$ (quarts Joe picked).

Page 311

Aim: To review computation and problem solving

Suggestion: Remind the pupils to review the formula on page 140 for the rate of interest charged in installment buying.

Key: **1.** \$7603.27. **2.** \$6. **3.** \$16,050. **4.** $36\frac{5}{12}$. **5.** 32.21. **6.** 21,666. **7.** \$3.57. **8.** \$13.68. **9.** \$.59. **10.** \$17,198.14. **11.** 74%. **12.** 75%. **13.** 55.8%. **14.** 25.0%. **15.** (1) \$607.50 ÷ \$13,500 = .045, or 4.5%; (2) \$13,500 − \$607.50 = \$12,892.50. **16.** $\frac{1}{4} \times \$80 = \$20$; $\$80 - \$20 = \$60$; $.02 \times \$60 = \$1.20$; $\$60 - \$1.20 = \$58.80$. **17.** 45° and 45°. **18.** (1) $V = \pi r^2 h$; (2) $V = \frac{22}{7} \times 5\frac{1}{4} \times 5\frac{1}{4} \times 24$, or 2079 (cu. ft.). **19.** \$830 is cost of bond; $.04 \times \$1000 = \$40$, interest; $\$40 \div \$830 = .0481$, or 4.8%. **20.** $13 \times \$14 = \$182$; $\$182 + \$20 = \$202$; $\$202 - \$185 = \$17$, carrying charge; $\$185 - \$20 = \$165$, unpaid balance; $C = \$17$, $P = \$165$, $n = 13$; $r = \dfrac{24 \times 17}{165(13 + 1)} = .176$, or 18%.

Pages 312–313

Aim: To show the meaning of a ratio, how to write a ratio, and how to change a ratio to another one of equal value

Suggestions: A ratio deals with the comparison of two numbers, this comparison *being made by division*. It is possible to compare two numbers by subtraction, but such a method of comparison is not called *ratio*. In teaching ratio and proportion, emphasize the fact that the comparisons must be made by division.

In ex. 2 on page 312 it should be noted that there are two ways of comparing 6 and 2 by division. One way is to divide 6 by 2, which gives 3. In this case the ratio is 3 and you say that 6 is 3 *times as large as* 2. The other way is to divide 2 by 6, which gives $\frac{2}{6}$ or $\frac{1}{3}$. In this case the ratio is $\frac{1}{3}$ and you say that 2 is $\frac{1}{3}$ of 6.

On page 313 you see that a ratio may be written as a common fraction. In fact, every common fraction may be considered a ratio. Since a ratio may be written as a common fraction, you may multiply or divide both terms of the ratio by the same number without changing its value, just as you multiply or divide both terms of a fraction by the same number. For example, the ratio $\frac{3}{8}$ has the same value as $\frac{6}{16}$. Likewise, $\frac{6}{2}$ has the same value as $\frac{3}{1}$.

Show the pupils the relationship between the work on these pages and the work in Chapter 2 on baseball standings. The standing of a team can be considered the ratio of the number of games won to the number of games played.

Workbook Reference: Arithmetic Workshop, Book 8, page 113

Key: Page 312 **5.** 2 times; 2; $\frac{1}{2}$; $\frac{1}{2}$. **6.** 5; $\frac{1}{5}$. **7.** $\frac{1}{3}$; 3. **8.** $1\frac{1}{3}$; $\frac{3}{4}$. **9.** $\frac{6}{7}$. **10.** $\frac{3}{8}$. **11.** 1. **12.** $1\frac{1}{2}$. **13.** $\frac{15}{16}$. **14.** 3. **15.** 2. **16.** $2\frac{1}{4}$. **17.** $\frac{1}{12}$. **18.** $\frac{1}{12}$. **19.** $\frac{1}{15}$. **20.** $\frac{1}{3}$. **21.** 6. **22.** $2\frac{1}{2}$. **23.** $4\frac{2}{3}$. **24.** $1\frac{1}{3}$.

Page 313 **3.** $\frac{5}{8}$, $5 \div 8$, 5:8; $\frac{3}{2}$, $3 \div 2$, 3:2; $\frac{1}{7}$, $1 \div 7$, 1:7; $\frac{3}{1}$, $3 \div 1$, 3:1. **5.** $\frac{8}{12}$; $\frac{16}{24}$; $\frac{6}{9}$. **6.** (1) 2 ft. = 24 in., $\frac{24}{15} = \frac{8}{5}$; (2) 6 ft. = 72 in., $\frac{27}{72} = \frac{3}{8}$. **7.** $\frac{1}{26}$; $\frac{1}{2}$. **8.** $\frac{1}{2}$. **9.** $\frac{3}{2}$. **10.** $\frac{4}{3}$. **11.** $\frac{1}{3}$. **12.** 1. **13.** $\frac{1}{3}$. **14.** $\frac{5}{9}$. **15.** 1 yd. = 36 in.; $\frac{9}{36} = \frac{1}{4}$. **16.** $\frac{1}{2}$. **17.** 1 lb. = 16 oz.; $\frac{6}{16} = \frac{3}{8}$. **18.** 1 qt. = 2 pt.; $\frac{2}{1} = 2$. **19.** 1 hr. = 60 min.; $\frac{60}{6} = 10$. **20.** $\frac{5}{2}$. **21.** $\frac{3}{5}$. **22.** $\frac{5}{2}$. **23.** 5. **24.** $\frac{4}{5}$. **25.** 1.

Page 314

Aim: To teach the meaning of proportion, the ways of writing a proportion, and an important principle relating to all proportions

Suggestions: When two ratios are equal, such as $\frac{2}{3} = \frac{8}{12}$, the four numbers are said to form a proportion. The four numbers, 2, 3, 8, and 12, are called *terms* of the proportion. A proportion is also an equation in which the two sides of the equation are equal ratios. Every proportion, therefore, consists of four terms that form two equal ratios.

The preferred way to write a proportion is to express each ratio in fractional form such as $\frac{2}{3} = \frac{8}{12}$. Another and older way to write a proportion is 2:3 = 8:12 which means $2 \div 3 = 8 \div 12$. The colon (:) is another way of writing the division sign (÷). See ex. 3 on this page.

An important principle which relates to any proportion is given in ex. 4 on this page. This principle states that *in any proportion, the cross-products are equal.* Years ago, when proportions were always written in the horizontal form

such as $2:3 = 8:12$, the end numbers, 2 and 12, were called the *extremes* and the middle numbers, 3 and 8, were called the *means*. In those days the principle mentioned above was stated as follows: *The product of the means equals the product of the extremes.* This principle is very useful in solving problems in proportion regardless of the way in which the principle is expressed. However, if pupils encounter a proportion written in the older form, it is simpler to rewrite it in fractional form before working with it.

Point out to the pupils that using the principle of cross-products is equivalent to using the multiplication principle of equations twice. For example, if both sides of the proportion $\frac{x}{4} = \frac{7}{2}$ are multiplied first by 2 and then by 4, you get $2x = 28$.

Key: 6. Yes; no; no; yes. **7.** Yes; yes. **8.** (1) $24:32 = 3:4$; (2) yes; (3) no.

Page 315

Aim: To show how to find an unknown term in a proportion

Suggestion: Ex. 1 should be carefully explained to the pupils since it shows how to solve the other exercises on this page.

Workbook Reference: Arithmetic Workshop, Book 8, page 114

Key: 2. $\frac{x}{15} = \frac{10}{30}$; $30x = 150$; $x = 5$. **3.** $\frac{x}{3} = \frac{12}{18}$; $18x = 36$; $x = 2$. **4.** $\frac{x}{3} = \frac{14}{21}$; $21x = 42$; $x = 2$. **5.** $\frac{x}{7} = \frac{12}{21}$; $21x = 84$; $x = 4$. **6.** $\frac{30}{x} = \frac{12}{10}$; $12x = 300$; $x = 25$. **7.** $\frac{x}{5} = \frac{2}{10}$; $10x = 10$; $x = 1$. **8.** 8. **9.** 4. **10.** 30. **11.** 40. **12.** 8. **13.** $17\frac{1}{2}$. **14.** 8. **15.** $\frac{x}{11} = \frac{5}{22}$; $22x = 55$; $x = 2\frac{1}{2}$. **16.** $\frac{x}{15} = \frac{12}{15}$; $15x = 180$; $x = 12$. **17.** $\frac{x}{4} = \frac{16}{32}$; $32x = 64$; $x = 2$. **18.** 3. **19.** 6. **20.** 18. **21.** 5. **22.** 2. **23.** $1\frac{1}{2}$. **24.** Let x = number of games won; $\frac{x}{18} = \frac{2}{3}$; $3x = 36$; $x = 12$ (games won).

Pages 316–317

Aim: To show how to use proportions to solve problems about enlarging pictures

Suggestions: If you take a picture with a camera and wish to have the picture enlarged, the length and the width of the enlarged picture will have the same ratio as the length and the width of the original picture. This statement is another way of saying that the sides of the enlarged picture are proportional to the sides of the original picture. In solving the proportion in ex. 1, the third step in the solution is $5x = 35$, which was obtained from the second step by finding the cross-products. In solving all the other problems on pages 316 and 317, the principle of the cross-products can also be used. It will add interest to the work

on these pages and make it easier to understand if you can obtain some pictures and their enlargements to use as illustrations. Perhaps the pupils can bring to class some illustrations.

Workbook Reference: Arithmetic Workshop, Book 8, page 115

Key: **2.** Let x = length in inches of enlarged picture; $\frac{x}{10} = \frac{5}{4}$; $4x = 50$; $x = 12\frac{1}{2}$ (length in inches). **3.** (1) Let x = length in inches of small picture; $\frac{x}{5} = \frac{12}{10}$; $10x = 60$; $x = 6$ (length in inches); (2) $\frac{x}{2} = \frac{12}{8}$; $8x = 24$; $x = 3$ (length in inches). **4.** Let x = width in inches of small picture; $\frac{x}{7} = \frac{18}{28}$; $28x = 126$; $x = 4\frac{1}{2}$ (width in inches). **5.** Let x = length in inches of enlarged picture; $\frac{x}{9} = \frac{3\frac{1}{4}}{2\frac{1}{4}}$; $2\frac{1}{4}x = 29\frac{1}{4}$; $x = 13$ (length in inches). **6.** If the length of a picture is doubled, its width is also doubled. **7.** Let x = length in inches of larger prints; $\frac{x}{4\frac{1}{2}} = \frac{4\frac{1}{4}}{3}$; $3x = 19\frac{1}{8}$; $x = 6\frac{3}{8}$ (length in inches). **8.** Let x = length in inches of the enlarged picture; $\frac{x}{9} = \frac{4\frac{1}{4}}{3}$; $3x = 38\frac{1}{4}$; $x = 12\frac{3}{4}$ (length in inches). **9.** (1) Let x = length in millimeters of larger print; $\frac{x}{72} = \frac{36}{24}$; $24x = 2592$; $x = 108$ (length in millimeters); (2) $2\frac{13}{16}''$ by $4\frac{1}{4}''$. **10.** Let x = length in inches of enlargement; $\frac{x}{6\frac{3}{4}} = \frac{3\frac{1}{4}}{2\frac{1}{4}}$; $2\frac{1}{4}x = 21\frac{15}{16}$; $x = 9\frac{3}{4}$ (length in inches).

Page 318

Aim: To show how to solve several types of problems by using proportions

Suggestions: The problems on page 318 are types of problems that may be conveniently solved by proportion. Ex. 1 should be carefully studied before the pupils attempt to work the other exercises on this page.

Key: **1.** 12 gal. **2.** Let x = number of miles on 6 gal.; $\frac{60}{x} = \frac{4}{6}$; $4x = 360$; $x = 90$ (miles on 6 gal.). **3.** Let x = number of miles in $7\frac{1}{2}$ hr.; $\frac{100}{x} = \frac{3}{7\frac{1}{2}}$; $3x = 750$; $x = 250$ (miles in $7\frac{1}{2}$ hr.). **4.** Let x = interest in dollars for 120 da.; $\frac{3}{x} = \frac{90}{120}$; $90x = 360$; $x = 4$ (interest in dollars for 120 da.). **5.** Let x = number of cups of sugar for smaller cake; $\frac{x}{2} = \frac{4}{6}$; $6x = 8$; $x = 1\frac{1}{3}$ (cups of sugar). **6.** Let x = number of examples in 6 min.; $\frac{6}{x} = \frac{4}{6}$; $4x = 36$; $x = 9$ (examples in 6 min.). **7.** Let

x = number of miles represented by 4 in.; $\frac{60}{x} = \frac{2\frac{1}{2}}{4}$; $2\frac{1}{2}x = 240$; $x = 96$ (number of miles represented by 4 in.). **8.** (1) Let x = number of cents charged for 6 pencils; $\frac{25}{x} = \frac{2}{6}$; $2x = 150$; $x = 75$ (number of cents for 6 pencils); (2) Let x = number of cents charged for 8 pencils; $\frac{25}{x} = \frac{2}{8}$; $2x = 200$; $x = 100$ (number of cents for 8 pencils).

Page 319

Aim: To teach how to write inequalities using the symbols $<$ and $>$, and to find solutions of inequalities

Suggestions: Begin the discussion of this work by writing many simple sentences such as "three is less than five" with numerals and the inequality symbols $<$ and $>$.

Emphasize the meaning of a *solution* of an inequality. In ex. 9 point out that *any* whole number greater than 3 makes the inequality $x + 2 > 5$ a true sentence and is, therefore, a solution of this inequality.

Key: **2.** $3 < 6$; $5 > 2$; $17 < 35$; $46 = 46$; $15628 < 16528$. **3.** $2 > 1$; $0 < 1$; $101 > 59$; $.5 = \frac{1}{2}$; $1002 < 1020$. **4.** $\frac{5}{6} < 1$; $2 > \frac{9}{5}$; $39 > 13 + 21$; $25 - 9 < 6 + 17$. **5.** $\frac{1}{4} > \frac{3}{16}$; $2^3 < 3^2$; $7 = 0 + 7$; $3^2 + 1 > 2^2 + 2^2$. **6.** $0 + 3 = 3 + 0$; $1 + 2 + 3 < 4^2 - 3^2$; $1 + 3 + 5 > 2^3$. **7.** MDCLV < MCM; $10_8 > 10_4$; $21_5 > 100_2$; 𓍢𓍢𓍢 > 𓍢𓍢𓎆𓎆𓎆𓏺𓏺𓏺𓏺. **9.** Any three whole numbers less than 19; any three whole numbers greater than 3; any three whole numbers less than 4; any three whole numbers greater than 12. **10.** Any three whole numbers greater than 9; any three whole numbers less than 11; any three whole numbers greater than 9; 0, 1, and 2.

Pages 320–321

Aim: To teach how to use inequalities to solve problems

Suggestions: Have pupils express the relationship given in a problem with a sentence, using words first and then mathematical symbols. You will need to check pupils' understanding of the verbal statement of the problem. In some cases diagrams will help pupils to see relationships. Lead pupils to see that a compact mathematical sentence is easier to read and to use than a more lengthy statement in words.

After an inequality for a problem has been obtained, have pupils try different values for the unknown quantity and determine which values are solutions. Emphasize that a solution must make the inequality a true mathematical sentence.

Key: Page 320 **2.** $x + 32 < 50$; from 1 to 17 stamps. **3.** $x - 8 < 20$; from 0 to 27 tickets. **4.** $x - 30 > 15$; from 46 to 60 papers.

Page 321 **5.** $x + 21 < 35$; from 0 to 13 records. **6.** $x > 22$; from 23 to 30 subscriptions. **7.** $x - 3 < 8$; from 3 to 10 rosebushes. **9.** All whole numbers less than 4; all whole numbers greater than 14; all whole numbers less than 55; all whole numbers greater than 15.

Page 322

Aim: To review the work of Chapter 9

Key: **1.** $n + 9$; $x - 5$. **2.** $x + 3x$. **3.** $5a - 6$. **4.** $4n + 5n$. **5.** $2n + 8$. **6.** $3a - 4$. **7.** $\frac{1}{3}n$ or $\frac{n}{3}$; $\frac{1}{2}n$ or $\frac{n}{2}$. **8.** $\frac{1}{4}n + 6$ or $\frac{n}{4} + 6$. **9.** ab; $3c$. **10.** 4. **11.** 17. **12.** 12. **13.** 8. **14.** 28. **15.** 4. **16.** 22. **17.** $2\frac{1}{3}$. **18.** 64. **19.** 15. **20.** 6. **21.** 5. **22.** 8. **23.** 14. **24.** 7. **25.** Let a = Grandfather's age in years; $3a + 1 = 250$; $3a = 249$; $a = 83$ (years). **26.** Let a = Jack's age in years; $2a - 8 = 16$; $2a = 24$; $a = 12$ (years). **27.** Let n = the number of Lucy's books; $\frac{1}{4}n = 23$; $n = 92$ (books). **28.** Let n = the number; $4n - 7 = 45$; $4n = 52$; $n = 13$.

Page 323

Aim: To present Problem Test 9

Workbook Reference: Arithmetic Workshop, Book 8, page 133

Key: **1.** Let x = the number; $2x + 8x = 120$; $10x = 120$; $x = 12$. **2.** 5% of loan = \$150; 1% of loan = \$150 ÷ 5, or \$30; 100% of loan = 100 × \$30, or \$3000. **3.** \$2.25 ÷ \$49 = .0459, or 4.6%. **4.** .10 × \$450 = \$45; \$450 − \$45 = \$405; .10 × \$405 = \$40.50; \$405 + \$40.50 = \$445.50; 12 × \$445.50 = \$5346.00. **5.** Let a = Grandfather's age in years; $2a - 20 = 100$; $2a = 120$; $a = 60$ (years). **6.** \$12,500 ÷ \$100 = 125 (hundreds); 125 × \$3.90 = \$487.50. **7.** 100% − 20% = 80%; 80% of list price = \$560; 1% of list price = \$560 ÷ 80, or \$7; 100% of list price = 100 × \$7, or \$700. **8.** \$10,000 ÷ \$1000 = 10 (thousands); 10 × \$26.15 = \$261.50; 5 × \$261.50 = \$1307.50. **9.** $V = \pi r^2 h$; $V = \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 8 = 77$ (cu. ft.); 77 × $7\frac{1}{2}$ gal. = $577\frac{1}{2}$ gal. **10.** Let x = number of dollars paid for net; $x - \$4$ = number of dollars paid for racket; $x + x - \$4 = \$20.50$; $2x = \$24.50$; $x = \$12.25$ (price of net); $x - \$4 = \$8.25$ (price of racket).

Page 324

Aim: To provide a diagnostic test with page references for practice

Suggestion: This test covers the use of the four principles in the solution of equations, and the use of equations to solve problems.

Key: **1.** 81; 45. **2.** 73; 80. **3.** 21; 25. **4.** 112; 63. **5.** 12; 22; 96. **6.** 20; 84; 100. **7.** 3; 5; 3. **8.** $1\frac{1}{4}$; $\frac{2}{3}$; $1\frac{1}{2}$. **9.** 3; 3. **10.** 9; 8. **11.** 7; 7. **12.** 12; 11. **13.** 19; 9. **14.** 9. **15.** $2\frac{1}{2}$ hr. **16.** \$98.

Chapter 10

Aims of Chapter 10. The major aims of Chapter 10 are to:

1. Develop an understanding of similar triangles.
2. Use similar triangles to find heights and distances by indirect measurement.
3. Discuss similar rectangles and their use in scale drawings.
4. Teach finding squares and square roots by means of a table.
5. Teach the Pythagorean formula and how to use it in problem solving.
6. Use the Pythagorean formula and square root in indirect measurement.
7. Provide a comprehensive review of problem solving covering the year's work.

Page 325

Aim: To give an interesting project using proportion

Suggestions: It will help the pupils to understand this page and see its relationship to the work which follows if you make diagrams to illustrate ex. 1–3. The pupils should see that a boy's legs and the length of his step, when viewed from the side, form approximately an isosceles triangle.

Key: **3.** (1) 3 ft. + 9 ft. = 12 ft.; (2) Let x = length of enlarged step in feet; $\frac{3}{12} = \frac{2\frac{1}{2}}{x}$; $3x = 30$; $x = 10$ (length in feet).

Page 326

Aims: To teach the meaning of similar triangles and to explain that their corresponding sides are proportional

Suggestions: In ex. 1 it must be made clear that similar triangles have the same shape but *not* necessarily the same size. Considerable care will have to be taken to pick out the corresponding sides in the two similar triangles in ex. 2. In ex. 3 you will have to proceed slowly to show that the ratio of any pair of corresponding sides equals the ratio of any other pair. In ex. 4 it is important to point out that *three* different proportions can be formed from the equal ratios of the sides in any pair of similar triangles. To explain clearly the work in ex. 2–4, you should draw on the chalkboard the two similar triangles in ex. 1, thus making it possible to point out the pairs of corresponding sides and the equal ratios. The work on this page is of basic importance since the exercises on pages 327–335 make constant use of similar triangles.

Key: **5.** S, $\frac{16}{12} = \frac{12}{9}$, $\frac{16}{12} = \frac{8}{6}$, $\frac{12}{9} = \frac{8}{6}$; T, $\frac{8}{16} = \frac{10}{20}$, $\frac{8}{16} = \frac{4}{8}$, $\frac{10}{20} = \frac{4}{8}$.

Page 327

Aim: To show how to find the length of an unknown side in one of two similar triangles

Suggestions: Ex. 1 on this page needs careful study since it shows how to find the length of the unknown side. The two triangles in ex. 1 should be drawn on the chalkboard so that you can point out the equal ratios.

Workbook Reference: Arithmetic Workshop, Book 8, page 116

Key: **2.** (1) $\frac{6}{x} = \frac{8}{12}$, $8x = 72$, $x = 9$ (feet); (2) $\frac{x}{6} = \frac{6}{8}$, $8x = 36$, $x = 4\frac{1}{2}$ (feet). **3.** (1) $\frac{x}{13} = \frac{12}{24}$, $24x = 156$, $x = 6\frac{1}{2}$ (feet); (2) $\frac{14}{x} = \frac{24}{15}$, $24x = 210$, $x = 8\frac{3}{4}$ (feet). **4.** (1) $\frac{32}{y} = \frac{24}{15}$, $24y = 480$, $y = 20$ (feet); (2) $\frac{y}{32} = \frac{15}{24}$. **5.** (1) Let x = length of MO in inches, $\frac{x}{18} = \frac{14}{24}$, $24x = 252$, $x = 10\frac{1}{2}$ (inches); (2) Let y = length of NO in inches, $\frac{y}{18} = \frac{14}{24}$, $24y = 252$, $y = 10\frac{1}{2}$ (inches).

Page 328

Aim: To teach that two triangles must be similar if two angles of one triangle are equal to two angles of the other triangle

Suggestions: Ex. 1 tells why it is important to have an easy way to tell whether two triangles are similar. If the pupil can show that two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar. This simple test is all that is necessary to prove that two triangles are similar. After showing in this way that two triangles are similar, it follows, of course, that the corresponding sides of these two triangles are proportional.

In ex. 2 the pupil should actually draw the two triangles as directed, using ruler and protractor. In each of these triangles, two angles and the included side are given. The method of drawing triangles of this type was described on page 176 of the text. After the triangles have been drawn, the pupil should measure the sides of each triangle, as directed in ex. 2, and see whether each pair of corresponding sides has the same ratio. In this way the pupil shows experimentally that two triangles are similar if two angles of one are equal to two angles of the other.

In answering ex. 4 on this page, use should always be made of the fact that the sum of the three angles of any triangle equals 180°, which was shown experimentally on pages 147 and 167 of the text. If two angles of one triangle are equal to two angles of another triangle, the third angles of these triangles must be equal since the sum of the angles in each triangle equals 180°.

Workbook Reference: Arithmetic Workshop, Book 8, page 117

Key: **3.** Angles C and F are equal. If two angles of one triangle are equal to two angles of another triangle, the third angles of these triangles must be equal because the sum of the angles in any triangle is always 180°. **4.** It is sufficient to know that two angles are equal.

Page 329

Aim: To present two more sets of improvement tests in multiplication and division

Suggestion: These sets of improvement tests should be administered in the same manner as previous sets in multiplication and division. See page 375 of the text.

Key: **1.** 31,628,466; 19,629,156; 56,893,662; 53,789,580; 46,176,480. **2.** 50,937,060; 15,556,275; 47,180,386; 6,058,416; 15,056,390. **3.** 6,193,908; 25,950,925; 29,596,126; 25,934,542; 19,201,520. **4.** 14.74; 47.66. **5.** 12.02; 63.21. **6.** 11.19; 71.03. **7.** 11.23; 58.33. **8.** 13.30; 48.68. **9.** 11.72; 74.26.

Pages 330–332

Aim: To teach the shadow method of measuring heights

Suggestions: In ex. 1 on page 330 the two triangles are similar because two angles of one triangle equal two angles of the other triangle. The angles of the two triangles can be shown equal as follows: In the large triangle the flagpole is perpendicular to the ground, and in the small triangle Mary is perpendicular to the ground; so each triangle has a right angle. Likewise, angle B in the large triangle equals angle A in the small triangle because the sun's rays are parallel and strike the ground at the same angle in each case. Since the triangles are similar, the proportion given in ex. 1 can be written, and the height x of the flagpole can be found.

The work of finding the height of a flagpole without measuring it directly is another illustration of *indirect measurement.* Indirect measurement plays an important part in applied mathematics, and pupils will be interested to know how such measurements are made. To give meaning to this work, you will find it helpful to take the pupils out of doors to measure the height of the school flagpole or the height of a building by means of shadows. In order to have a second object with which to compare the flagpole, a pole of known height must be held vertically on the ground and its shadow measured.

The pupils should make a sufficient number of diagrams for the problems on pages 331 and 332 to insure a thorough understanding of the shadow method.

Workbook Reference: Arithmetic Workshop, Book 8, page 118

Key: Pages 330–331 **2.** Let x = height, in feet, of flagpole; $\frac{x}{5} = \frac{49}{7}$; $7x = 245$; $x = 35$ (feet). **3.** Let x = height, in feet, of tree; $\frac{x}{40} = \frac{4}{3}$; $3x = 160$; $x = 53\frac{1}{3}$ (feet). **4.** Let x = height, in feet, of smokestack; $\frac{x}{5} = \frac{50}{2}$; $2x = 250$; $x = 125$ (feet). **5.** Let x = height, in feet, of mast; $\frac{x}{6} = \frac{15}{3}$; $3x = 90$; $x = 30$ (feet). **6.** Let x = height, in feet, of store; $\frac{x}{4} = \frac{48}{6}$; $6x = 192$; $x = 32$ (feet). **7.** Let x = height,

in feet, of tree; 5 ft. 8 in. = $5\frac{2}{3}$ ft.; $\frac{x}{5\frac{2}{3}} = \frac{42}{7}$; $7x = 238$; $x = 34$ (feet). **8.** No. The length of the shadow will change throughout the day; the ratio, therefore, is not constant.

Page 332 **1.** Yes. **2.** Shadows at noon are shorter than at 3 P.M.; no. **3.** Yes. **4.** Let x = height, in feet, of tree; $\frac{x}{7} = \frac{12}{2}$; $2x = 84$; $x = 42$ (feet). **5.** 40 ft. **6.** 52 ft. **7.** 35 ft. **8.** 51 ft. **9.** 32 ft. **10.** 25 ft. **11.** 100 ft. **12.** 10 ft. **13.** 45 ft.

Page 333

Aim: To teach another method of measuring the heights of trees and other similar heights

Suggestions: This method of measuring the height of a tree gives only the *approximate* height. In this method the *horizontal* distance AF from the eye to the ruler should be measured once for each pupil, since this distance remains the same if the ruler is *always held at arm's length.* The distance AF, however, differs from person to person; hence, each person should measure this distance in his own case. In ex. 1 the distance AF is assumed to be 2 ft., but in each pupil's case it may be more or less than 2 ft.

This method is based on a theorem in geometry which is not presented to the pupils at this time. This theorem states that *corresponding altitudes of two similar triangles have the same ratio as any two corresponding sides of the triangles.* The large triangle ADE is similar to the small triangle ABC. These triangles are similar because two angles of one triangle equal two angles of the other triangle. Lines BC and DE are parallel since each line is perpendicular to the ground. Because these lines are parallel, angle B = angle D and angle C = angle E. Side BC of the small triangle corresponds to side DE of the large triangle. The altitude of the small triangle is AF, and the length of the corresponding altitude of the large triangle equals GE. Using the above theorem, we get the proportion $AF:GE = BC:DE$. Caution the pupils to express the distance BC in feet when using this proportion.

The pupils should have considerable practice in using this method to measure the heights of trees, flagpoles, and similar objects. It is not necessary for you to explain to all pupils the geometry on which this method is based, but you may wish to explain it to the more capable pupils.

Key: **2.** Let x = height of tree in feet; $\frac{2}{60} = \frac{\frac{5}{6}}{x}$; $2x = 50$; $x = 25$ (feet). **3.** 30 ft. **4.** 30 ft. **5.** 24 ft. **6.** $41\frac{1}{4}$ ft. **7.** $52\frac{1}{2}$ ft. **8.** 18 ft. **9.** 33 ft.

Pages 334–335

Aim: To teach a method of estimating distances

Suggestions: The method described on page 334 is to be used for estimating distances straight ahead of you. To understand this method it will be necessary to read page 334 with great care. Several of the steps in this method should be emphasized. First, the right arm must be extended *at full length* and held in that position without moving it until you have sighted with both the right and left eyes. Second, the distance CS, along which the finger F seems to move when you close one eye and open the other, has to be estimated very carefully. This estimate is then multiplied by 10 to get the *approximate* distance from you to the distant object. Everything depends, therefore, upon your estimate of the distance CS. This method makes use of the fact that it is easier to estimate a short distance such as CS, which is perpendicular to the line of sight, than to estimate a long distance such as FC, which is straight ahead.

In order that this method of estimating distances may mean something to the pupils, it is necessary for them to have some practice in using it out of doors. This method is applied in ex. 1 and 2 on page 335. In each of these cases remember to multiply by 10 the distance your finger seems to move when you close one eye and open the other.

Key: Page 335 **1.** 3×350 ft. = 1050 ft.; 10×1050 ft. = 10,500 ft.; 10,500 ft. is about 2 times 5280 ft., or about 2 mi. **2.** 2×800 ft. = 1600 ft.; 10×1600 ft. = 16,000 ft.; 16,000 ft. is about 3 times 5280 ft., or about 3 mi. **3.** The distance will be too large provided you use the same distance for AB as you use when the ruler is held vertically. The distance AB remains the same for each individual person; hence, AB is usually measured only once. **4.** 47 ft.

Pages 336–337

Aim: To define similar rectangles and to show that scale drawings of floor plans are an application of similar rectangles

Suggestions: Similar rectangles are defined in ex. 1 and 2 on page 336. Make sure that the pupil understands that two rectangles are not necessarily similar just because the angles of one equal the angles of the other.

Key: Page 336 **3.** Yes, because $\frac{12}{15} = \frac{8}{10}$; no, because $\frac{12}{15}$ does not equal $\frac{9}{12}$. Page 337 **2.** 4 in.; 240 in.; $\frac{1}{60}$; $\frac{1}{60}$; yes; yes; yes.

Pages 338–339

Aim: To show how to use a table of squares and square roots

Suggestions: The method of using the table on page 338 is described in ex. 4 on page 339. The square roots of most of the numbers given in this table are only approximately correct. For example, the square root of 20 is 4.472, which is correct to the nearest thousandth. Some whole numbers such as 16, 25, 36, and 49 have square roots that are also whole numbers, but most of the whole numbers have square roots that can be expressed only approximately.

Some pupils will be interested in finding the square root of fractions having numerators and denominators that are perfect squares, such as $\frac{4}{9}$ and $\frac{9}{25}$.

Workbook Reference: Arithmetic Workshop, Book 8, page 119

Key: Page 339 **1.** 64; 49; 144. **2.** 6; 9; 5; 10; 2. **5.** 225, 3.873; 1849, 6.557; 6561, 9.000; 8281, 9.539; 14,641, 11.000; 12,544, 10.583; 11,664, 10.392. **6.** 1225, 5.916; 2500, 7.071; 5184, 8.485; 4225, 8.062; 22,201, 12.207; 18,225, 11.619; 22,500, 12.247.

Pages 340–341

Aim: To teach how to find, to the nearest tenth, the square roots of numbers that are not in the table

The Modern Treatment of Square Root. The great majority of people engaged in nontechnical or nonprofessional work have little need for square root. Engineers, scientists, statisticians, research workers, and others who have occasion to find square roots generally make use of a table for this purpose, such as *Barlow's Tables, Squares and Cubes* (The Charles T. Powner Co., Chicago, Ill.) or similar tables that can be found in engineering or mathematical handbooks. The same practice is followed in finding cube roots. Roots are found also by logarithms, by the slide rule, and by calculating machines. Hence, the learning of an elaborate method of computing square roots is no longer regarded as essential either in the eighth grade or in the senior high school.

Some years ago it was the general practice, in finding square roots, to make use of a rather complicated arithmetical procedure. While this method gave the desired square root, most pupils found it quite difficult to apply; they followed the procedure more or less mechanically, with little real understanding of what they were doing. Moreover, they did not use the method often enough to remember it accurately. After getting the root by this method, they seldom knew what it meant. To remedy this difficulty, several simple methods of finding square roots have been introduced in recent years, the simplest being that given in ex. 3 on page 340 and in ex. 1 and 3 on page 341 of the text. This method has the great advantage of keeping prominently in the pupil's mind the meaning of the term *square root*. This method also emphasizes the important fact that the square roots of most whole numbers can be obtained only approximately, since most of our given numbers are *not* perfect squares. Hence, this method constantly applies important ideas concerning approximate computation and the degree of precision of results.

Workbook Reference: Arithmetic Workshop, Book 8, pages 120 and 121

Key: Page 340

4. (1) 234 lies between 225 (15^2) and 256 (16^2). Since 234 is closer to 225 than to 256, the square root of 234 is closer to 15 than to 16. Square 15.1, 15.2, 15.3, and 15.4 as shown at the right. 234.09 is closest to 234, so 15.3 is the square root of 234 to the nearest tenth.

$15.1^2 = 228.01$
$15.2^2 = 231.04$
$15.3^2 = 234.09$
$15.4^2 = 237.16$

(2) 408 lies between 400 (20^2) and 441 (21^2) and is closer to 400 than to 441. The square root of 408 is a little over 20. 408.04 is closest to 408, so 20.2 is the square root of 408.

$20.1^2 = 404.01$
$20.2^2 = 408.04$
$20.3^2 = 412.09$

(3) 300 lies between 289 (17^2) and 324 (18^2) and is closer to 289. Square 17.1, 17.2, 17.3, and 17.4. 299.29 is closest to 300, so 17.3 is the square root of 300.

$17.1^2 = 292.41$
$17.2^2 = 295.84$
$17.3^2 = 299.29$
$17.4^2 = 302.76$

(4) 14.2; (5) 13.3; (6) 16.3; (7) 12.3; (8) 13.1; (9) 22.3; (10) 23.3; (11) 94.2. **6.** 24.2; 18.4; 19.2.

Page 341 **2.** 12.9; 13.8; 14.8; 15.9. **4.** 18.5; 19.6; 20.4; 21.5; 13.6; 37.4; 44.8; 36.5; 134.9. **5.** 9.5 ft. **6.** 18; 77; 84; 136; 149.

Pages 342–343

Aim: To show, in a right triangle, how to find the length of the hypotenuse if the other two sides are given

Suggestions: The Pythagorean formula, which was discovered by Pythagoras about 540 B.C., is a very important geometrical fact which applies only to right triangles. The Pythagorean formula is explained in ex. 2 on page 342. This formula is stated in words in ex. 3 on page 342 as follows: *In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.* The formula can be written also in letters as is shown in ex. 1 on page 344. In using this formula the pupil must be made familiar with the terms *hypotenuse* and *legs.* In a right triangle the hypotenuse is the side opposite the right angle, and it is always the *longest side* of the triangle. The other two sides, which are called *legs,* are shorter than the hypotenuse. In the Pythagorean formula the phrase "square of the hypotenuse" means the *area of the square on the hypotenuse.* Likewise, the "sum of the squares of the other two sides" means the *sum of the areas of the squares on the other two sides.* In ex. 3 on page 342, the pupils should actually draw the triangle as directed, using large sheets of paper. Ex. 4 on page 342 shows how to use the Pythagorean formula to find the length of the hypotenuse when the lengths of the two legs are given; this explanation shows that it is necessary to use square root in order to find the length of the hypotenuse.

In ex. 1 on page 343 it should be noted that the baseball diamond with which you start is divided into two right triangles by drawing the dotted line shown in the figure. The Pythagorean formula, of course, applies to each of the right triangles. Likewise, in ex. 2 and 3 on page 343 you start with a rectangle in each case and get right triangles by drawing a diagonal line between the opposite corners of the rectangle, as shown in the figure for ex. 3. In ex. 5 on page 343 the pupil should make the scale drawing as directed; this drawing serves as a check on the arithmetical solution to this exercise.

Workbook Reference: Arithmetic Workshop, Book 8, page 123

Key: Page 342 **4.** 13 in. **5.** (1) $9^2 + 12^2 = 81 + 144 = 225$; $\sqrt{225} = 15$ (in.); (2) $7^2 + 24^2 = 49 + 576 = 625$; $\sqrt{625} = 25$ (in.); (3) $12^2 + 35^2 = 144 + 1225 = 1369$; $\sqrt{1369} = 37$ (in.).

Page 343 **1.** (1) $90^2 + 90^2 = 8100 + 8100 = 16{,}200$; $\sqrt{16{,}200} = 127.3$ (ft.); (2) second base to home plate is same distance, 127.3 ft. **2.** (1) $30^2 + 20^2 = 900$

$+ 400 = 1300$; $\sqrt{1300} = 36.1$ (ft.); (2) 36.1 ft. + 4 ft. = 40.1 ft. **3.** (1) $60^2 + 45^2 = 3600 + 2025 = 5625$; $\sqrt{5625} = 75$ (ft.); (2) the distance is also 75 ft. **4.** $60^2 + 100^2 = 3600 + 10{,}000 = 13{,}600$; $\sqrt{13{,}600} = 116.6$ (ft.); 60 ft. + 100 ft. = 160 ft.; 160 ft. − 116.6 ft. = 43.4 ft. **5.** $12^2 + 16^2 = 144 + 256 = 400$; $\sqrt{400} = 20$ (mi.); 12 mi. + 16 mi. = 28 mi.; 28 mi. − 20 mi. = 8 mi.

Page 344

Aim: To show, in a right triangle, how to find the length of one leg of the triangle if the hypotenuse and the other leg are given

Suggestions: In ex. 1 the Pythagorean formula is written in letters as follows: $h^2 = a^2 + b^2$. It should be noted that the formula written in this brief form is an equation. Any one of the letters in this equation can be considered to represent an unknown quantity that can be found provided the other two quantities are known. Ex. 2 shows how to use this formula when one of the legs of a right triangle is unknown. In this case the hypotenuse and the other leg are given. In ex. 2 it should be noted that the distance x, which is the distance from the window to the ground, is found *indirectly*. You did not actually measure x; instead, you computed it by the help of the Pythagorean formula. This example shows that the Pythagorean formula can be used in indirect measurement. Ex. 3 and 4 on this page are similar to ex. 2. In ex. 3 and 4 it will be helpful to have the pupils draw the triangle in each case before doing the work.

Key: **3.** $23^2 = 7^2 + x^2$; $529 = 49 + x^2$; $480 = x^2$; $\sqrt{480} = x$; $x = 21.9$ (ft.). **4.** $25^2 = 12^2 + x^2$; $625 = 144 + x^2$; $481 = x^2$; $\sqrt{481} = x$; $x = 21.9$ (ft.).

Pages 345–346

Aim: To show other uses of the Pythagorean formula and of square root

Suggestions: In ex. 1 on page 345, the Pythagorean formula is applied in an interesting problem of indirect measurement; namely, to find the approximate height of a kite above the ground. The degree of accuracy with which the height is found by this method depends upon the accuracy with which the distance AB can be measured. Since point B is a point on the ground *directly under the kite*, the location of this point has to be fixed approximately because of the motion of the kite. The sag of the kite string can be disregarded, since a measurement of this kind is necessarily very approximate. Ex. 3 on page 345 has nothing to do with the Pythagorean formula, but it is a new use for square root. In the explanation of this exercise and also of ex. 2 and ex. 3 on page 346, the methods of solving equations learned in Chapter 9 are applied to the formulas used. Ex. 1 on page 346 illustrates another use of square root; namely, to find the length of the side of a square when its area is given. Ex. 4 on page 346 shows an interesting way of telling whether a triangle is a right triangle or not *without drawing it to find out*. To apply this test, the lengths of the three sides of the triangle must be given. The method of drawing a triangle when the three sides

are given was described in ex. 1 on page 166 of the text. In connection with ex. 4, point out to the pupils that if the square of the longest side of any triangle is *greater* than the sum of the squares of the other two sides, then the angle opposite the longest side is an *obtuse angle*. If the square of the longest side is *less* than the sum of the squares of the other two sides, then the angle opposite the longest side is an *acute angle*.

Key: Page 345 **1.** $150^2 = 115^2 + x^2$; $22{,}500 = 13{,}225 + x^2$; $9275 = x^2$; $\sqrt{9275} = x$; $x = 96.3$ (ft.). **2.** (1) $150^2 = 90^2 + x^2$; $22{,}500 = 8100 + x^2$; $14{,}400 = x^2$; $\sqrt{14{,}400} = x$; $x = 120$ (ft.); (2) $125^2 = 100^2 + x^2$; $15{,}625 = 10{,}000 + x^2$; $5625 = x^2$; $\sqrt{5625} = x$; $x = 75$ (ft.); (3) $175^2 = 140^2 + x^2$; $30{,}625 = 19{,}600 + x^2$; $11{,}025 = x^2$; $\sqrt{11{,}025} = x$; $x = 105$ (ft.). **4.** (1) $154 = 3\frac{1}{7}r^2$; $49 = r^2$; $\sqrt{49} = r$; $r = 7$ (in.); (2) $1386 = 3\frac{1}{7}r^2$; $441 = r^2$; $r = 21$ (in.); (3) $2464 = 3\frac{1}{7}r^2$; $784 = r^2$; $\sqrt{784} = r$; $r = 28$ (in.); (4) $3850 = 3\frac{1}{7}r^2$; $1225 = r^2$; $\sqrt{1225} = r^2$; $r = 35$ (in.); (5) $5544 = 3\frac{1}{7}r^2$; $1764 = r^2$; $\sqrt{1764} = r$; $r = 42$ (in.). **5.** $37^2 = 35^2 + x^2$; $1369 = 1225 + x^2$; $144 = x^2$; $\sqrt{144} = x$; $x = 12$ (ft.). **6.** 14 in. **7.** 60 ft. **8.** 18 in. **9.** 16 ft. **10.** 24 in. **11.** 20 ft. **12.** 24 in. **13.** 48 ft.

Page 346 **1.** $208^2 = 43{,}264$; $209^2 = 43{,}681$; $208.7^2 = 43{,}555.69$; $208.8^2 = 43{,}597.44$; so $\sqrt{43{,}560} = 208.7$ (ft.). **2.** 6 in. **3.** $r^2 = 25$; $r = 5$ (ft.). **4.** Yes; yes. **5.** (1) Does $19^2 = 15^2 + 8^2$? $361 > 225 + 64$, or 289; so the triangle is not a right triangle; (2) Does $36^2 = 35^2 + 12^2$? $1296 < 1225 + 144$, or 1369; so the triangle is not a right triangle. **6.** No. **7.** Yes. **8.** Yes. **9.** No. **10.** Yes. **11.** No.

Page 347

Aim: To give mixed practice on computation and problems

Key: **1.** $19.8 \div 132 = .15$, or 15%. **2.** 75% of the number is 477; 1% of the number = $477 \div 75$, or 6.36; 100% of the number = 100×6.36, or 636. **3.** 3.142. **4.** 330. **5.** 2934; 1467. **6.** 3120. **7.** $4.72 + 219.39 = 224.11$; $224.11 - 175.29 = 48.82$. **8.** $2 \times 2 \times 7 \times 11$; $3 \times 3 \times 3 \times 5 \times 5$. **9.** \$3.60; \$2.88. **10.** $15 + 4\frac{1}{8} + 23\frac{1}{2} + 6\frac{1}{4} + 11\frac{5}{8} + 9\frac{1}{2} + 17 = 87$; $87 \div 7 = 12\frac{3}{7}$. **11.** 35.13. **12.** 1.95; 7.4%. **13.** \$432; \$.03. **14.** $\frac{6}{8}$; $\frac{9}{12}$. **15.** 5%. **16.** $41° + 90° = 131°$; $180° - 131° = 49°$. **17.** $.01 \times \$600 = \$6.00$; $\$600 + \$6 = \$606$. **18.** $\$17{,}500 - \$3500 = \$14{,}000$; $\$14{,}000 \div \$1000 = 14$ (thousands); $14 \times \$6.60 = \$92.40$. **19.** $\$18{,}750 \div 7500 = \$2.50$.

Pages 348–351

Aim: To provide a comprehensive review of problem solving covering the topics presented in Grade 8

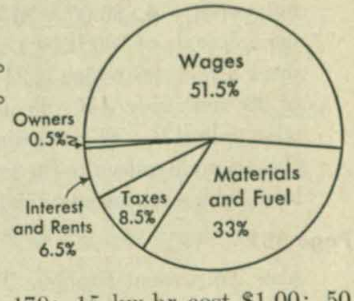
Suggestion: Many of the problems on these pages can be used to introduce and motivate a review of topics covered during the year.

Workbook Reference: Arithmetic Workshop, Book 8, pages 130 and 131

Key: Page 348 **1.** Let x = taxes on property assessed at \$12,000; $\frac{8500}{12,000} = \frac{246.50}{x}$; $8500x = 2,958,000$; $x = 348$ (dollars). **2.** $6\frac{1}{4}$ lb. + 5 lb. = $11\frac{1}{4}$ lb.; $18\frac{3}{4}$ lb. − $11\frac{1}{4}$ lb. = $7\frac{1}{2}$ lb.; $6\frac{1}{4} \times \$1 = \$6.25$; $5 \times \$.85 = \$4.25$; $7\frac{1}{2} \times \$1.25 = \$9.37\frac{1}{2}$, which is called \$9.38; \$6.25 + \$4.25 + \$9.38 = \$19.88. **3.** 8 oz. = $\frac{1}{2}$ lb.; 2.2 lb. = 1000 g.; 1000 g. ÷ 2.2 = 454.5 g.; $\frac{1}{2} \times 454.5$ g. = 227.25 g., or about 227 g. **4.** $12 \times \$16.75 = \$201$; $6 \times \$32.50 = \$195$; $8 \times \$10 = \$80$; \$201 + \$195 + \$80 = \$476; $.20 \times \$476 = \$95.20$; \$476 − \$95.20 = \$380.80; $.10 \times \$380.80 = \$38.08$; \$380.80 − \$38.08 = \$342.72; $.02 \times \$342.72 = \$6.8544$, which is called \$6.85; \$342.72 − \$6.85 = \$335.87. **5.** 38 + 15 = 53 (games), 38 ÷ 53 = .7169, or .717 (Cubs); 36 + 14 = 50 (games), 36 ÷ 50 = .7200, or .720 (Bears); the Bears have the better standing. **6.** $A = \pi r^2$; $A = \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1} = 154$ (sq. ft.), area of flower bed; 14 ft. + $3\frac{1}{2}$ ft. + $3\frac{1}{2}$ ft. = 21 ft., diameter of outside edge of walk; $A = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 346\frac{1}{2}$ (sq. ft.), area of flower bed and walk together; $346\frac{1}{2}$ sq. ft. − 154 sq. ft. = $192\frac{1}{2}$ sq. ft., area of walk. **7.** $192\frac{1}{2}$ sq. ft. − 154 sq. ft. = $38\frac{1}{2}$ sq. ft.; so the walk is larger by $38\frac{1}{2}$ sq. ft. **8.** 100% − 40% = 60%, cost; 60% of S.P. = \$2115, 1% of S.P. = \$2115 ÷ 60, or \$35.25, 100% of S.P. = $100 \times \$35.25$, or \$3525; \$3525 ÷ 300 = \$11.75. **9.** $A = \frac{1}{2}h(a + b)$; $A = \frac{1}{2} \times 200 \times 575 = 57,500$ (sq. ft.); 57,500 sq. ft. ÷ 43,560 sq. ft. = 1.320 (A.), or 1.32 A.

Page 349 **1.** From 6:30 A.M. to 12:30 P.M. = 6 hr.; 15.50 in. ÷ 6 = 2.583 in., or 2.58 in. **2.** Let x = height, in feet, of the silo; $\frac{x}{6} = \frac{40}{5}$; $5x = 240$; $x = 48$ (feet). **3.** (1) 49¢ − 37¢ = 12¢; (2) 37¢ + 39¢ + 43¢ + 45¢ + 42¢ + 40¢ + 41¢ + 49¢ + 49¢ = 385¢; 385¢ ÷ 9 = $42\frac{7}{9}$¢, or 43¢.

4. $.515 \times 360° = 185.4°$, which rounds off to 185°
$.33 \times 360° = 118.8°$, which rounds off to 119°
$.085 \times 360° = 30.6°$, which rounds off to 31°
$.065 \times 360° = 23.4°$, which rounds off to 23°
$.005 \times 360° = 1.8°$, which rounds off to 2°



5. 6818 − 6574 = 244 (kw-hr); 244 = 15 + 50 + 179; 15 kw-hr cost \$1.00; $50 \times \$.04 = \$2.00$; $179 \times \$.02\frac{1}{2} = \$4.47\frac{1}{2}$, which is called \$4.48; \$1.00 + \$2.00 + \$4.48 = \$7.48. **6.** (1) $1500 \times \$1.56308 = \$2344.62$; (2) \$2344.62 − \$1500 = \$844.62.

Page 350 **1.** $.80 \times \$16,000 = \$12,800$; \$12,800 ÷ \$100 = 128 (hundreds); $128 \times \$.24 = \$30.72$. **2.** (1) $12 \times \$28.68 = \$344.16$; $20 \times \$344.16 = \$6883.20$; \$6883.20 − \$4000 = \$2883.20; (2) \$2883.20 ÷ 20 = \$144.16. **3.** 16 da. (Dec.) + 31 da. (Jan.) + 28 da. (Feb.) + 31 da. (Mar.) + 30 da. (Apr.) + 14 da. (May) = 150 da. (or 151 da. in leap year). **4.** (1) $180^2 + 240^2 = 32,400 + 57,600$

$= 90{,}000$; $\sqrt{90{,}000} = 300$ (ft.); $180 + 240 = 420$ (ft.); 420 ft. − 300 ft. = 120 ft.; (2) $120 \div 2\frac{1}{2} = 48$ (steps). **5.** $\frac{2}{3} \times \$18{,}000 = \$12{,}000$; $\$18{,}000 - \$12{,}000 = \$6000$. **6.** $12 \times \$73.62 = \$883.44$; $15 \times \$883.44 = \$13{,}251.60$; $\$13{,}251.60 - \$9000 = \$4251.60$. **7.** Let x = cost of tennis balls; $x + \$7.31$ = cost of tennis racket; $x + x + \$7.31 = \$11.81$; $2x = \$4.50$; $x = \$2.25$ (cost of tennis balls); $x + \$7.31 = \$9.56$ (cost of racket). **8.** Let x = cost of 1 week at camp; $3x$ = cost of 3 weeks at camp; $3x + \$10 = \$95.50$; $3x = \$85.50$; $x = \$28.50$ (cost of 1 week at camp). **9.** Let x = the number; $6x + 5 = 47$; $6x = 42$; $x = 7$.

Page 351 **1.** 12:27 P.M. E.D.S.T. is 9:27 A.M. P.D.S.T.; 3 hr. 23 min. before 9:27 A.M. is 6:04 A.M. **2.** 3 hr. 23 min. = 3.38 hr.; $2460 \div 3.38 = 727.8$, or 728 mi. **3.** Interest at 2% compounded semiannually is found by computing the interest each 6 mo. at 1%.

$.01 \times \$30 = \$.30$;	$\$30 + \$.30 = \$30.30$, amt. end of $\frac{1}{2}$ yr.
$.01 \times \$30 = \$.30$;	$\$30.30 + \$.30 = \$30.60$, amt. end of 1 yr.
$.01 \times \$30 = \$.30$;	$\$30.60 + \$.30 = \$30.90$, amt. end of $1\frac{1}{2}$ yr.
$.01 \times \$30 = \$.30$;	$\$30.90 + \$.30 = \$31.20$, amt. end of 2 yr.
$.01 \times \$31 = \$.31$;	$\$31.20 + \$.31 = \$31.51$, amt. end of $2\frac{1}{2}$ yr.
$.01 \times \$31 = \$.31$;	$\$31.51 + \$.31 = \$31.82$, amt. end of 3 yr.
$.01 \times \$31 = \$.31$;	$\$31.82 + \$.31 = \$32.13$, amt. end of $3\frac{1}{2}$ yr.
$.01 \times \$32 = \$.32$;	$\$32.13 + \$.32 = \$32.45$, amt. end of 4 yr.

4. (1) $\$19 + (\frac{1}{2} \times \$49.25) = \$43.625$, which is called \$43.63; $2 \times \$43.63 = \$87.26$, brokerage; (2) $200 \times \$49.25 = \$9850$; $\$9850 - \$87.26 = \$9762.74$, net amount. **5.** (1) $\$39 + (\frac{1}{10} \times \$60.75) = \$45.075$, which is called \$45.08, brokerage; (2) $100 \times \$50 = \$5000$, $\$5000 + \$40 = \$5040$, cost of stock; $100 \times \$60.75 = \$6075$, $\$6075 - \$45.08 = \$6029.92$, receipt from sale of stock; $\$6029.92 - \$5040 = \$989.92$, net profit. **6.** $\$6.00 \div \$150.00 = .04$, or 4%. **7.** Draw the triangle to scale, using a scale of 100 ft. = 1 in. Then 325 ft. = $3\frac{1}{4}$ in. For the base of the triangle, draw a line AB which is $3\frac{1}{4}$ in. long. From point A draw a line making an angle of 90° with side AB. From point B draw another line, making an angle of 40° with side AB. The two lines will intersect at C to form the triangle ABC. In this triangle, measure the length of AC with your ruler. It should be about $2\frac{3}{4}$ in. long, which represents 275 ft., since 1 in. = 100 ft. See page 178 of the textbook.

Page 352

Aim: To present Problem Test 10

Key: **1.** Let x = height, in feet, of flagpole; $\frac{x}{5\frac{1}{4}} = \frac{20}{4}$; $4x = 105$; $x = 26\frac{1}{4}$ (height in feet). **2.** $8^2 + 6^2 = 64 + 36 = 100$; $\sqrt{100} = 10$ (ft.). **3.** 67¢ − 49¢ = 18¢; 18¢ ÷ 49¢ = .3673, or 36.7%. **4.** (1) 13 da. (May) + 30 da. (June) + 20 da. (July) = 63 da.; $i = \frac{\$900}{1} \times \frac{4}{100} \times \frac{63}{360} = \$6.30$, bank discount; (2) $\$900 - \$6.30 = \$893.70$, proceeds. **5.** Let x = Joe's earnings per week; $7x - \$42.50 = \$6.50$; $7x = \$49.00$; $x = \$7.00$ (earned per week). **6.** 4% interest per year is 2% semiannually.

$.02 \times \$10{,}000 = \$200$; $\$10{,}000 + \$200 = \$10{,}200$, amt. end of $\frac{1}{2}$ yr.
$.02 \times \$10{,}200 = \$204$; $\$10{,}200 + \$204 = \$10{,}404$, amt. end of 1 yr.
$.02 \times \$10{,}404 = \$208.08$; $\$10{,}404 + \$208.08 = \$10{,}612.08$, amt. end of $1\frac{1}{2}$ yr.
$.02 \times \$10{,}612 = \$212.24$; $\$10{,}612.08 + \$212.24 = \$10{,}824.32$, amt. end of 2 yr.

7. 6 ft. $- \frac{1}{2}$ ft. $= 5\frac{1}{2}$ ft.; $80 \times 48 \times 5\frac{1}{2} = 21{,}120$ (cu. ft.); $21{,}120 \times 7\frac{1}{2}$ gal. $= 158{,}400$ gal. **8.** \$920 is cost of bond; $.04 \times \$1000 = \$40$, interest; $\$40 \div \$920 = .0434$, or 4.3%. **9.** $\$8{,}323{,}000 \div \$253{,}750{,}000 = .0328$, or \$3.28 per \$100. **10.** \$12,000 $\div$ \$1000 = 12 (thousands); $12 \times \$23.51 = \$282.12$.

Pages 353–354

Aim: To provide a comprehensive diagnostic test with references to practice pages

Suggestions: Since this is the end of the year, the diagnostic test comprises two pages. The work of Chapters 9 and 10 is emphasized, but other important topics are also included. Make assignments for needed practice when weaknesses are found. Some pupils may need to have procedures re-explained before practice is assigned.

Key: Page 353 **1.** 32; 3; 21. **2.** $2\frac{3}{4}$; 36. **3.** 1521; 2116; 2704; 7056; 16,129. **4.** 6.164; 8.426; 9.165; 9.849; 12.166. **5.** 29; 74; 93; 131. **6.** 30.2; 44.2; 66.1; 96.4. **7.** 26.6; 44.7; 62.8; 96.9. **8.** 31.5; 45.6; 63.4; 85.6. **9.** All whole numbers less than 26; all whole numbers greater than 33; all whole numbers less than 37. **10.** $x = 16$ in.; $y = 18$ in. **11.** $x = 10\frac{1}{2}$ in.; $y = 12$ in. **12.** $x = 31\frac{1}{2}$ in.; $y = 14$ in.

Page 354 **1.** $\frac{3}{4}$; $4\frac{1}{2}$; $81\frac{1}{8}$. **2.** $\frac{5}{6}$; $2\frac{2}{5}$; $\frac{3}{10}$. **3.** 11.2; 647.1; 360.8; 5.5. **4.** \$16.07; \$22.28. **5.** \$1.31; \$4.18. **6.** \$129.41; \$835.45. **7.** 53; 114. **8.** 8; 35; 15. **9.** 11; 6. **10.** $V = \pi r^2 h$; $V = 3\frac{1}{7} \times 3\frac{1}{2} \times 3\frac{1}{2} \times 8$; $V = 308$ (cu. ft.). **11.** $V = \frac{1}{3}Bh$; $V = \frac{1}{3} \times 9 \times 3$; $V = 9$ (cu. ft.). **12.** $V = \frac{1}{3}\pi r^2 h$; $V = \frac{1}{3} \times 3\frac{1}{7} \times 7 \times 7 \times 8$; $V = 410\frac{2}{3}$ (cu. ft.). **13.** $V = \frac{4}{3}\pi r^3$; $V = \frac{4}{3} \times 3\frac{1}{7} \times 35 \times 35 \times 35$; $V = 179{,}666\frac{2}{3}$ (cu. ft.). **14.** (1) $\$1380 - \$1200 = \$180$; $\$180 \div \$1200 = .15$, or 15% (increase); (2) $\$24{,}000 - \$23{,}880 = \$120$; $\$120 \div \$24{,}000 = .005$, or .5% (decrease). **15.** (1) $\$1960 - \$1715 = \$245$; $\$245 \div \$1960 = .125$, or $12\frac{1}{2}\%$ (decrease); (2) $\$59{,}020 - \$56{,}750 = \$2270$; $\$2270 \div \$56{,}750 = .04$, or 4% (increase).

Pages 355–357

Aim: To provide diagnostic tests on the fundamental addition, subtraction, multiplication, and division facts

Suggestions: Some pupils entering Grade 8 show weaknesses in the fundamental facts. Therefore it is important to give these tests at the beginning of the year in order to discover any such weaknesses. A mastery of these facts is necessary for success in the year's work. These tests can be given also at any other time during the year. See the suggestions on page 377 of the text for giving these diagnostic tests.

Key: Page 355 (Addition) **1.** 4; 12; 17; 8; 9; 8; 13; 4; 11; 12. **2.** 6; 10; 4; 5; 8; 12; 3; 11; 16; 8. **3.** 7; 7; 8; 11; 1; 9; 10; 6; 15; 11. **4.** 13; 9; 15; 13; 9; 16; 11; 2; 3; 14. **5.** 3; 10; 5; 10; 8; 7; 5; 12; 2; 4. **6.** 13; 6; 7; 9; 13; 5; 8; 14; 9; 17. **7.** 12; 3; 11; 10; 9; 14; 7; 14; 5; 4. **8.** 9; 13; 5; 9; 8; 6; 7; 1; 15; 8. **9.** 2; 6; 18; 7; 12; 7; 15; 10; 16; 10. **10.** 10; 10; 11; 0; 6; 6; 12; 9; 11; 14. (Multiplication) **1.** 3; 32; 72; 0; 20; 15; 40; 0; 30; 27. **2.** 8; 9; 0; 6; 15; 35; 2; 18; 63; 0. **3.** 10; 6; 12; 24; 0; 18; 16; 0; 54; 28. **4.** 40; 0; 56; 36; 14; 64; 18; 1; 0; 49. **5.** 2; 21; 0; 16; 16; 6; 6; 27; 0; 3. **6.** 42; 0; 12; 14; 36; 4; 7; 48; 0; 72. **7.** 32; 0; 30; 24; 8; 45; 0; 48; 4; 4. **8.** 8; 42; 0; 20; 7; 9; 10; 0; 56; 12. **9.** 0; 5; 81; 12; 35; 0; 54; 25; 63; 21. **10.** 9; 24; 24; 0; 8; 5; 36; 18; 28; 45.

Page 356 **1.** 1; 8; 5; 9; 2; 6; 3; 7; 3; 0. **2.** 2; 0; 4; 8; 0; 1; 7; 5; 3; 0. **3.** 0; 1; 9; 4; 8; 7; 6; 2; 5 ; 5. **4.** 5; 7; 4; 7; 3; 6; 4; 9; 2; 4. **5.** 8; 6; 5; 0; 4; 8; 7; 4; 4; 5. **6.** 3; 2; 9; 5; 6; 0; 8; 2; 6; 3. **7.** 8; 6; 9; 1; 6; 2; 8; 6; 9; 0. **8.** 9; 7; 1; 7; 9; 5; 4; 1; 1; 3. **9.** 3; 2; 5; 0; 6; 9; 2; 1; 1; 1. **10.** 7; 4; 2; 3; 7; 9; 0; 8; 3; 8.

Page 357 **1.** 2; 0; 1; 6; 2; 8. **2.** 2; 2; 0; 7; 5; 7. **3.** 0; 9; 8; 4; 6; 5. **4.** 4; 1; 9; 6; 6; 3. **5.** 3; 8; 4; 7; 3; 5. **6.** 3; 3; 7; 4; 8; 8. **7.** 0; 1; 2; 6; 9; 4. **8.** 1; 0; 9; 6; 5; 4. **9.** 5; 0; 5; 8; 5; 3. **10.** 0; 1; 6; 9; 4; 4. **11.** 2; 0; 6; 8; 2; 7. **12.** 1; 6; 5; 7; 3; 9. **13.** 7; 0; 7; 9; 7; 2. **14.** 1; 1; 8; 3; 3; 8. **15.** 4; 1; 9; 2; 5; 9.

Pages 358–372

Aim: To give practice in the fundamental operations with whole numbers, fractions, decimals, and percentage

Suggestions: These sets of practice exercises are correlated with the work on certain pages of Chapters 1 and 2 where more practice may be needed. On these pages you will find a red circle containing a white number which refers to one of the sets of practice exercises.

These practice exercises are to be assigned in accordance with the individual need of each pupil. They can be assigned in connection with the work of Chapters 1 and 2, or at any time during the year that practice is found necessary.

Key: Page 358 **1.** 631. **2.** 543. **3.** 545. **4.** 493. **5.** 587. **6.** 3016. **7.** 3509. **8.** \$2349.06. **9.** \$3207.15. **10.** \$2704.10. **11.** 3798; 5289; 5158; 5697; 43,959; 51,989; 47,559. **12.** 5887; 5757; 4099; 7076; 57,887; 61,098; 51,449. **13.** \$25,986.87; \$21,405.79; \$28,839.35; \$29,491.89; \$43,997.87.

Page 359 **1.** 4739; 1295; 8841; 9339; 359,815; \$457.43. **2.** 4569; 5855; 8798; 32,578; 353,843; \$279.87. **3.** 1373; 5729; 5806; 27,457; 101,347; \$546.59. **4.** 2760; 6565; 5268; 18,972; 317,964; \$88.75. **5.** 4977; 2464; 4278; 3608; 379,875; \$336.15. **6.** 1961; 10,036; 76,648; 109,869; 300,700. **7.** 4005; 16,720; 104,192; 540,892; 142,600. **8.** 9016; 9139; 239,888; 662,196; 154,050. **9.** 4624; 11,592; 196,857; 170,533; 305,624. **10.** 1054; 15,660; 206,752; 265,680; 43,659. **11.** 1073; 11,400; 362,952; 418,592; 232,000. **12.** 3450; 9592; 346,020; 121,946; 133,875. **13.** 7470; 32,630; 145,254; 613,089; 204,120. **14.** 92,412; 4,685,616; 874,140; 12,283,110. **15.** 479,014; 1,266,228; 6,689,334; 58,596,790.

16. 435,079; 4,771,280; 3,349,350; 19,281,158. **17.** 580,110; 977,075; 1,145,430; 44,922,556. **18.** 157,572; 3,782,086; 844,444; 35,904,320. **19.** 351,360; 6,365,790; 6,318,564; 7,583,136. **20.** 768,145; 371,416; 4,831,236; 14,667,840. **21.** 217,022; 6,464,640; 2,563,290; 24,329,728.

Page 360 **1.** 2647; 386 R3; 5076; 2963 R1. **2.** 2299 R1; 709; 5074; 6382 R2. **3.** 1027 R2; 445; 6253; 6075. **4.** 1429; 654; 7359; 2054 R3. **5.** $75\frac{3}{43}$; $503\frac{8}{13}$; $84\frac{4}{109}$; 469. **6.** $235\frac{12}{41}$; $238\frac{62}{65}$; $46\frac{19}{711}$; $725\frac{1}{12}$. **7.** $65\frac{7}{81}$; $923\frac{20}{27}$; 73; 521. **8.** 29; $626\frac{26}{83}$; 59; $780\frac{1}{9}$. **9.** 350; $723\frac{4}{41}$; $36\frac{2}{51}$; 971. **10.** 90; $187\frac{37}{64}$; 93; $307\frac{1}{8}$. **11.** 37; 1293; 86; $1804\frac{3}{25}$. **12.** $93\frac{48}{53}$; $873\frac{6}{25}$; 67; $972\frac{1}{4}$. **13.** $36\frac{9}{55}$; $261\frac{52}{53}$; 54; 518. **14.** $63\frac{9}{44}$; $394\frac{43}{74}$; 68; 1176. **15.** 50 R17; 920 R6; 72 R161; 975. **16.** 62; 530 R13; 67 R92; 398. **17.** 27 R8; 560 R31; 51 R9; 819. **18.** 27 R29; 853 R7; 28; 634. **19.** 217 R3; 304 R22; 77; 484. **20.** 58 R2; 750; 367 R28; 128. **21.** 34 R17; 846; 212 R45; 361. **22.** 85; 457 R10; 143 R96; 719 R201. **23.** 207 R7; 803 R22; 85 R8; 460. **24.** 70 R9; 2002 R6; 305 R50; 525 R240. **25.** 209 R13; 2388 R24; 292 R220; 375 R230.

Page 361 **1.** 852 R19; 681 R2; 87 R50; 645 R25. **2.** 553 R8; 480 R11; 49 R80; 1079 R12. **3.** 174; 1067; 69; 236. **4.** 248; 643; 163 R16; 334. **5.** 384; 159; 308; 312. **6.** 625; 813; 95 R230; 793. **7.** $5\frac{1}{2}$; 2; $5\frac{2}{3}$; 1; $2\frac{1}{4}$; $6\frac{1}{2}$; 5; $3\frac{1}{3}$; $3\frac{1}{2}$; $2\frac{5}{8}$. **8.** $4\frac{2}{3}$; 3; $4\frac{1}{5}$; 1; 4; $5\frac{1}{2}$; 7; $5\frac{1}{4}$; $3\frac{1}{12}$; $2\frac{2}{9}$. **9.** $\frac{10}{3}$; $\frac{13}{2}$; $\frac{43}{8}$; $\frac{9}{4}$; $\frac{22}{5}$; $\frac{23}{6}$; $\frac{31}{4}$; $\frac{20}{3}$; $\frac{49}{12}$. **10.** $\frac{31}{5}$; $\frac{49}{6}$; $\frac{11}{4}$; $\frac{57}{8}$; $\frac{41}{6}$; $\frac{28}{5}$; $\frac{31}{8}$; $\frac{9}{5}$; $\frac{37}{16}$. **11.** $13\frac{1}{3}$; $8\frac{1}{2}$; $13\frac{2}{3}$; $10\frac{5}{6}$; $14\frac{1}{12}$; $14\frac{5}{16}$; $13\frac{1}{6}$; 23. **12.** $10\frac{1}{8}$; $11\frac{5}{8}$; $8\frac{1}{2}$; $10\frac{1}{3}$; $11\frac{1}{5}$; $16\frac{9}{16}$; $15\frac{1}{12}$; $7\frac{3}{5}$. **13.** 2; $1\frac{1}{2}$; $3\frac{1}{6}$; $\frac{7}{8}$; $4\frac{11}{12}$; $\frac{13}{16}$; $2\frac{5}{6}$; $6\frac{2}{5}$. **14.** $5\frac{2}{3}$; $2\frac{7}{12}$; $1\frac{7}{8}$; $2\frac{7}{10}$; $2\frac{11}{16}$; $5\frac{1}{2}$; $3\frac{1}{2}$; $4\frac{7}{12}$. **15.** 2; $3\frac{1}{3}$; $4\frac{3}{5}$; $5\frac{1}{8}$; $4\frac{9}{16}$; $2\frac{1}{4}$; $\frac{1}{4}$; $2\frac{1}{2}$.

Page 362 **1.** $3\frac{11}{12}$; $15\frac{7}{24}$; $8\frac{1}{12}$; $12\frac{7}{24}$; $6\frac{1}{12}$; $11\frac{5}{24}$; $10\frac{17}{24}$; $13\frac{1}{20}$. **2.** $9\frac{23}{24}$; $5\frac{19}{24}$; $6\frac{5}{12}$; $9\frac{7}{12}$; $8\frac{13}{24}$; $14\frac{7}{12}$; $10\frac{19}{20}$; $11\frac{11}{24}$. **3.** $4\frac{13}{24}$; $15\frac{1}{8}$; $12\frac{5}{24}$; $13\frac{13}{24}$; $11\frac{11}{24}$; $6\frac{3}{4}$; $16\frac{1}{8}$; $9\frac{19}{20}$. **4.** $19\frac{1}{4}$; $10\frac{1}{24}$; $15\frac{3}{4}$; $6\frac{11}{12}$; $4\frac{5}{8}$; $16\frac{1}{15}$; $16\frac{19}{20}$; $11\frac{13}{20}$. **5.** $21\frac{1}{4}$; $26\frac{17}{24}$; $22\frac{1}{6}$; $11\frac{1}{24}$; $15\frac{5}{8}$; $15\frac{3}{4}$; $18\frac{13}{20}$; $24\frac{1}{3}$. **6.** $\frac{11}{12}$; $4\frac{1}{12}$; $1\frac{5}{24}$; $2\frac{7}{12}$; $3\frac{5}{12}$; $2\frac{7}{24}$; $3\frac{7}{15}$; $3\frac{11}{20}$. **7.** $\frac{23}{24}$; $3\frac{1}{24}$; $3\frac{5}{12}$; $8\frac{11}{24}$; $\frac{13}{24}$; $2\frac{5}{24}$; $\frac{33}{40}$; $3\frac{19}{24}$. **8.** $5\frac{5}{12}$; $\frac{13}{24}$; $4\frac{11}{24}$; $6\frac{7}{12}$; $2\frac{17}{24}$; $2\frac{17}{24}$; $1\frac{19}{20}$; $4\frac{11}{20}$. **9.** $5\frac{7}{12}$; $4\frac{7}{12}$; $2\frac{7}{24}$; $6\frac{11}{24}$; $1\frac{5}{12}$; $2\frac{23}{24}$; $4\frac{23}{24}$; $1\frac{9}{20}$. **10.** $1\frac{7}{24}$; $3\frac{11}{12}$; $3\frac{11}{12}$; $2\frac{1}{24}$; $\frac{5}{12}$; $\frac{11}{12}$; $3\frac{9}{20}$; $3\frac{7}{24}$.

Page 363 **1.** $1\frac{7}{8}$; $1\frac{3}{10}$; $68\frac{1}{4}$; 7; 22. **2.** $\frac{7}{10}$; $3\frac{1}{2}$; $28\frac{1}{2}$; $11\frac{1}{4}$; 50. **3.** $\frac{8}{15}$; 1; 21; 15; $50\frac{2}{5}$. **4.** $\frac{3}{10}$; $2\frac{2}{3}$; $38\frac{3}{4}$; $4\frac{2}{5}$; $40\frac{1}{2}$. **5.** $6\frac{2}{3}$; $4\frac{9}{10}$; $157\frac{1}{2}$; $5\frac{5}{8}$; $8\frac{2}{3}$. **6.** $\frac{15}{32}$; $2\frac{1}{12}$; 253; $2\frac{7}{10}$; $15\frac{3}{4}$. **7.** $\frac{1}{3}$; $\frac{27}{40}$; 153; $9\frac{4}{5}$; 34. **8.** $\frac{9}{40}$; $1\frac{1}{3}$; 387; $10\frac{1}{2}$; 39. **9.** $\frac{3}{4}$; 1; 99; $2\frac{11}{20}$; $3\frac{1}{2}$. **10.** $7\frac{1}{2}$; $2\frac{3}{4}$; 30; $3\frac{9}{10}$; 6. **11.** $10\frac{2}{3}$; $\frac{7}{10}$; 156; $7\frac{1}{3}$; $2\frac{1}{2}$. **12.** 15; $4\frac{2}{5}$; $83\frac{1}{3}$; $3\frac{3}{10}$; $1\frac{1}{3}$. **13.** $\frac{5}{8}$; $1\frac{3}{5}$; $38\frac{1}{2}$; 15; $\frac{3}{4}$. **14.** $1\frac{1}{2}$; $1\frac{3}{8}$; 48; $2\frac{1}{4}$; 1. **15.** $5\frac{1}{4}$; $\frac{15}{16}$; $32\frac{1}{2}$; $1\frac{3}{4}$; 2. **16.** $1\frac{1}{2}$; $1\frac{2}{5}$; $103\frac{1}{2}$; 12; $1\frac{11}{16}$. **17.** $3\frac{1}{3}$; 3; 234; $3\frac{3}{4}$; $1\frac{1}{8}$. **18.** $3\frac{3}{5}$; 2; 100; $16\frac{1}{2}$; $\frac{5}{8}$. **19.** $\frac{1}{4}$; 2; 70; $8\frac{3}{4}$; $\frac{9}{16}$. **20.** $3\frac{3}{8}$; $\frac{1}{2}$; 105; 10; 1. **21.** $11\frac{1}{5}$; $\frac{15}{32}$; $31\frac{1}{2}$; $7\frac{1}{2}$; $\frac{1}{2}$. **22.** $13\frac{1}{2}$; $\frac{7}{8}$; $20\frac{5}{6}$; $3\frac{3}{4}$; $\frac{3}{4}$. **23.** $10\frac{1}{2}$; $\frac{12}{25}$; 38; 10; $\frac{1}{2}$. **24.** $18\frac{3}{4}$; $1\frac{1}{3}$; $82\frac{1}{2}$; $2\frac{2}{5}$; $\frac{3}{4}$. **25.** 22; $1\frac{1}{4}$; 33; $4\frac{1}{6}$; $2\frac{2}{5}$.

Page 364 **1.** 10; 7; 27; $\frac{2}{7}$; $2\frac{1}{2}$. **2.** 18; $1\frac{7}{9}$; 40; $\frac{2}{3}$; $\frac{1}{2}$. **3.** $17\frac{1}{2}$; 4; 28; $\frac{4}{5}$; $4\frac{1}{2}$. **4.** $\frac{5}{8}$; $9\frac{3}{8}$; 42; 4; $\frac{5}{6}$. **5.** $2\frac{1}{12}$; $1\frac{1}{2}$; 51; $\frac{2}{7}$; $2\frac{1}{4}$. **6.** 4; 14; 32; $4\frac{1}{2}$; $\frac{1}{4}$. **7.** $\frac{4}{5}$; 8; 22; $\frac{3}{4}$; $\frac{5}{6}$. **8.** $\frac{5}{6}$; 6; 40; $1\frac{7}{8}$; $\frac{1}{8}$. **9.** 10; 22; 60; $1\frac{1}{20}$; $18\frac{2}{3}$. **10.** $3\frac{1}{3}$; 9; $26\frac{2}{3}$; $1\frac{1}{5}$; $16\frac{2}{3}$. **11.** $\frac{3}{8}$; $4\frac{2}{3}$; 32; $\frac{5}{54}$; $5\frac{1}{3}$. **12.** $2\frac{1}{2}$; 16; 24; $1\frac{2}{3}$; $3\frac{3}{4}$. **13.** $\frac{1}{16}$; $\frac{2}{9}$; $2\frac{3}{4}$; 6; $1\frac{1}{3}$. **14.** $\frac{1}{16}$; $\frac{1}{4}$; $1\frac{1}{2}$; $3\frac{1}{3}$; $\frac{3}{4}$. **15.** $\frac{3}{16}$; $\frac{2}{25}$; $3\frac{5}{9}$; $5\frac{5}{9}$; $\frac{9}{20}$. **16.** $\frac{1}{6}$; $\frac{1}{4}$; $1\frac{7}{8}$; $7\frac{1}{2}$; $\frac{8}{9}$. **17.** $\frac{1}{3}$; $\frac{3}{16}$; $2\frac{7}{9}$; 10; $1\frac{1}{2}$. **18.** $\frac{1}{12}$; $\frac{3}{20}$; $1\frac{3}{25}$; 4; $1\frac{1}{2}$. **19.** $\frac{1}{3}$; $\frac{3}{10}$; $2\frac{7}{9}$; $3\frac{3}{4}$; $1\frac{5}{8}$. **20.** $\frac{1}{27}$; $\frac{5}{6}$; $1\frac{3}{5}$; $3\frac{1}{3}$; 2. **21.** $1\frac{7}{9}$; $\frac{1}{8}$; $1\frac{1}{5}$; $7\frac{1}{2}$; $2\frac{2}{3}$. **22.** 4; $\frac{2}{3}$; $1\frac{1}{5}$; $9\frac{1}{3}$; $\frac{5}{6}$. **23.** $1\frac{1}{3}$; $\frac{3}{4}$; $2\frac{22}{25}$; $4\frac{1}{2}$; $\frac{4}{5}$. **24.** $1\frac{1}{4}$; $\frac{1}{6}$; $1\frac{3}{4}$; $4\frac{4}{5}$; $\frac{14}{15}$.

Page 365 **1.** 2.46; 208.2; 2.079; 329.45; 32.234; 255.617. **2.** 4.6; 3.17; .750; 328.1; 18.82; 2.137. **3.** 41.1; 3.22. **4.** 27.548; .934. **5.** 4.91; .239. **6.** 163.4; 3.56. **7.** 26.37; 2.89. **8.** 148.829; 2.24. **9.** 17.375; .244. **10.** 20.9250; 1.47. **11.** 49.73; 55.7. **12.** 318.6; 169.0; 35.25; 4.347; .02816. **13.** 22.86; 243.6; .943; .1691; 147.00. **14.** 1.810; 35.88; 2.736; .0522; 7.5768. **15.** 585.2; 510.6; .1170; .0063; 35.014. **16.** .125; 41.86; 6.24; .0584; 174.46. **17.** .0035; 1.638; .975; .0032; .0689. **18.** .0030; .0192; 3.293; .364; .04662. **19.** 1.296; .0570; .2262; .0711; 1.0620. **20.** .0522; 29.05; 9.25; .0048; 35.028. **21.** .0009; 58.90; 4.958; .0280; .05952. **22.** .0056; 38.54; 1.235; .0375; .89082.

Page 366 **1.** 3.27; .982; 1.24; .679; 2.38. **2.** 29.4; 7.09; 36.7; 17.6; .997. **3.** .845; 63.3; .318; 31.6; 5.43. **4.** 97.75; 23.4375; .674; 56.1875; 74.56. **5.** 4.65; 13.76; 4.38; 89.25; 83.375. **6.** 9.26; .825; 56.5; 71.5; 3.925. **7.** 58.5; 18.4375; 47.3125; 9.975; 3.45. **8.** 6.52; 17.25; 82.5; .73; 97.5. **9.** .1775; 8.64; 8.05; 9.3; .8136. **10.** 6.6; 3.8; 8.6; 7.5; 9.275. **11.** .3; .425; .3125; .85; .92; .4375; .38; .2; .96875; .6875. **12.** .2375; .9375; .72; .0625; .275; .28125; .34; .625; .8125; .59375. **13.** .37; .79; .31; .30; .18; .39; .23; .89; .62; .73. **14.** .51; .74; .53; .69; .42; .89; .38; .83; .21; .81. **15.** .33; .63; .45; .62; .24; .46; .44; .57; .46; .64. **16.** .917; .267; .417; .463; .212; .808; .621; .167; .395; .638. **17.** .913; .731; .273; .607; .533; .367; .895; .714; .627; .554. **18.** .806; .378; .769; .165; .164; .389; .707; .778; .754; .308.

Page 367 **1.** .161; 137; 7760; .018; .13. **2.** 7.5; 51; 7.9; 4.23; 65. **3.** .96; 7.1; 84.25; 31,600; 5. **4.** .021; 186; 32; 532; 7. **5.** 230; 206; 18; 96.75; 6.25. **6.** .07; 3.365; 35.9; 8.2; 2.36. **7.** .08; 720; 8.8; 7.85; 4.75. **8.** .079; 6960; 32.5; .021; 3400. **9.** .076; .07; 1830; .007; 2500. **10.** 1.2; 11.2; 8.2; .6; .3. **11.** 6.4; 14.9; 24.9; 1.2; 1.7. **12.** 4.8; 34.1; 7.0; .6; .4. **13.** 2.8; 9.5; 24.6; .6; .7. **14.** 3.83; 46.07; 8.83; 1.65; .21. **15.** 6.33; .71; 4.78; 49.15; .48. **16.** .87; 24.64; 8.19; 11.86; .18. **17.** 8.22; .91; 5.04; 27.90; .01. **18.** 6.17; 2.82; .84; 482.11; .01.

Page 368 **1.** $43.50; $.27; $52; $2.20. **2.** $44.80; $.44; $69; $4.51. **3.** $153.72; $.20; $295; $16.15. **4.** $376.50; $.29; $325; $18.88. **5.** $195; $.37; $168; $7.89. **6.** $81; $.07; $73; $5.23. **7.** $26.25; $.11; $3.48; $105.48. **8.** $42; $.28; $1.26; $144. **9.** $91.30; $.52; $1.20; $227.34. **10.** $147; $.12; $.83; $113.04. **11.** $92; $.09; $3.12; $81.39. **12.** $55; $.50; $3.25; $255. **13.** .182; .114; .734; .2716; .059; .0268. **14.** .465; .191; .619; .8336; .084; .0193. **15.** 65.2%; 24.9%; 7.4%; 44.1%; 4.5%; 3.36%. **16.** 84.6%; 58.3%; 9.6%; 8.2%; 72.4%; 7.14%. **17.** 114; 16; 177. **18.** 27; 14; 403. **19.** 87; 48; 298. **20.** 70; 23; 454. **21.** 33; 35; 778. **22.** 139; 25; 2046.

Page 369 **1.** $40, $37.62; $1200, $1197; $2000, $1952. **2.** $500, $503.37; $900, $893.55; $6000, $5933.40. **3.** $20, $19.08; $600, $585.55; $2000, $2049.18. **4.** $225, $227.97; $700, $710.60; $600, $579.50. **5.** $200, $205.40; $200, $193.60; $7000, $6873. **6.** $200, $196.68; $1000, $986.87; $900, $896.80. **7.** $500, $502.20; $400, $412.75; $3500, $3524.40. **8.** $2.44; $11; $4.90. **9.** $2.00; $6.72; $32. **10.** $2.30; $9.20; $2.80. **11.** $2.70; $15.30; $12. **12.** $3.75; $12; $15. **13.** $6.00; $52.50; $14.55. **14.** $.35; $12.60; $38.20. **15.** $1180; $475;

\$67.84. **16.** \$1260; \$4429; \$132.53. **17.** \$426; \$245; \$64.69. **18.** \$324.80; \$570; \$159.63. **19.** \$425; \$270; \$154.16. **20.** \$458.64; \$1710; \$42.44. **21.** \$328.32; \$192.50; \$69.79. **22.** \$806.25; \$1207.50; \$143.92. **23.** \$377; \$526.40; \$127.94.

Page 370 **1.** 15%; 22%; 200%; 12½%; 70%. **2.** 28%; 53⅓%; 125%; 16½%; 75%. **3.** 12½%; 42½%; 75%; 8%; 60%. **4.** 6¼%; 62%; 33⅓%; 15¾%; 100%. **5.** 25%; 84%; 350%; 7½%; 213%. **6.** 5%; 37½%; 17½%; 34%; 81¼%. **7.** 27%; 55%; 14%; 142%; 146%. **8.** 26%; 45%; 23%; 177%; 259%. **9.** 19%; 40%; 36%; 57%; 273%. **10.** 26%; 24%; 39%; 66%; 310%. **11.** 31% (increase); 99% (increase); 3% (increase); 25% (increase). **12.** 4% (decrease); 17% (increase); 52% (increase); 55% (increase). **13.** 31% (increase); 48% (decrease); 19% (decrease); 27% (decrease). **14.** 62% (increase); 6% (decrease); 8% (increase); 81% (increase). **15.** 12% (decrease); 18% (increase); 6% (decrease); 17% (decrease). **16.** 33⅓% (increase); 12% (decrease); 10% (increase); 250% (increase). **17.** 90% (increase); 11% (increase); 25% (decrease); 175% (increase). **18.** 20% (decrease); 150% (increase); 8% (increase); 112½% (increase). **19.** 5¼% (decrease); 25% (decrease); 15% (increase); 40% (decrease). **20.** 75% (increase); 20% (decrease); 10% (decrease); 37½% (decrease).

Page 371 **1.** \$129, \$516; \$120, \$840; \$85, \$1615; \$1.50, \$6.00. **2.** \$90, \$270; \$92, \$184; \$76.50, \$2473.50; \$.94, \$8.46. **3.** \$120, \$680; \$183, \$305; \$112.50, \$1762.50; \$.69, \$3.91. **4.** \$238, \$442; \$90, \$450; \$66, \$3234; \$2.20, \$6.60. **5.** \$255, \$595; \$22, \$154; \$114, \$2736; \$.69, \$6.21. **6.** \$19; \$224; \$581; \$32.85. **7.** \$23; \$288; \$290; \$22.20. **8.** \$32.90; \$385; \$290; \$31.95. **9.** \$74.40; \$165; \$230.10; \$17.01. **10.** \$71.25; \$572; \$311.88; \$22.44. **11.** \$35, \$465; \$98, \$882; \$480, \$7520; \$1.02, \$24.48. **12.** \$34, \$646; \$155, \$620; \$300, \$7200; \$.89, \$16.91. **13.** \$22.50, \$727.50; \$168, \$392; \$340, \$6460; \$2.79, \$43.71. **14.** \$55.20, \$864.80; \$149, \$447; \$230, \$4370; \$2.37, \$76.63. **15.** \$34, \$816; \$42.50, \$382.50; \$286.50, \$9263.50; \$1.33, \$65.17. **16.** 20%; 9%; 25%; 6%. **17.** 25%; 8½%; 15%; 4%. **18.** 33⅓%; 8%; 20%; 6%. **19.** 12½%; 5½%; 20%; 5%. **20.** 16⅔%; 15%; 15%; 7%.

Page 372 **1.** 400; 650; 6600. **2.** 150; 420; 4320. **3.** 150; 450; 2500. **4.** 300; 450; 3425. **5.** 140; 225; 2700. **6.** \$50. **7.** \$30. **8.** \$120. **9.** \$60. **10.** \$25. **11.** \$20. **12.** \$10. **13.** \$20. **14.** \$15. **15.** \$5. **16.** \$150. **17.** \$121. **18.** \$38. **19.** \$51.25. **20.** \$50. **21.** \$142. **22.** \$64. **23.** \$43.75. **24.** \$139. **25.** \$112.50. **26.** \$172.50. **27.** \$900. **28.** \$1300. **29.** \$1500. **30.** \$550. **31.** \$13.75. **32.** \$8. **33.** \$13.10. **34.** \$20. **35.** \$14. **36.** \$5.50. **37.** \$14.20.

Pages 373–374

Aim: To provide ready-reference pages of tables of measures

Pages 375–377

Aim: To present "Suggestions to Teachers"

Audio-Visual Aids

Films: The following 16 mm. motion pictures are listed by producer or distributor. The film catalogues of these firms, which will be sent to you on request, give the objectives of these films and outline briefly the material covered by them.

Coronet Films, Coronet Building, 65 East South Water Street, Chicago 1, Illinois.

Angles and Their Measurement
Arithmetic: Estimating and Checking Answers
Arithmetic: Understanding the Problem
Brushing Up on Division
Brushing Up on Multiplication
Geometry and You
Graphing Linear Equations
How to Find the Answer
Meaning of Pi, The
Number System and Its Structure, The
Percent in Everyday Life
Principles of Scale Drawing
Ratio and Proportion in Mathematics
Story of Weights and Measures
Symbols in Algebra
Triangles: Types and Uses
Volume and Its Measurement

Delta Film Productions, Inc., 1821 University Avenue, St. Paul 4, Minnesota.

Solids in the World Around Us
Surface Areas of Solids I
Surface Areas of Solids II
Volumes of Cubes, Prisms, and Cylinders
Volumes of Pyramids, Cones, and Spheres

Encyclopaedia Britannica Films, Inc., 1150 Wilmette Avenue, Wilmette, Illinois.

Weights and Measures
What Are Decimals?
What Are Fractions?

Film Associates of California, 11014 Santa Monica Boulevard, Los Angeles 25, California.

Adventure in Science: The Size of Things
Big Numbers . . . Little Numbers
Sets, Crows, and Infinity

Johnson Hunt Productions, Film Center, La Cañada, California.

Introduction to Fractions
How to Add Fractions
How to Subtract Fractions
How to Change Fractions
How to Multiply Fractions
How to Divide Fractions
Decimal Fractions
Percentage

Modern Learning Aids, A Division of Modern Talking Pictures, Inc., 3 East 54 Street, New York 22, New York.

Addition and Multiplication
Introduction to Fractions
Manipulating Fractions
Measuring Angles
Operating with Decimals
Subtraction and Division
Whole Numbers

NET Film Service, Indiana University, Audio-Visual Center, Bloomington, Indiana.

Understanding Numbers (7 Films)

Filmstrips: The following filmstrips are listed by producer or distributor. These firms will send you on request catalogues and sales information which will provide you with descriptions and brief outlines of the material covered by these filmstrips.

Don H. Parsons Associates, Instructional Materials and Equipment, 1415 Westwood Boulevard, Los Angeles 24, California.

The Language of Sets
Open Sentences
The Number Line
Negative Numbers
The Closure, Commutative, and Associative Properties

DuKane Corporation, St. Charles, Illinois.

Area and Volume:
Measuring the Squares
Studies in Square Inches and Square Feet
Problems in Area
Introduction to Volume
Using the Cubic Inch
Problems in Volume

Filmstrip House, 347 Madison Avenue, New York 17, New York.

Dividing with Fractions
Early Counting
Early Measuring
Geometric Figures
Multiplying with Fractions

Society for Visual Education, Inc., 1345 Diversey Parkway, Chicago 14, Illinois.

Using and Understanding Numbers — Decimals and Measurements:
Meaning and Reading of Decimals
Addition and Subtraction of Decimals
Multiplication of Decimals
Division of Decimals
Changing Fractions to Decimals — Decimals to Fractions
Advancing in Linear Measurements
Advancing in Quantity Measurements

Text-Film Department, McGraw-Hill Book Company, 330 West 42 Street, New York 36, New York.

History of Measures Series:
History of Area Measure
History of Our Calendar
History of Telling Time
History of Linear Measure
History of Weight and Volume Measure
History of Our Number System

Equipment and Supplies: Instruction in mathematics is made more effective by the use of charts, models, measuring instruments, devices, and other learning aids. Many of these should be made by the teacher and the pupils. When pupils construct their own aids, they are engaging in valuable learning activities. It is also desirable, however, for teachers to make use of some of the excellent mathematics equipment that can be purchased. From the firms listed below you can obtain protractors, plastic triangles, chalkboard drawing instruments, parallel rulers, steel tapes, geometric surfaces and solids, slide rules, and other equipment. These firms are also sources of mathematics supplies such as graph paper and large cardboard protractors. You can obtain from them some of the materials you will need for the construction of homemade models and devices. Most firms will be glad to provide you with literature describing their products and with the addresses of their nearest offices or distributors.

Milton Bradley Company, 74 Park Street, Springfield 2, Massachusetts
J. L. Hammett Company, Kendall Square, Cambridge 42, Massachusetts
Keuffel and Esser Company, 303 Adams Street, Hoboken, New Jersey

Lapine Scientific Company, 6001 South Knox Avenue, Chicago 29, Illinois
Math-Master Labs Incorporated, Box 310, Big Springs, Texas
F. A. Owen Publishing Company, Dansville, New York
Pickett and Eckel, Inc., 1109 South Fremont Avenue, Alhambra, California
W. M. Welch Manufacturing Company, 1515 North Sedgwick Street, Chicago 10, Illinois
Yoder Instruments, East Palestine, Ohio

More information on films, filmstrips, and other audio-visual aids can be obtained from the National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C.

AMERICAN BOOK COMPANY

AMERICAN ARITHMETIC, 2ND EDITION